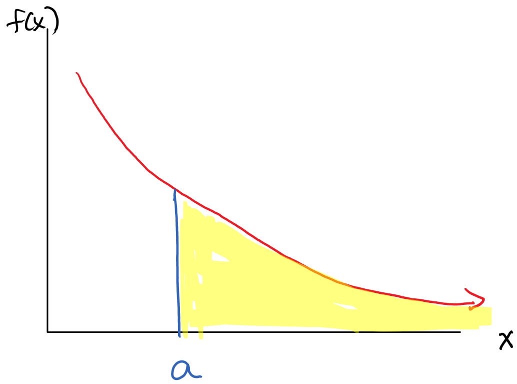
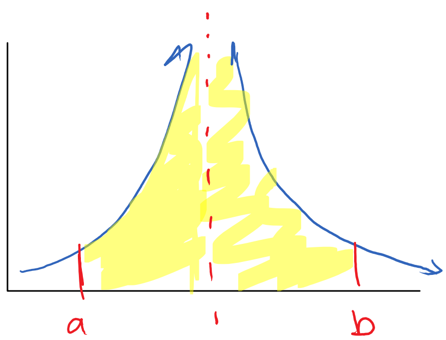
9.4 Day 1 — Improper Integrals

Wednesday, January 29, 2020 8:01 AM

2 types

① 

$\int_a^{\infty} f(x)\,dx$ with an infinite limit(s) of integration

② 

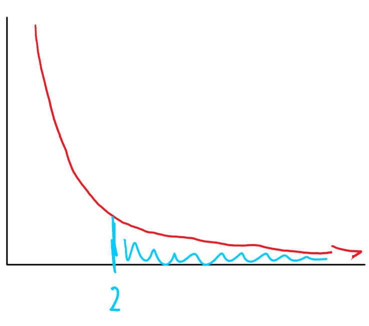
$\int_a^{b} f(x)\,dx$ with an infinite disc. somewhere $[a,b]$

Evaluate:

Ex 1: $\int_2^{\infty} \frac{1}{x^3}\,dx = \lim_{b\to\infty} \int_2^{b} \frac{1}{x^3}\,dx = \lim_{b\to\infty} \left[\frac{-1}{2x^2}\right]_2^{b} = \lim_{b\to\infty} \left[\frac{-1}{2b^2} - \frac{-1}{2(2)^2}\right]$

$= 0 + \frac{1}{8}$

converges $\frac{1}{8}$



Ex 2: $\int^{\infty} dx = \lim \int^{b} \frac{1}{x}\,dx = \lim \ln x \Big|^{b} = \lim \left[\ln b - \ln 2\right]$

Ex 2: $\int_2^\infty \frac{dx}{x} = \lim_{b\to\infty} \int_2^b \frac{1}{x}\,dx = \lim_{b\to\infty} \ln x \Big|_2^b = \lim_{b\to\infty} [\ln b - \ln 2]$

$\infty - \ln 2$

diverges

Ex 3: $\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0^+} 2x^{1/2}\Big|_a^1 = \lim_{a\to 0^+}\left[2 - 2a^{1/2}\right] = 2 - 0 = 2$

$u = -x^2 \quad u(b) = -b^2$

$u(1) = -1$

Ex 4: $\int_1^\infty x e^{-x^2}\,dx = \lim_{b\to\infty} \int_1^b x e^{-x^2}\,dx$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\lim_{b\to\infty} -\frac{1}{2}\int_{-1}^{-b^2} e^u\,du = \lim_{b\to\infty} -\frac{1}{2}e^u \Big|_{-1}^{-b^2} = \lim_{b\to\infty}\left[\frac{-1}{2}e^{-b^2} + \frac{1}{2}e^{-1}\right]$

$0 + \frac{1}{2}e^{-1} = \frac{1}{2e}$

converges to