9.7 Day 3

Wednesday, February 26, 2020 8:32 AM

APQ: Let f be the function given by $f(x) = \sin(5x + \pi/4)$, and let $P_3(x)$ be the 3rd degree Poly for f and centered @ $x = 0$.

a. $P_3(x)$

$f(0) = \frac{\sqrt{2}}{2}$

$f'(x) = 5\cos(5x + \pi/4)$ $\quad f'(0) = \frac{5\sqrt{2}}{2}$

$f''(x) = -25\sin(5x + \pi/4)$ $\quad f''(0) = \frac{-25\sqrt{2}}{2}$

$f'''(x) = -125\cos(5x + \pi/4)$ $\quad f'''(0) = \frac{-125\sqrt{2}}{2}$

$$\star\ P_3(x) = \frac{\sqrt{2}}{2} + \frac{\frac{5\sqrt{2}}{2}}{1!}x - \frac{\frac{25\sqrt{2}}{2}}{2!}x^2 - \frac{\frac{125}{2}}{3!}x^3$$

$$= \frac{\sqrt{2}}{2} + \frac{5\sqrt{2}\,x}{2} - \frac{25\sqrt{2}x^2}{4} - \frac{125x^3}{12}$$

b. Use the LaGrang Error Bound to show that $\left|f\left(\frac{1}{10}\right) - P_3\left(\frac{1}{10}\right)\right| \le \frac{1}{100}$

$$\left|R_3\left(\tfrac{1}{10}\right)\right| \le \left|\frac{f^{(4)}(z)\left(\frac{1}{10}\right)^4}{4!}\right| = \frac{625}{24 \cdot 10000}$$

$$= \frac{1}{24(16)} < \frac{1}{100}$$

Think about $f(x) = \ln x$

$f'(x) = x^{-1}$

$f''(x) = -x^{-2}$

$f'''(x) = -2 \cdot -1\, x^{-3}$

$f^{(4)}(x) = -3 \cdot -2 \cdot -1\, x^{-4}$

$f^{(5)}(x) = -4 \cdot -3 \cdot -2 \cdot -1\, x^{-5}$

$$f^{(n+1)}(x) = \frac{(-1)^{n+1}\, n!}{x^{n+1}}$$

Ex: Determine the order of the Taylor Polynomial $P_n(x)$ expanded about $x = 1$ that should be used to approx. ln(1.2) so that the error is less than 0.001.

$[1, 1.2]$

$f(x) = \ln x$ $\quad (-1)^{n+1}$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2}$

$f'''(x) = \frac{2}{x^3}$ $\quad 2!$

$f^{(4)}(x) = \frac{-6}{x^4}$ $\quad 3!$

$f^{(5)}(x) = \frac{24}{x^5}$ $\quad 4!$

$$f^{(n+1)}(x) = \frac{(-1)^{n+1} \cdot n!}{x^{n+1}}$$

$[1, 1.2]$

max when $x = 1$

$$|R_n(1.2)| \leq \left| \frac{f^{(n+1)}(z) \cdot (1.2-1)^{n+1}}{(n+1)!} \right|$$

$$\leq \left| \frac{(-1)^{n+1} \cdot \cancel{n!}\,(0.2)^{n+1}}{(n+1)!} \right| < 0.001$$

$(n+1)\cancel{n!}$

$$\frac{(0.2)^{n+1}}{n+1} < 0.001$$

Y_1 $\quad Y_2$

$n = 3$

$\boxed{\text{Order } 3}$