Precalculus Honors
Some review questions 5.1-5.4

Name Key

Prove the following identities:

1. $\sin 4x + \sin 2x = 2\sin 3x \cos x$

$\sin(3x+x) + \sin(3x-x)$

$\sin 3x\cos x + \sin x\cos 3x + \sin 3x\cos x - \sin x\cos 3x$

$2\sin 3x\cos x$

2. $\cos 4\theta = 1 - 8\sin^2\theta + 8\sin^4\theta$

$\cos(2(2\theta))$

$2\cos^2(2\theta) - 1$

$2(\cos(2\theta))^2 - 1$

$2(1 - 2\sin^2(2\theta))^2 - 1$

$2(1 - 4\sin^2 2\theta + 4\sin^4\theta) - 1$

$2 - 8\sin^2\theta + 8\sin^4\theta - 1$

$1 - 8\sin^2\theta + 8\sin^4\theta$

3. $\dfrac{\sin x - \cos x}{\sin x + \cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x\cos x}$

$= \dfrac{(\sin x - \cos x)(\sin x + \cos x)}{(\sin x + \cos x)(\sin x + \cos x)}$

$= \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x + 2\sin x\cos x + \cos^2 x}$

$= \dfrac{\sin^2 x - (1 - \sin^2 x)}{1 + 2\sin x\cos x}$

$= \dfrac{2\sin^2 x - 1}{1 + 2\sin x\cos x}$

4. $\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$

$\sin\frac{\pi}{2}\cos x - \sin x\cos\frac{\pi}{2}$

$1\cdot\cos x - \sin x\cdot 0$

$\cos x$

Find the exact value.

5. $11\sin\left(\dfrac{5\pi}{12}\right)\cos\left(\dfrac{5\pi}{12}\right)$

$\frac{11}{2}\left(2\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right)\right)$

$\frac{11}{2}\sin\left(2\cdot\frac{5\pi}{12}\right) \Rightarrow \frac{11}{2}\sin\frac{5\pi}{6}$

$\frac{11}{2}\cdot\frac{1}{2} = 11/4$

6. $\cos 157.5^\circ$

$\cos\frac{315}{2} = +\sqrt{\frac{1+\cos(315)}{2}}$

$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{1}{2}+\frac{1}{2\sqrt{2}}}$

$= \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

Solve the following:

7. $3\sin x = 2\cos^2 x$ for all values of x

$3\sin x = 2(1-\sin^2 x)$

$3\sin x = 2 - 2\sin^2 x$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = 1/2$

$x = \frac{\pi}{6} + 2\pi k$

$x = \frac{5\pi}{6} + 2\pi k$

8. $2\sin^4 x - 3\sin^2 x + 1 = 0$ over the interval $0, 2\pi$

$(2\sin^2 x - 1)(\sin^2 x - 1) = 0$

$2\sin^2 x - 1 = 0$ $\quad \sin^2 x - 1 = 0$

$\sin^2 x = 1/2$ $\quad \sin x = \pm 1$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$

$\frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$