

Evaluate each indefinite integral by using U-Substitution, Algebra, or basic integral rules.

$$1. \int \sec(2x) \tan(2x) dx \quad u = 2x$$

$$\frac{1}{2} \int \sec u \tan u du \quad du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$= \frac{1}{2} \sec u + C = \boxed{\frac{1}{2} \sec(2x) + C}$$

$$2. \int \frac{1}{\sqrt{1-x^2}} dx = \boxed{\sin^{-1} x + C}$$

$$3. \int \frac{1}{(1-x)^2} dx \quad u = 1-x$$

$$du = -dx$$

$$-du = dx$$

$$-\int u^{-2} du = u^{-1} + C$$

$$= \boxed{\frac{1}{1-x} + C}$$

$$4. \int 3(\sin x)^{-2} dx = 3 \int \frac{1}{\sin^2 x} dx$$

$$= 3 \int \csc^2 x dx$$

$$= \boxed{-3 \cot x + C}$$

$$5. \int \frac{(\ln x)^6}{x} dx \quad u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int u^6 du$$

$$= \frac{1}{7} u^7 + C = \boxed{\frac{1}{7} (\ln x)^7 + C}$$

$$6. \int \cot^7\left(\frac{x}{2}\right) \csc^2\left(\frac{x}{2}\right) dx \quad u = \cot\left(\frac{x}{2}\right)$$

$$-2 \int u^7 du \quad du = -\frac{1}{2} \csc^2\left(\frac{x}{2}\right) dx$$

$$-2 du = \csc^2\left(\frac{x}{2}\right) dx$$

$$= -\frac{1}{4} u^8 + C$$

$$= \boxed{-\frac{1}{4} \cot^8\left(\frac{x}{2}\right) + C}$$

$$7. \int \frac{x^3 + 4x^2 - 7x + 1}{x+1} dx$$

-1	1	4	-7	1
		-1	-3	10
	1	3	-10	11

$$\int (x^2 + 3x - 10 + \frac{11}{x+1}) dx$$

$$= \boxed{\frac{1}{3} x^3 + \frac{3}{2} x^2 - 10x + 11 \ln|x+1| + C}$$

$$8. \int \frac{4x}{3x^2+2} dx \quad u = 3x^2 + 2$$

$$du = 6x dx$$

$$\frac{2}{3} du = 4x dx$$

$$= \frac{2}{3} \int \frac{1}{u} du$$

$$= \frac{2}{3} \ln|u| + C$$

$$= \boxed{\frac{2}{3} \ln(3x^2+2) + C}$$

Evaluate each definite integral.

$$9. \int_0^{\frac{\pi}{4}} \tan x \sec^2 x dx \quad u = \tan x \quad u(0) = 0$$

$$du = \sec^2 x dx \quad u\left(\frac{\pi}{4}\right) = 1$$

$$= \int_0^1 u du = \left. \frac{1}{2} u^2 \right|_0^1$$

$$= \frac{1}{2} (1) - \frac{1}{2} (0)^2$$

$$= \boxed{\frac{1}{2}}$$

$$10. \int_0^1 \sqrt{t^5 + 2t} (5t^4 + 2) dt \quad u = t^5 + 2t$$

$$du = (5t^4 + 2) dt$$

$$u(1) = 3$$

$$u(0) = 0$$

$$= \int_0^3 u^{\frac{1}{2}} du$$

$$= \left. \frac{2}{3} u^{\frac{3}{2}} \right|_0^3$$

$$= \frac{2}{3} (3^{\frac{3}{2}}) - 0 = \frac{2}{3} \cdot 3\sqrt{3} = \boxed{2\sqrt{3}}$$

