9.7 Day 3

Sunday, January 12, 2020 9:21 AM

Pg 654 #19,30,44,50,51,58

Actual Book Pg 510 #20, 22

19. $f(x) = xe^x$ $\quad n=4 \quad$ Maclaurin $\quad x=c=0$

$f(x) = xe^x \qquad f(0)=0$

$f'(x) = xe^x + e^x = e^x(x+1) \qquad f'(0) = 1$

$f''(x) = e^x + e^x(x+1) = e^x(1+x+1) = e^x(x+2) \qquad f''(0) = 2$

$f'''(x) = e^x(x+3) \qquad f'''(0) = 3$

$f^4(x) = e^x(x+4) \qquad f^{(4)}(0) = 4$

$$\boxed{P_4(x) = x + \frac{2x^2}{2!} + \frac{3x^3}{3!} + \frac{4x^4}{4!} = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!}}$$

30 $f(x) = x^2\cos x \quad n=2 \quad \underline{c=\pi}$

$f(\pi) = \underline{-\pi^2}$

$f'(x) = -x^2\sin x + 2x\cos x \qquad f'(\pi) = -\pi^2(0) + 2\pi(-1) = \underline{-2\pi}$

$f''(x) = -x^2\cos x - 2x\sin x + {}^-2x\sin x + 2\cos x \qquad f''(\pi) = \underline{\pi^2 - 2}$

$$P_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{(\pi^2-2)(x-\pi)^2}{2!}$$

44. $f(x) = x^2\cos x \qquad f(7\pi/8)$

($P_2(x)$ found in #30)

$$P_2\left(\tfrac{7\pi}{8}\right) = -\pi^2 - 2\pi\left(\tfrac{7\pi}{8} - \pi\right) + \frac{(\pi^2-2)\left(\frac{7\pi}{8}-\pi\right)^2}{2} \approx -6.795$$

50. $\cos(0.1) \qquad g(x) = \cos x \qquad M = 1$ (upper bound) b/c $g^{(n+1)}(x)$ will be ≤ 1

$\left| \overset{M}{\downarrow}{1}\,(0.1)^{n+1} \right| < 0.001$

$$\left|\frac{M(0.1)^{n+1}}{(n+1)!}\right| \le 0.001$$

b/c $g^{(n+1)}(x)$ will be ≤ 1

using a table $\boxed{n=2}$

51. $e^{0.6}$ $f(x)=e^x$ $x=c$ center $=0$ Maclaurin

$f^{(n+1)}(x)=e^x$ $R_n \le \frac{e^{0.6}}{(n+1)!}(0.6)^{n+1} \le 0.001$

looking @ table $\boxed{n=5}$

58. $f(x)=e^{-2x}\approx 1-2x+2x^2-\frac{4}{3}x^3$

$$|\text{error}| \le \left|\frac{f^{(4)}(x)\cdot x^4}{4!}\right| \le 0.001$$

$$\left|\frac{16e^{-2x}\cdot x^4}{4!}\right| \le 0.001$$

y_1 y_2

$f' = -2e^{-2x}$

$f'' = 4e^{-2x}$

$f''' = -8e^{-2x}$

$f^{(4)} = 16e^{-2x}$

$-0.180 \le x \le 0.220$

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20. $P_2(x) = 1-\frac{x^2}{2}$ $\cos(0.5)$

$M=1$

b. $\frac{f^{(3)}(x)\cdot 0.5^3}{3!} = \frac{0.5^3}{3!} \approx 0.021$

22.