Wednesday, March 4, 2020 6:52 AM

Section 9.9

pg 662: 5, 11, 15, 16, 19, 21

5. $f(x) = \frac{1}{3-x}$ $c = 1$

$f(x) = \frac{1}{-1+3-(x-1)} = \frac{1}{2-(x-1)} \cdot \frac{\frac{1}{2}}{(\frac{1}{2})}$

$f(x) = \frac{\frac{1}{2}}{1-\frac{(x-1)}{2}}$ $a = 1/2$ $r = \frac{x-1}{2}$

* power series => $\frac{1}{2} + \frac{x-1}{4} + \frac{(x-1)^2}{8} + \ldots + = \sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{x-1}{2}\right)^n$ or $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}(x-1)^n$

$\left|\frac{x-1}{2}\right| < 1$ $-1 < \frac{x-1}{2} < 1$ $-2 < x-1 < 2$ $-1 < x < 3$

* interval of convergence = $(-1, 3)$

11. $f(x) = \frac{3}{3x+4}$, $c = 0$

$f(x) = \frac{3}{(4+3x)} \cdot \frac{\frac{1}{4}}{1/4}$ $f(x) = \frac{3/4}{1+\frac{3}{4}x}$ $a = 3/4$ $r = (-3/4\, x)$

* power series => $\frac{3}{4} + -\frac{9x^2}{16} + \frac{27}{64}x^3 + \ldots = \sum_{n=0}^{\infty} \frac{3}{4}\left(-\frac{3}{4}x\right)^n$

$\left|-\frac{3}{4}x\right| < 1$ $-1 < \frac{3}{4}x < 1$ $-4 < 3x < 4$ $-\frac{4}{3} < x < \frac{4}{3}$

* interval of convergence $(-4/3, 4/3)$

15. $f(x) = \frac{2}{1-x^2}$ $a = 2$ $r = x^2$

power series

* $2 + 2x^2 + 2x^4 + 2x^6 + \ldots = \sum_{n=0}^{\infty} 2x^{2n}$

$|x^2| < 1$ $-1 < x < 1$

* interval of convergence $(-1, 1)$

16. $f(x) = \frac{5}{5+x^2}$ $\quad f(x) = \frac{1}{1+\frac{x^2}{5}}$ $\quad a_1 = 1$ $\quad r = \left(\frac{-x^2}{5}\right)$

* power series $1 - \frac{x^2}{5} + \frac{x^4}{25} + \frac{x^6}{125} + \cdots$ $\quad \sum_{n=0}^{\infty} \left(\frac{-x^2}{5}\right)^n$

$\left|\frac{-x^2}{5}\right| < 1$ $\quad \frac{x^2}{5} < 1$ $\quad x^2 < 5$

$-\sqrt{5} < x < \sqrt{5}$

* interval of convergence
$(-\sqrt{5}, 5)$

$$\boxed{\text{For \#19 \& 21} \quad \frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \ *}$$

19. $f(x) = -\frac{1}{(x+1)^2} = \frac{d}{dx}\left(\frac{1}{x+1}\right)$

$* \sum_{n=0}^{\infty} (-1)^n n x^{n-1}$ $\qquad \sum_{n=0}^{\infty} (-1)^{n+1} (n+1) x^n$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+2}(n+2)x^{n+1}}{(-1)^{n+1}(n+1)(x^n)} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+2}(n+2)x}{(-1)^{n+1}(n+1)} \right|$$

$|x| < 1$ $\quad -1 < x < 1$

interval of convergence $(-1, 1)$

21. $f(x) = \ln(x+1) = \int \frac{1}{x+1}\, dx$

$$C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$$

Find C since the center is @ $x=0$

$C = \ln(0+1) \quad C = 0$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+2}}{n+2} \cdot \frac{n+1}{(-1)^n \cdot x^{n+1}} \right| = \lim_{n\to\infty} \left| \frac{-1\,x(n+1)}{(n+2} \right|$$

$$|x| < 1 \qquad -1 < x < 1$$

check endpts

$x=-1 \quad \sum_{n=0}^{\infty} \frac{(-1)^n (-1)^{n+1}}{n+1} \qquad \sum_{n=0}^{\infty} \frac{-1}{n+1}$ diverges

$x=1 \quad \sum_{n=0}^{\infty} \frac{(-1)^n (1)^{n+1}}{n+1}$

alt series

$\frac{1}{n+1} \geq 0$ ✓

$\frac{1}{n+1} \geq \frac{1}{n+2}$ ✓

$\lim_{n\to\infty} \frac{1}{n+1} = 0$ ✓

★ conv. cond @ $x=1$

Converges $(-1, 1]$

converges abs $(-1, 1)$