4.2 Day 2 — Implicit Diff. w/ Higher order Derivatives.

Thursday, September 19, 2019 8:09 AM

Let's revisit $x^2+y^2=16$

$2x+2yy'=0$

$2yy'=-2x$

$y'=\frac{-x}{y}$

Recall $y'=\frac{-x}{y}$

Find $\frac{d^2y}{dx^2}$ (second derivative) $=\frac{y(-1)-(-x)y'}{y^2}=\frac{-y+xy'}{y^2}$

$=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}=\frac{\left(-y+\frac{-x^2}{y}\right)}{(y^2)}\frac{y}{y}$

FRQ $\frac{d^2y}{dx^2}=\frac{-y^2-x^2}{y^3}=\frac{-(y^2+x^2)}{y^3}=\frac{-16}{y^3}$ MC

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ $\quad x^2=y^2-x \Rightarrow x^2+x=y^2$

$2x=2yy'-1$

$2x+1=2yy'$

$y'=\frac{2x+1}{2y}=\frac{dy}{dx}$

$y''=\frac{2y(2)-(2x+1)\cdot 2y'}{4y^2}=\frac{4y-2y'(2x+1)}{4y^2}$

$$y'' = \frac{4y - 2\left(\frac{2x+1}{2y}\right)(2x+1)}{4y^2} = \left(\frac{4y - \frac{2(2x+1)^2}{2y}}{4y^2}\right)\frac{y}{y}$$

$$= \frac{4y^2 - (2x+1)^2}{4y^3} \quad \text{FRO} \quad = \frac{4y^2 - (4x^2+4x+1)}{4y^3}$$

$$= \frac{4y^2 - (4(x^2+x)+1)}{4y^3} = \frac{4y^2 - 4y^2 - 1}{4y^3} = \frac{-1}{4y^3}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{4y^3}$$ mc