8.1/8.2 Grading rubric (AP Problems)

AB-3 1997

3. Let f be the function given by $f(x) = \sqrt{x-3}$.

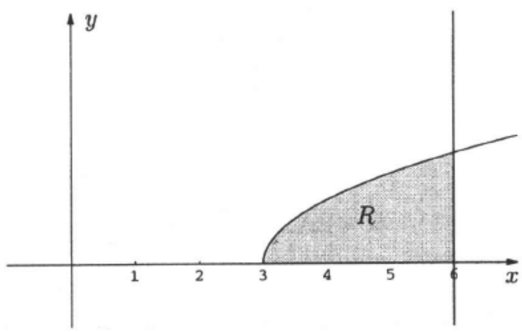
(a) On the axes provided below, sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x = 6$.

(b) Find the area of the region R described in part (a).

(c) Rather than using the line $x = 6$ as in part (a), consider the line $x = w$, where w can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of f, the x-axis, and the vertical line $x = w$. Write an integral expression for $A(w)$.

(d) Let $A(w)$ be as described in part (c). Find the rate of change of A with respect to w when $w = 6$.

(a)



$$2\begin{cases} 1: \text{graph of } f, \text{ (domain is } x \geq 3, \\ \quad \text{goes through } (3,0), \text{ is increasing,} \\ \quad \text{positive, and concave down)} \\ 1: \text{correct region relative to graph of } f \end{cases}$$

(b) $\text{area} = \displaystyle\int_3^6 \sqrt{x-3}\,dx = \frac{2}{3}(x-3)^{3/2}\Big]_3^6$

$= 2\sqrt{3} = 3.464$

$$3\begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \\ \quad 0/1 \text{ if second point is not earned} \end{cases}$$

(c) $A(w) = \displaystyle\int_3^w \sqrt{x-3}\,dx$

$$2\begin{cases} 1: \text{limits} \\ 1: \text{integrand} \end{cases}$$

(d) $\dfrac{dA}{dw} = \sqrt{w-3}$

$\left.\dfrac{dA}{dw}\right|_{w=6} = \sqrt{3} = 1.732$

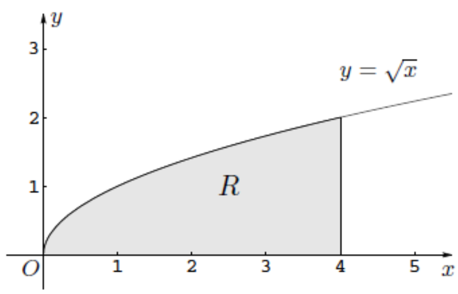
$$2\begin{cases} 1: \dfrac{dA}{dw} \\ 1: \text{evaluation at } 6 \end{cases}$$

$0/2$ if $\dfrac{dA}{dw}$ is constant

1998 AP Calculus AB Scoring Guidelines

1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   (a) Find the area of the region R.
   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
   (c) Find the volume of the solid generated when R is revolved about the x–axis.
   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

(a)



$$A = \int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3} \quad \text{or} \quad 5.333$$

$$\mathbf{2}\begin{cases} 1: & A = \int_0^4 \sqrt{x}\,dx \\ 1: & \text{answer} \end{cases}$$

(b) $\displaystyle\int_0^h \sqrt{x}\,dx = \frac{8}{3}$

$\displaystyle\frac{2}{3}h^{3/2} = \frac{8}{3}$

–or–

$\displaystyle\int_0^h \sqrt{x}\,dx = \int_h^4 \sqrt{x}\,dx$

$\displaystyle\frac{2}{3}h^{3/2} = \frac{16}{3} - \frac{2}{3}h^{3/2}$

$h = \sqrt[3]{16}$ or 2.520 or 2.519

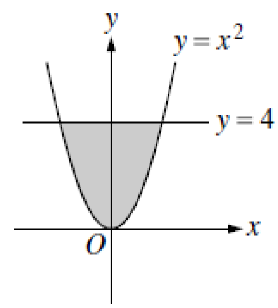
$$\mathbf{2}\begin{cases} 1: & \text{equation in } h \\ 1: & \text{answer} \end{cases}$$

2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated by revolving R about the x–axis.

   (c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.



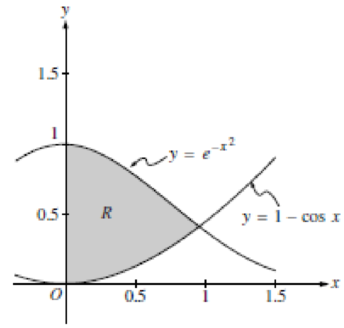
(a) $\text{Area} = \displaystyle\int_{-2}^{2} (4 - x^2)\,dx$

$= 2\displaystyle\int_{0}^{2} (4 - x^2)\,dx$

$= 2\left[4x - \dfrac{x^3}{3}\right]_0^2$

$= \dfrac{32}{3} = 10.666$ or 10.667

$2 \begin{cases} 1\text{: integral} \\ 1\text{: answer} \end{cases}$



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Region R $e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$	1 : Correct limits in an integral in (a), (b), or (c).
(a) Area $= \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$ $= 0.590$ or 0.591	$2 \begin{cases} 1: \text{ integrand} \\ 1: \text{ answer} \end{cases}$

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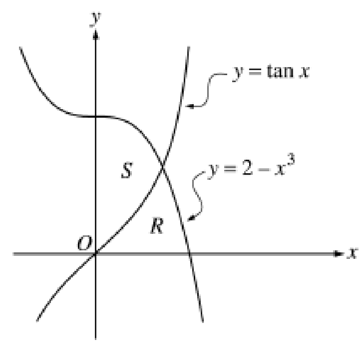
2001 SCORING GUIDELINES

Question 1

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.

Point of intersection

$2 - x^3 = \tan x$ at $(A, B) = (0.902155, 1.265751)$

(a) Area $R = \int_0^A \tan x\,dx + \int_A^{\sqrt[3]{2}} (2 - x^3)\,dx = 0.729$

or

Area $R = \int_0^B \left((2 - y)^{1/3} - \tan^{-1} y\right) dy = 0.729$

or

Area $R = \int_0^{\sqrt[3]{2}} (2 - x^3)\,dx - \int_0^A (2 - x^3 - \tan x)\,dx = 0.729$

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) Area $S = \int_0^A (2 - x^3 - \tan x)\,dx = 1.160$ or 1.161

or

Area $S = \int_0^B \tan^{-1} y\,dy + \int_B^2 (2 - y)^{1/3}\,dy = 1.160$ or 1.161

or

Area S

$= \int_0^2 (2 - y)^{1/3}\,dy - \int_0^B \left((2 - y)^{1/3} - \tan^{-1} y\right) dy$

$= 1.160$ or 1.161

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 1

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

(a) Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ or 1.223	$2 \begin{cases} 1: \text{ integral} \\ 1: \text{ answer} \end{cases}$