9.5 Day 1

Wednesday, February 12, 2020 8:05 AM

The series $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot a_n = a_1 - a_2 + a_3 - a_4 + a_5 - \ldots$

converges if the following 3 conditions hold:

1. $a_n \geq 0$
2. $a_n \geq a_{n+1}$ (Terms are decreasing)
3. $\lim_{n \to \infty} a_n = 0$

Ex: $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n}\right)$ alternating harmonic series

① $\frac{1}{n} \geq 0$ ✓

② $\frac{1}{n} > \frac{1}{n+1}$ ✓

③ $\lim_{n \to \infty} \frac{1}{n} = 0$ ✓

$\sum_{n=1}^{\infty} (-1)^{n+1}$ converges by alt. series test

Ex: $\sum_{n=1}^{\infty} \frac{n}{(-3)^{n-1}} = \sum_{n=1}^{\infty} \frac{n}{(-1)^{n-1}(3)^{n-1}}$

$a_n = \frac{n}{3^{n-1}}$ $a_1 = \frac{1}{1}$ $a_2 = \frac{2}{3}$ $a_3 = 3/9$

① $\frac{n}{3^{n-1}} \geq 0$ ✓

② $\frac{n}{3^{n-1}} > \frac{n+1}{3^{(n+1)-1}} = \frac{n+1}{3^n}$ ✓

③ $\lim_{n \to \infty} \frac{n}{3^{n-1}}$ $\to \infty$, $\to \infty$ $\Rightarrow \lim_{n \to \infty} \frac{1}{3^{n-1} \cdot \ln 3} = 0$ ✓

Converges by alt. series test.

Ex: $\sum^{\infty} \frac{(-1)^{n+1}(n+2)}{}$ $\Longrightarrow$ Diverges by n^{th} term test

Ex: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2)}{n} \Longrightarrow$ Diverges by n^{th} term test

① $\frac{n+2}{n} \geq 0$ ✓

② $\frac{n+2}{n} \geq \frac{n+3}{n+1}$

③ $\lim_{n\to\infty} \frac{n+2}{n} = 1$ ☹

$\varepsilon BM = 1$

Try: $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^{n+1}}$

① $\frac{1}{e^{n+1}} \geq 0$ ✓

② $\frac{1}{e^{n+1}} \geq \frac{1}{e^{n+2}}$ ✓

③ $\lim_{n\to\infty} \frac{1}{e^{n+1}} = 0$ ✓

☺ converges by alt. series test

factorials

$$5! = 5 \cdot (5-1)(5-2)(5-3)(5-4)$$
$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$\frac{(n+2)!}{(n-1)!} = \frac{(n+2)(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}$$