us $\left(x - \frac{1}{2}\right)\left(2x^2 - \frac{3}{2}x + \frac{3}{4}\right) + \frac{7}{8}$
$= 2x^3 - \frac{5}{4}x^2 + \frac{3}{2}x + \frac{1}{2}.$

**76.** Use the zero or root finder feature to locate the zero near $x = -3$. Then regraph the function in a smaller window, such as $[0, 2]$ by $[-0.5, 0.5]$, and locate the other three zeros of the function.

**77. (a)** $g(x) = 3f(x)$, so the zeros of $f$ and the zeros of $g$ are identical. If the coefficients of a polynomial are rational, we may multiply that polynomial by the least common multiple (LCM) of the denominators of the coefficients to obtain a polynomial, with integer coefficients, that has the same zeros as the original.

**(b)** The zeros of $f(x)$ are the same as the zeros of $6f(x) = 6x^3 - 7x^2 - 40x + 21$. Possible rational zeros: $\frac{\pm1, \pm3, \pm7, \pm21}{\pm1, \pm2, \pm3, \pm6}$, or $\pm1, \pm3, \pm7, \pm21$, $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}, \pm\frac{1}{3}, \pm\frac{7}{3}, \pm\frac{1}{6}, \pm\frac{7}{6}$.
The actual zeros are $-7/3$, $1/2$, and 3.

**(c)** The zeros of $f(x)$ are the same as the zeros of $12f(x) = 12x^3 - 30x^2 - 37x + 30$.
Possible rational zeros:
$\frac{\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30}{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}$, or
$\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$,
$\pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{6}$,
$\pm\frac{5}{6}, \pm\frac{1}{12}, \pm\frac{5}{12}$.
There are no rational zeros.

**78.** Let $f(x) = x^2 - 2 = (x + \sqrt{2})(x - \sqrt{2})$. Notice that $\sqrt{2}$ is a zero of $f$. By the Rational Zeros Theorem, the only possible rational zeros of $f$ are $\pm1$ and $\pm2$. Because $\sqrt{2}$ is none of these, it must be irrational.

**79. (a)** Approximate zeros: $-3.126, -1.075, 0.910, 2.291$

**(b)** $f(x) \approx g(x)$
$= (x + 3.126)(x + 1.075)(x - 0.910)(x - 2.291)$

**(c)** Graphically: Graph the original function and the approximate factorization on a variety of windows and observe their similarity. Numerically: Compute $f(c)$ and $g(c)$ for several values of $c$.

## ■ Section 2.5 Complex Zeros and the Fundamental Theorem of Algebra

### Exploration 1

**1.** $f(2i) = (2i)^2 - i(2i) + 2 = -4 + 2 + 2 = 0$;
$f(-i) = (-i)^2 - i(-i) + 2 = -1 - 1 + 2 = 0$; no.

**2.** $g(i) = i^2 - i + (1 + i) = -1 - i + 1 + i = 0$;
$g(1 - i) = (1 - i)^2 - (1 - i) + (1 + i) =$
$-2i + 2i = 0$; no.

**3.** The Complex Conjugate Zeros Theorem does not necessarily hold true for a polynomial function with *complex* coefficients.

### Quick Review 2.5

**1.** $(3 - 2i) + (-2 + 5i) = (3 - 2) + (-2 + 5)i$
$= 1 + 3i$

**2.** $(5 - 7i) - (3 - 2i) = (5 - 3) + (-7 - (-2))i$
$= 2 - 5i$

**3.** $(1 + 2i)(3 - 2i) = 1(3 - 2i) + 2i(3 - 2i)$
$= 3 - 2i + 6i - 4i^2$
$= 7 + 4i$

**4.** $\frac{2 + 3i}{1 - 5i} = \frac{2 + 3i}{1 - 5i} \cdot \frac{1 + 5i}{1 + 5i}$
$= \frac{2 + 10i + 3i + 15i^2}{1^2 + 5^2}$
$= \frac{-13 + 13i}{26}$
$= -\frac{1}{2} + \frac{1}{2}i$

**5.** $(2x - 3)(x + 1)$

**6.** $(3x + 1)(2x - 5)$

**7.** $x = \frac{5 \pm \sqrt{25 - 4(1)(11)}}{2} = \frac{5 \pm \sqrt{-19}}{2}$
$= \frac{5}{2} \pm \frac{\sqrt{19}}{2}i$

**8.** $x = \frac{-3 \pm \sqrt{9 - 4(2)(7)}}{4} = \frac{-3 \pm \sqrt{-47}}{4}$
$= -\frac{3}{4} \pm \frac{\sqrt{47}}{4}i$

**9.** $\frac{\pm1, \pm2}{\pm1, \pm3}$, or $\pm1, \pm2, \pm\frac{1}{3}, \pm\frac{2}{3}$

**10.** $\frac{\pm1, \pm3}{\pm1, \pm2, \pm4}$, or $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

### Section 2.5 Exercises

**1.** $(x - 3i)(x + 3i) = x^2 - (3i)^2 = x^2 + 9$. The factored form shows the zeros to be $x = \pm3i$. The absence of real zeros means that the graph has no $x$-intercepts.

**2.** $(x + 2)(x - \sqrt{3}\,i)(x + \sqrt{3}\,i) = (x + 2)(x^2 + 3)$
$= x^3 + 2x^2 + 3x + 6$. The factored form shows the zeros to be $x = -2$ and $x = \pm\sqrt{3}i$. The real zero $x = -2$ is the $x$-intercept of the graph.

**3.** $(x - 1)(x - 1)(x + 2i)(x - 2i)$
$= (x^2 - 2x + 1)(x^2 + 4)$
$= x^4 - 2x^3 + 5x^2 - 8x + 4$. The factored form shows the zeros to be $x = 1$ (multiplicity 2) and $x = \pm2i$. The real zero $x = 1$ is the $x$-intercept of the graph.

**4.** $x(x - 1)(x - 1 - i)(x - 1 + i)$
$= (x^2 - x)[x - (1 + i)][x - (1 - i)]$
$= (x^2 - x)[x^2 - (1 - i + 1 + i)\,x + (1 + 1)]$
$= (x^2 - x)(x^2 - 2x + 2) = x^4 - 3x^3 + 4x^2 - 2x$.
The factored form shows the zeros to be $x = 0$, $x = 1$, and $x = 1 \pm i$. The real zeros $x = 0$ and $x = 1$ are the $x$-intercepts of the graph.

In #5–16, any constant multiple of the given polynomial is also an answer.

**5.** $(x - i)(x + i) = x^2 + 1$

**6.** $(x - 1 + 2i)(x - 1 - 2i) = x^2 - 2x + 5$

**7.** $(x - 1)(x - 3i)(x + 3i) = (x - 1)(x^2 + 9)$
$= x^3 - x^2 + 9x - 9$

**8.** $(x + 4)(x - 1 + i)(x - 1 - i)$
$= (x + 4)(x^2 - 2x + 2) = x^3 + 2x^2 - 6x + 8$

**9.** $(x - 2)(x - 3)(x - i)(x + i)$
$= (x - 2)(x - 3)(x^2 + 1)$
$= x^4 - 5x^3 + 7x^2 - 5x + 6$

**10.** $(x + 1)(x - 2)(x - 1 + i)(x - 1 - i)$
$= (x + 1)(x - 2)(x^2 - 2x + 2)$
$= x^4 - 3x^3 + 2x^2 + 2x - 4$

**11.** $(x - 5)(x - 3 - 2i)(x - 3 + 2i)$
$= (x - 5)(x^2 - 6x + 13) = x^3 - 11x^2 + 43x - 65$

**12.** $(x + 2)(x - 1 - 2i)(x - 1 + 2i)$
$= (x + 2)(x^2 - 2x + 5) = x^3 + x + 10$

**13.** $(x - 1)^2(x + 2)^3 = x^5 + 4x^4 + x^3 - 10x^2 - 4x + 8$

**14.** $(x + 1)^3(x - 3) = x^4 - 6x^2 - 8x - 3$

**15.** $(x - 2)^2(x - 3 - i)(x - 3 + i)$
$= (x - 2)^2(x^2 - 6x + 10)$
$= (x^2 - 4x + 4)(x^2 - 6x + 10)$
$= x^4 - 10x^3 + 38x^2 - 64x + 40$

**16.** $(x + 1)^2(x + 2 + i)(x + 2 - i)$
$= (x + 1)^2(x^2 + 4x + 5)$
$= (x^2 + 2x + 1)(x^2 + 4x + 5)$
$= x^4 + 6x^3 + 14x^2 + 14x + 5$

In #17–20, note that the graph crosses the $x$-axis at odd-multiplicity zeros, and "kisses" (touches but does not cross) the $x$-axis where the multiplicity is even.

**17.** (b)

**18.** (c)

**19.** (d)

**20.** (a)

In #21–26, the number of complex zeros is the same as the degree of the polynomial; the number of real zeros can be determined from a graph. The latter always differs from the former by an even number (when the coefficients of the polynomial are real).

**21.** 2 complex zeros; none real.

**22.** 3 complex zeros; all 3 real.

**23.** 3 complex zeros; 1 real.

**24.** 4 complex zeros; 2 real.

**25.** 4 complex zeros; 2 real.

**26.** 5 complex zeros; 1 real.

In #27–32, look for real zeros using a graph (and perhaps the Rational Zeros Test). Use synthetic division to factor the polynomial into one or more linear factors and a quadratic factor. Then use the quadratic formula to find complex zeros.

**27.** Inspection of the graph reveals that $x = 1$ is the only real zero. Dividing $f(x)$ by $x - 1$ leaves $x^2 + x + 5$ (below). The quadratic formula gives the remaining zeros of $f(x)$.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 4 & -5 \\ & & 1 & 1 & 5 \\ \hline & 1 & 1 & 5 & 0 \end{array}$$

Zeros: $x = 1,\ x = -\frac{1}{2} \pm \frac{\sqrt{19}}{2} i$

$f(x)$

$= (x - 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{19}}{2} i\right)\right]\left[x - \left(-\frac{1}{2} + \frac{\sqrt{19}}{2} i\right)\right]$

$= \frac{1}{4}(x - 1)(2x + 1 + \sqrt{19}\, i)(2x + 1 - \sqrt{19}\, i)$

**28.** Zeros: $x = 3$ (graphically) and $x = \frac{7}{2} \pm \frac{\sqrt{43}}{2} i$ (applying the quadratic formula to $x^2 - 7x + 23$).

$$\begin{array}{r|rrrr} 3 & 1 & -10 & 44 & -69 \\ & & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & 0 \end{array}$$

$f(x)$

$= (x - 3)\left[x - \left(\frac{7}{2} - \frac{\sqrt{43}}{2} i\right)\right]\left[x - \left(\frac{7}{2} + \frac{\sqrt{43}}{2} i\right)\right]$

$= \frac{1}{4}(x - 3)(2x - 7 + \sqrt{43}\, i)(2x - 7 - \sqrt{43}\, i)$

**29.** Zeros: $x = \pm 1$(graphically) and $x = -\frac{1}{2} \pm \frac{\sqrt{23}}{2} i$ (applying the quadratic formula to $x^2 + x + 6$).

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 5 & -1 & -6 \\ & & 1 & 2 & 7 & 6 \\ \hline & 1 & 2 & 7 & 6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 7 & 6 \\ & & -1 & -1 & -6 \\ \hline & 1 & 1 & 6 & 0 \end{array}$$

$f(x) = (x - 1)(x + 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{23}}{2} i\right)\right]$
$\left[x - \left(-\frac{1}{2} + \frac{\sqrt{23}}{2} i\right)\right]$

$= \frac{1}{4}(x - 1)(x + 1)(2x + 1 + \sqrt{23}\, i)\ (2x + 1 - \sqrt{23}\, i)$

**30.** Zeros: $x = -2$ and $x = \frac{1}{3}$ (graphically) and $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$ (applying the quadratic formula to $3x^2 + 3x + 3 = 3(x^2 + x + 1)$).

$$\begin{array}{r|rrrrr} -2 & 3 & 8 & 6 & 3 & -2 \\ & & -6 & -4 & -4 & 2 \\ \hline & 3 & 2 & 2 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1/3 & 3 & 2 & 2 & -1 \\ & & 1 & 1 & 1 \\ \hline & 3 & 3 & 3 & 0 \end{array}$$

$f(x) = (x + 2)(3x - 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)\right]$
$\left[x - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)\right]$

$= \frac{1}{4}(x + 2)(3x - 1)(2x + 1 + \sqrt{3}\,i)$
$(2x + 1 - \sqrt{3}\,i)$

**31.** Zeros: $x = -\frac{7}{3}$ and $x = \frac{3}{2}$ (graphically) and $x = 1 \pm 2i$ (applying the quadratic formula to $6x^2 - 12x + 30 = 6(x^2 - 2x + 5)$).

$$\begin{array}{r|rrrrr} -7/3 & 6 & -7 & -1 & 67 & -105 \\ & & -14 & 49 & -112 & 105 \\ \hline & 6 & -21 & 48 & -45 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3/2 & 6 & -21 & 48 & -45 \\ & & 9 & -18 & 45 \\ \hline & 6 & -12 & 30 & 0 \end{array}$$

$f(x) = (3x + 7)(2x - 3)[x - (1 - 2i)]$
$[x - (1 + 2i)]$
$= (3x + 7)(2x - 3)(x - 1 + 2i)(x - 1 - 2i)$

**32.** Zeros: $x = -\frac{3}{5}$ and $x = 5$ (graphically) and $x = \frac{3}{2} \pm i$
$= (5x + 3)(x - 5)(2x - 3 + 2i)(2x - 3 - 2i)$
(applying the quadratic formula to $20x^2 - 60x + 65 = 5(4x^2 - 12x + 13)$).

$$\begin{array}{r|rrrrr} 5 & 20 & -148 & 269 & -106 & -195 \\ & & 100 & -240 & 145 & 195 \\ \hline & 20 & -48 & 29 & 39 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3/5 & 20 & -48 & 29 & 39 \\ & & -12 & 36 & -39 \\ \hline & 20 & -60 & 65 & 0 \end{array}$$

$f(x) = (5x + 3)(x - 5)[2x - (3 - 2i)]$
$[2x - (3 + 2i)]$
$= (5x + 3)(x - 5)(2x - 3 + 2i)(2x - 3 - 2i)$

In #33–36, since the polynomials' coefficients are real, for the given zero $z = a + bi$, the complex conjugate $\bar{z} = a - bi$ must also be a zero. Divide $f(x)$ by $x - z$ and $x - \bar{z}$ to reduce to a quadratic.

**33.** First divide $f(x)$ by $x - (1 + i)$ (synthetically). Then divide the result, $x^3 + (-1 + i)x^2 - 3x + (3 - 3i)$, by $x - (1 - i)$. This leaves the polynominal $x^2 - 3$. Zeros: $x = \pm\sqrt{3}$, $x = 1 \pm i$

$$\begin{array}{r|rrrrr} 1+i & 1 & -2 & -1 & 6 & -6 \\ & & 1+i & -2 & -3-3i & 6 \\ \hline & 1 & -1+i & -3 & 3-3i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1-i & 1 & -1+i & -3 & 3-3i \\ & & 1-i & 0 & -3+3i \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$f(x) = (x - \sqrt{3})(x + \sqrt{3})[x - (1 - i)][x - (1 + i)]$
$= (x - \sqrt{3})(x + \sqrt{3})(x - 1 + i)(x - 1 - i)$

**34.** First divide $f(x)$ by $x - 4i$. Then divide the result, $x^3 + 4ix^2 - 3x - 12i$, by $x + 4i$. This leaves the polynomial $x^2 - 3$. Zeros: $x = \pm\sqrt{3}$, $x = \pm 4i$

$$\begin{array}{r|rrrrr} 4i & 1 & 0 & 13 & 0 & -48 \\ & & 4i & -16 & -12i & 48 \\ \hline & 1 & 4i & -3 & -12i & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4i & 1 & 4i & -3 & -12i \\ & & -4i & 0 & 12i \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - 4i)(x + 4i)$

**35.** First divide $f(x)$ by $x - (3 - 2i)$. Then divide the result, $x^3 + (-3 - 2i)x^2 - 2x + 6 + 4i$, by $x - (3 + 2i)$. This leaves $x^2 - 2$. Zeros: $x = \pm\sqrt{2}$, $x = 3 \pm 2i$

$$\begin{array}{r|rrrrr} 3-2i & 1 & -6 & 11 & 12 & -26 \\ & & 3-2i & -13 & -6+4i & 26 \\ \hline & 1 & -3-2i & -2 & 6+4i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3+2i & 1 & -3-2i & -2 & 6+4i \\ & & 3+2i & 0 & -6-4i \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$f(x) = (x - \sqrt{2})(x + \sqrt{2})[x - (3 - 2i)]$
$[x - (3 + 2i)]$
$= (x - \sqrt{2})(x + \sqrt{2})(x - 3 + 2i)(x - 3 - 2i)$

**36.** First divide $f(x)$ by $x - (1 + 3i)$. Then divide the result, $x^3 + (-1 + 3i)x^2 - 5x + 5 - 15i$, by $x - (1 - 3i)$. This leaves $x^2 - 5$. Zeros: $x = \pm\sqrt{5}$, $x = 1 \pm 3i$

$$\begin{array}{r|rrrrr} 1+3i & 1 & -2 & 5 & 10 & -50 \\ & & 1+3i & -10 & -5-15i & 50 \\ \hline & 1 & -1+3i & -5 & 5-15i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1-3i & 1 & -1+3i & -5 & 5-15i \\ & & 1-3i & 0 & -5+15i \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$f(x) = (x - \sqrt{5})(x + \sqrt{5})[x - (1 - 3i)]$
$[x - (1 + 3i)]$
$= (x - \sqrt{5})(x + \sqrt{5})(x - 1 + 3i)(x - 1 - 3i)$

For #37–42, find real zeros graphically, then use synthetic division to find the quadratic factors. Only the synthetic divison step is shown.

**37.** $f(x) = (x - 2)(x^2 + x + 1)$

**38.** $f(x) = (x - 2)(x^2 + x + 3)$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -1 & -2 \\ & & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

**39.** $f(x) = (x - 1)(2x^2 + x + 3)$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

**40.** $f(x) = (x - 1)(3x^2 + x + 2)$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & 2 & -3 \\ & & 2 & 1 & 3 \\ \hline & 2 & 1 & 3 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 3 & -2 & 1 & -2 \\ & & 3 & 1 & 2 \\ \hline & 3 & 1 & 2 & 0 \end{array}$$

**41.** $f(x) = (x - 1)(x + 4)(x^2 + 1)$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 3 | −3 | 3 | −4 |
| | | 1 | 4 | 1 | 4 |
| | 1 | 4 | 1 | 4 | 0 |

| | | | | |
|---|---|---|---|---|
| −4 | 1 | 4 | 1 | 4 |
| | | −4 | 0 | −4 |
| | 1 | 0 | 1 | 0 |

**42.** $f(x) = (x - 3)(x + 1)(x^2 + 4)$

| | | | | | |
|---|---|---|---|---|---|
| 3 | 1 | −2 | 1 | −8 | −12 |
| | | 3 | 3 | 12 | 12 |
| | 1 | 1 | 4 | 4 | 0 |

| | | | | |
|---|---|---|---|---|
| −1 | 1 | 1 | 4 | 4 |
| | | −1 | 0 | −4 |
| | 1 | 0 | 4 | 0 |

**43.** Solve for $h$: $\frac{\pi}{3}(15h^2 - h^3)(62.5) = \frac{4}{3\pi}(125)(20)$, so that $15h^2 - h^3 = 160$. Of the three solutions (found graphically), only $h \approx 3.776$ ft makes sense in this setting.

**44.** Solve for $h$: $\frac{\pi}{3}(15h^2 - h^3)(62.5) = \frac{4}{3}\pi(125)(45)$, so that $15h^2 - h^3 = 360$. Of the three solutions (found graphically), only $h \approx 6.513$ ft makes sense in this setting.

**45.** Yes: $(x + 2)(x^2 + 1) = x^3 + 2x^2 + x + 2$ is one such polynomial. Another example is $(x + 2)^3 = x^3 + 6x^2 + 2x + 8$. Other examples can be obtained by multiplying any other quadratic with no real zeros by $(x + 2)$.

**46.** No: by the Complex Conjugate Zeros Theorem, for such a polynomial, if $2i$ is a zero, so is $-2i$.

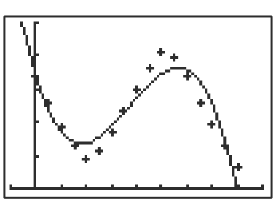
**47.** No: if all coefficients are real, $1 - 2i$ and $1 + i$ must also be zeros, giving 5 zeros for a degree 4 polynomial.

**48.** Yes: $f(x) = (x - 1 - 3i)(x - 1 + 3i)(x - 1 - i)(x - 1 + i) = x^4 - 4x^3 + 16x^2 - 24x + 20$ is one such polynomial; all other examples would be multiples of this polynomial.

**49.** $f(x)$ must have the form $a(x - 3)(x + 1)(x - 2 + i)(x - 2 - i)$; since $f(0) = a(-3)(1)(-2 + i)(-2 - i) = -15a = 30$, we know that $a = -2$. Multiplied out, this gives $f(x) = -2x^4 + 12x^3 - 20x^2 - 4x + 30$.

**50.** $f(x)$ must have the form $a(x - 1 - 2i)(x - 1 + 2i)(x - 1 - i)(x - 1 + i)$; since $f(0) = a(-1 - 2i)(-1 + 2i)(-1 - i)(-1 + i)$ $= a(5)(2) = 10a = 20$, we know that $a = 2$. Multiplied out, this gives $f(x) = 2x^4 - 8x^3 + 22x^2 - 28x + 20$.

**51.** **(a)** The model is $D \approx -0.0820t^3 + 0.9162t^2 - 2.5126t + 3.3779$.

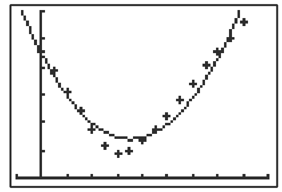


[−1, 9] by [0, 5]

**(b)** Sally walks toward the detector, turns and walks away (or walks backward), then walks toward the detector again.

**(c)** The model "changes direction" at $t \approx 1.81$ sec ($D \approx 1.35$ m) and $t \approx 5.64$ sec (when $D \approx 3.65$ m).

**52.** **(a)** $D \approx 0.2434t^2 - 1.7159t + 4.4241$



[−1, 9] by [0, 6]

**(b)** Jacob walks toward the detector, then turns and walks away (or walks backward).

**(c)** The model "changes direction" at $t \approx 3.52$ (when $D \approx 1.40$ m).

**53.** False. Complex, nonreal solutions always come in conjugate pairs, so that if $1 - 2i$ is a zero, then $1 + 2i$ must also be a zero.

**54.** False. All three zeros could be real. For instance, the polynomial $f(x) = x(x - 1)(x - 2) = x^3 - 3x^2 + 2x$ has degree 3, real coefficients, and no non-real zeros. (The zeros are 0, 1, and 2.)

**55.** Both the sum and the product of two complex conjugates are real numbers, and the absolute value of a complex number is always real. The square of a complex number, on the other hand, need not be real. The answer is E.

**56.** Allowing for multiplicities other than 1, then, the polynomial can have anywhere from 1 to 5 distinct real zeros. But it cannot have no real zeros at all. The answer is A.

**57.** Because the complex, non-real zeros of a real-coefficient polynomial always come in conjugate pairs, a polynomial of degree 5 can have either 0, 2, or 4 non-real zeros. The answer is C.

**58.** A polynomial with real coefficients can never have an odd number of non-real complex zeros. The answer is E.

**59.** **(a)**

| Power | Real Part | Imaginary Part |
|---|---|---|
| 7 | 8 | −8 |
| 8 | 16 | 0 |
| 9 | 16 | 16 |
| 10 | 0 | 32 |

**(b)** $(1 + i)^7 = 8 - 8i$
$(1 + i)^8 = 16$
$(1 + i)^9 = 16 + 16i$
$(1 + i)^{10} = 32i$

**(c)** Reconcile as needed.

**60.** **(a)** $(a + bi)(a + bi) = a^2 + 2abi + bi^2$
$= a^2 + 2abi - b^2$.

**(b)** $a^2 - b^2 = 0$

$2abi = i$, so $2ab = 1$.

**(c)** From (1), we have:
$a^2 - b^2 = 0$
$(a + b)(a - b) = 0$
$a = -b, a = b$.

Substituting into (2), we find:

$a = b$: $2ab = 1$ $\qquad$ $a = -b$: $2ab = 1$

$2b^2 = 1$ $\qquad$ $-2b^2 = 1$

$b^2 = \frac{1}{2}$ $\qquad$ $b^2 = -\frac{1}{2}$

$b = \pm\sqrt{\frac{1}{2}}$ $\qquad$ $b = \pm\sqrt{-\frac{1}{2}}$

$b = \pm i\sqrt{\frac{1}{2}}$.

Since $a$ and $b$ must be real, we have

$$(a, b) = \left\{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)\right\}.$$

**(d)** Checking $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ first

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2(1 + i)^2$$
$$= \frac{1}{2}(1 + 2i + i^2) = \frac{1}{2}(1 - 1 + 2i) = \frac{2i}{2} = i.$$

Checking $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$$\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)^2 = \left(-\frac{\sqrt{2}}{2}\right)(1 + i)^2$$
$$= \frac{1}{2}(1 + 2i + i^2) = \frac{1}{2}(1 - 1 + 2i) = \frac{2i}{2} = i.$$

**(e)** The two square roots of $i$ are:

$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$ and $\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

**61.** $f(i) = i^3 - i(i)^2 + 2i(i) + 2 = -i + i - 2 + 2 = 0$. One can also take the last number of the bottom row from synthetic division.

**62.** $f(-2i) = (-2i)^3 - (2 - i)(-2i)^2 + (2 - 2i)(-2i) - 4$
$= 8i + (2 - i)(4) - (2 - 2i)(2i) - 4 =$
$8i + 8 - 4i - 4i - 4 - 4 = 0$.
One can also take the last number of the bottom row from synthetic division.

**63.** Synthetic division shows that $f(i) = 0$ (the remainder), and at the same time gives $f(x) \div (x - i) = x^2 + 3x - i = h(x)$, so $f(x) = (x - i)(x^2 + 3x - i)$.

$$\begin{array}{r|rrrr} i & 1 & 3-i & -4i & -1 \\ & & i & 3i & 1 \\ \hline & 1 & 3 & -i & 0 \end{array}$$

**64.** Synthetic division shows that $f(1 + i) = 0$ (the remainder), and at the same time gives $f(x) \div (x - 1 - i) = x^2 + 1 = h(x)$, so $f(x) = (x - 1 - i)(x^2 + 1)$.

$$\begin{array}{r|rrrr} 1+i & 1 & -1-i & 1 & -1-i \\ & & 1+i & 0 & 1+i \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

**65.** From graphing (or the Rational Zeros Test), we expect $x = 2$ to be a zero of $f(x) = x^3 - 8$. Indeed, $f(2) = 8 - 8 = 0$. So, $x = 2$ *is* a zero of $f(x)$. Using synthetic division we obtain:

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$f(x) = (x - 2)(x^2 + 2x + 4)$. We then apply the quadratic formula to find that the cube roots of $x^3 - 8$ are $2, -1 + \sqrt{3}i$, and $-1 - \sqrt{3}i$.

**66.** From graphing (or the Rational Zeros Test), we expect $x = -4$ to be a zero of $f(x) = x^3 + 64$. Indeed $f(-4) = -64 + 64 = 0$, so $x = -4$ is a zero of $f(x)$. Using synthetic division, we obtain:

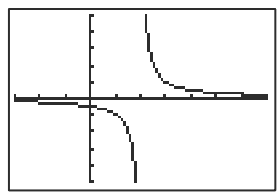
$$\begin{array}{r|rrrr} -4 & 1 & 0 & 0 & 64 \\ & & -4 & 16 & -64 \\ \hline & 1 & -4 & 16 & 0 \end{array}$$

$f(x) = (x + 4)(x^2 - 4x + 16)$. We then apply the quadratic formula to find that the cube roots of $x^3 + 64$ are $-4, 2 + 2\sqrt{3}i$, and $2 - 2\sqrt{3}i$.

## ■ Section 2.6 Graphs of Rational Functions

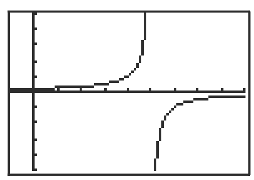
### Exploration 1

**1.** $g(x) = \frac{1}{x - 2}$



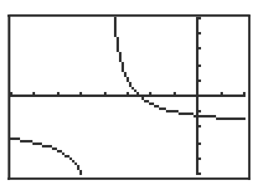
$[-3, 7]$ by $[-5, 5]$

**2.** $h(x) = -\frac{1}{x - 5}$



$[-1, 9]$ by $[-5, 5]$

**3.** $k(x) = \frac{3}{x + 4} - 2$



$[-8, 2]$ by $[-5, 5]$

### Quick Review 2.6

**1.** $f(x) = (2x - 1)(x + 3) \Rightarrow x = -3$ or $x = \frac{1}{2}$

**2.** $f(x) = (3x + 4)(x - 2) \Rightarrow x = -\frac{4}{3}$ or $x = 2$

**3.** $g(x) = (x + 2)(x - 2) \Rightarrow x = \pm 2$

**4.** $g(x) = (x + 1)(x - 1) \Rightarrow x = \pm 1$

**5.** $h(x) = (x - 1)(x^2 + x + 1) \Rightarrow x = 1$

**6.** $h(x) = (x - i)(x + i) \Rightarrow$ no real zeros

**7.**
$$\begin{array}{r} 2 \\ x - 3\overline{)2x + 1} \\ \underline{2x - 6} \\ 7 \end{array}$$
Quotient: 2, Remainder: 7

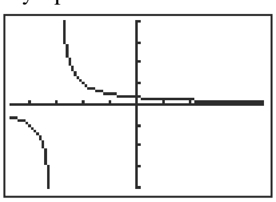
**8.**
$$\begin{array}{r} 2 \\ 2x - 1\overline{)4x + 3} \\ \underline{4x - 2} \\ 5 \end{array}$$
Quotient: 2, Remainder: 5

**9.**
$$\begin{array}{r} 3 \\ 2x\overline{)3x - 5} \\ \underline{3x\phantom{ - 5}} \\ -5 \end{array}$$
Quotient: 3, Remainder: −5

**10.**
$$\begin{array}{r} \frac{5}{2} \\ 2x\overline{)5x - 1} \\ \underline{5x\phantom{ - 1}} \\ -1 \end{array}$$
Quotient: $\frac{5}{2}$, Remainder: –1

### Section 2.6 Exercises

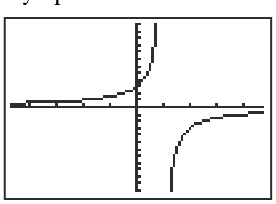
**1.** The domain of $f(x) = 1/(x + 3)$ is all real numbers $x \neq -3$. The graph suggests that $f(x)$ has a vertical asymptote at $x = -3$.



[–4.7, 4.7] by [–4, 4]

As $x$ approaches −3 from the left, the values of $f(x)$ decrease without bound. As $x$ approaches −3 from the right, the values of $f(x)$ increase without bound. That is, $\lim_{x\to-3^-} f(x) = \infty$ and $\lim_{x\to-3^+} f(x) = \infty$.

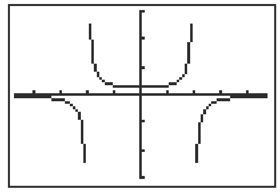
**2.** The domain of $f(x) = -3/(x - 1)$ is all real numbers $x \neq 1$. The graph suggests that $f(x)$ has a vertical asymptote at $x = 1$.



[–4.7, 4.7] by [–12, 12]

As $x$ approaches 1 from the left, the values of $f(x)$ increase without bound. As $x$ approaches 1 from the right, the values of $f(x)$ decrease without bound. That is, $\lim_{x\to1^-} f(x) = \infty$ and $\lim_{x\to1^+} f(x) = -\infty$.

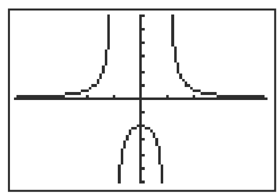
**3.** The domain of $f(x) = -1/(x^2 - 4)$ is all real numbers $x \neq -2, 2$. The graph suggests that $f(x)$ has vertical asymptotes at $x = -2$ and $x = 2$.



[–4.7, 4.7] by [–3, 3]

As $x$ approaches −2 from the left, the values of $f(x)$ decrease without bound, and as $x$ approaches −2 from the right, the values of $f(x)$ increase without bound. As $x$ approaches 2 from the left, the values of $f(x)$ increase without bound, and as $x$ approaches 2 from the right, the values of $f(x)$ decrease without bound. That is, $\lim_{x\to-2^-} f(x) = -\infty$, $\lim_{x\to-2^+} f(x) = \infty$, $\lim_{x\to2^-} f(x) = \infty$, and $\lim_{x\to2^+} f(x) = -\infty$.

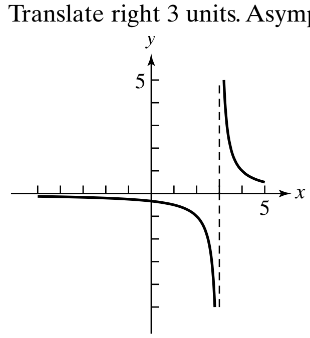
**4.** The domain of $f(x) = 2/(x^2 - 1)$ is all real numbers $x \neq -1, 1$. The graph suggests that $f(x)$ has vertical asymptotes at $x = -1$ and $x = 1$.



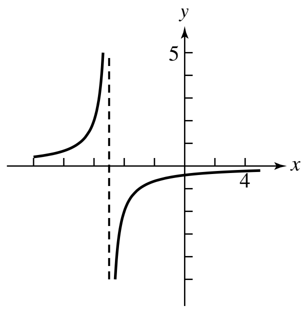
[–4.7, 4.7] by [–6, 6]

As $x$ approaches −1 from the left, the values of $f(x)$ increase without bound, and as $x$ approaches −1 from the right, the values of $f(x)$ decrease without bound. As $x$ approaches 1 from the left, the values of $f(x)$ decrease without bound, and as $x$ approaches 1 from the right, the values of $f(x)$ increase without bound. That is, $\lim_{x\to-1^-} f(x) = \infty$, $\lim_{x\to-1^+} f(x) = -\infty$, $\lim_{x\to1^-} f(x) = -\infty$, and $\lim_{x\to1^+} f(x) = \infty$.

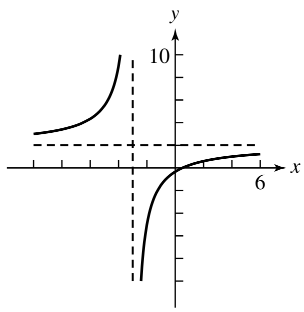
**5.** Translate right 3 units. Asymptotes: $x = 3$, $y = 0$.



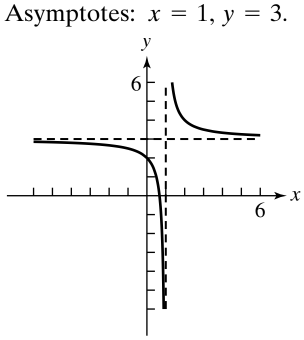
**6.** Translate left 5 units, reflect across $x$-axis, vertically stretch by 2. Asymptotes: $x = -5$, $y = 0$.



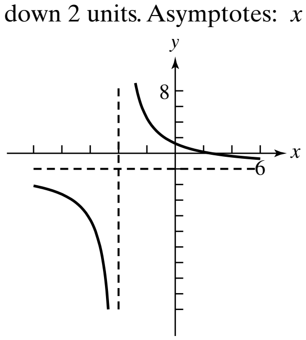

**7.** Translate left 3 units, reflect across $x$-axis, vertically stretch by 7, translate up 2 units.
Asymptotes: $x = -3$, $y = 2$.



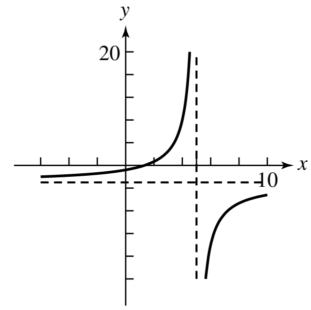

**8.** Translate right 1 unit, translate up 3 units.
Asymptotes: $x = 1$, $y = 3$.




**9.** Translate left 4 units, vertically stretch by 13, translate down 2 units. Asymptotes: $x = -4$, $y = -2$.




**10.** Translate right 5 units, vertically stretch by 11, reflect across $x$-axis, translate down 3 units. Asymptotes: $x = 5$, $y = -3$.




**11.** $\lim_{x\to 3^-} f(x) = \infty$

**12.** $\lim_{x\to 3^+} f(x) = -\infty$

**13.** $\lim_{x\to \infty} f(x) = 0$

**14.** $\lim_{x\to -\infty} f(x) = 0$

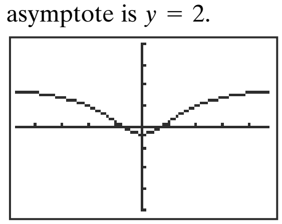
**15.** $\lim_{x\to -3^+} f(x) = \infty$

**16.** $\lim_{x\to -3^-} f(x) = -\infty$

**17.** $\lim_{x\to -\infty} f(x) = 5$

**18.** $\lim_{x\to \infty} f(x) = 5$

**19.** The graph of $f(x) = (2x^2 - 1)/(x^2 + 3)$ suggests that there are no vertical asymptotes and that the horizontal asymptote is $y = 2$.



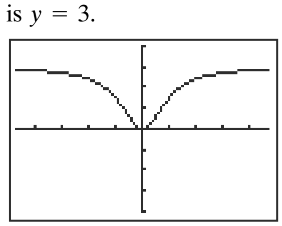
[–4.7, 4.7] by [–4, 4]

The domain of $f(x)$ is all real numbers, so there are indeed no vertical asymptotes. Using polynomial long division, we find that

$$f(x) = \frac{2x^2 - 1}{x^2 + 3} = 2 - \frac{7}{x^2 + 3}.$$

When the value of $|x|$ is large, the denominator $x^2 + 3$ is a large positive number, and $7/(x^2 + 3)$ is a small positive number, getting closer to zero as $|x|$ increases. Therefore, $\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = 2$, so $y = 2$ is indeed a horizontal asymptote.

**20.** The graph of $f(x) = 3x^2/(x^2 + 1)$ suggests that there are no vertical asymptotes and that the horizontal asymptote is $y = 3$.



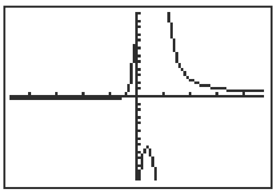
[–4.7, 4.7] by [–4, 4]

The domain of $f(x)$ is all real numbers, so there are indeed no vertical asymptotes. Using polynomial long division, we find that

$$f(x) = \frac{3x^2}{x^2 + 1} = 3 - \frac{3}{x^2 + 1}$$

When the value of $|x|$ is large, the denominator $x^2 + 1$ is a large positive number, and $3/(x^2 + 1)$ is a small positive number, getting closer to zero as $|x|$ increases. Therefore, $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 3$, so $y = 3$ is indeed a horizontal asymptote.

**21.** The graph of $f(x) = (2x + 1)/(x^2 - x)$ suggests that there are vertical asymptotes at $x = 0$ and $x = 1$, with $\lim_{x\to 0^-} f(x) = \infty$, $\lim_{x\to 0^+} f(x) = -\infty$, $\lim_{x\to 1^-} f(x) = -\infty$, and $\lim_{x\to 1^+} f(x) = \infty$, and that the horizontal asymptote is $y = 0$.



[–4.7, 4.7] by [–12, 12]

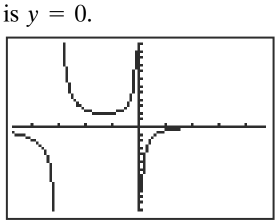
The domain of $f(x) = (2x + 1)/(x^2 - x) = (2x + 1)/[x(x - 1)]$ is all real numbers $x \neq 0, 1$, so there are indeed vertical asymptotes at $x = 0$ and $x = 1$. Rewriting one rational expression as two, we find that

$$f(x) = \frac{2x + 1}{x^2 - x} = \frac{2x}{x^{2-x}} + \frac{1}{x^{2-x}}$$

$$= \frac{2}{x - 1} + \frac{1}{x^2 - x}.$$

When the value of $|x|$ is large, both terms get close to zero. Therefore,
$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$,
so $y = 0$ is indeed a horizontal asymptote.

**22.** The graph of $f(x) = (x - 3)/(x^2 + 3x)$ suggests that there are vertical asymptotes at $x = -3$ and $x = 0$, with $\lim_{x\to -3^-} f(x) = -\infty$, $\lim_{x\to -3^+} f(x) = \infty$, $\lim_{x\to 0^-} f(x) = \infty$, and $\lim_{x\to 0^+} f(x) = -\infty$, and that the horizontal asymptote is $y = 0$.



[–4.7, 4.7] by [–4, 4]

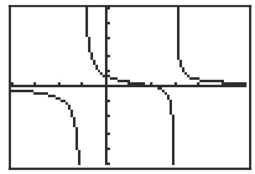
The domain of $f(x) = (x - 3)/(x^2 + 3x) = (x - 3)/[x(x + 3)]$ is all real numbers $x \neq -3, 0$, so there are indeed vertical asymptotes at $x = -3$ and $x = 0$. Rewriting one rational expression as two, we find that

$$f(x) = \frac{x - 3}{x^2 + 3x} = \frac{x}{x^2 + 3x} - \frac{3}{x^2 + 3x}$$

$$= \frac{1}{x + 3} - \frac{3}{x^2 + 3x}.$$

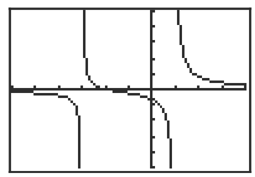
When the value of $|x|$ is large, both terms get close to zero. Therefore,
$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$,
so $y = 0$ is indeed a horizontal asymptote.

**23.** Intercepts: $\left(0, \frac{2}{3}\right)$ and $(2, 0)$. Asymptotes: $x = -1$, $x = 3$, and $y = 0$.



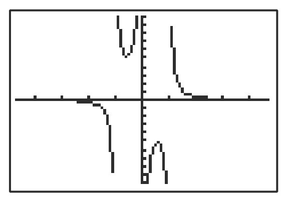
[–4, 6] by [–5, 5]

**24.** Intercepts: $\left(0, -\frac{2}{3}\right)$ and $(-2, 0)$. Asymptotes: $x = -3$, $x = 1$, and $y = 0$.



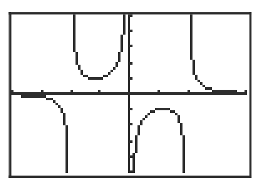
[–6, 4] by [–5, 5]

**25.** No intercepts. Asymptotes: $x = -1$, $x = 0$, $x = 1$, and $y = 0$.



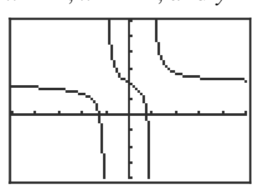
[−4.7, 4.7] by [−10, 10]

**26.** No intercepts. Asymptotes: $x = -2$, $x = 0$, $x = 2$, and $y = 0$.



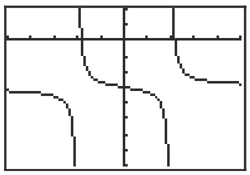
[–4, 4] by [–5, 5]

**27.** Intercepts: $(0, 2)$, $(-1.28, 0)$, and $(0.78, 0)$. Asymptotes: $x = 1$, $x = -1$, and $y = 2$.



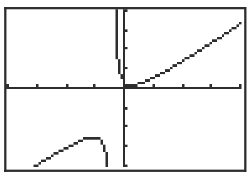
[–5, 5] by [–4, 6]

**28.** Intercepts: $(0, -3)$, $(-1.84, 0)$, and $(2.17, 0)$. Asymptotes: $x = -2$, $x = 2$, and $y = -3$.



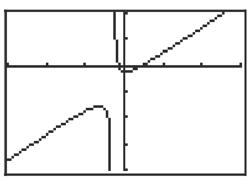
[–5, 5] by [–8, 2]

**29.** Intercept: $\left(0, \frac{3}{2}\right)$. Asymptotes: $x = -2$, $y = x - 4$.



[–20, 20] by [–20, 20]

**30.** Intercepts: $\left(0, -\frac{7}{3}\right)$, $(-1.54, 0)$, and $(4.54, 0)$.
Asymptotes: $x = -3$, $y = x - 6$.



[–30, 30] by [–40, 20]

**31.** (d); Xmin $= -2$, Xmax $= 8$, Xscl $= 1$, and Ymin $= -3$, Ymax $= 3$, Yscl $= 1$.

**32.** (b); Xmin $= -6$, Xmax $= 2$, Xscl $= 1$, and Ymin $= -3$, Ymax $= 3$, Yscl $= 1$.

**33.** (a); Xmin $= -3$, Xmax $= 5$, Xscl $= 1$, and Ymin $= -5$, Ymax $= 10$, Yscl $= 1$.

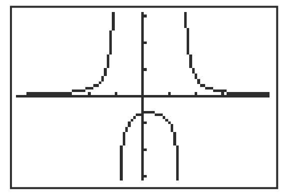
**34.** (f); Xmin $= -6$, Xmax $= 2$, Xscl $= 1$, and Ymin $= -5$, Ymax $= 5$, Yscl $= 1$.

**35.** (e); Xmin $= -2$, Xmax $= 8$, Xscl $= 1$, and Ymin $= -3$, Ymax $= 3$, Yscl $= 1$.

**36.** (c); Xmin $= -3$, Xmax $= 5$, Xscl $= 1$, and Ymin $= -3$, Ymax $= 8$, Yscl $= 1$.

**37.** For $f(x) = 2/(2x^2 - x - 3)$, the numerator is never zero, and so $f(x)$ never equals zero and the graph has no $x$-intercepts. Because $f(0) = -2/3$, the $y$-intercept is $-2/3$. The denominator factors as $2x^2 - x - 3 = (2x - 3)(x + 1)$, so there are vertical asymptotes at $x = -1$ and $x = 3/2$. And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that
$\lim_{x \to -1^-} f(x) = \infty$, $\lim_{x \to -1^+} f(x) = -\infty$, $\lim_{x \to (3/2)^-} f(x) = -\infty$, and $\lim_{x \to (3/2)^+} f(x) = \infty$.

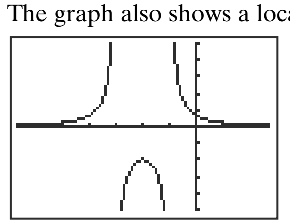
The graph also shows a local maximum of $-16/25$ at $x = 1/4$.



[–4.7, 4.7] by [–3.1, 3.1]

Intercept: $\left(0, -\frac{2}{3}\right)$
Domain: $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$
Range: $\left(-\infty, -\frac{16}{25}\right) \cup (0, \infty)$
Continuity: All $x \neq -1, \frac{3}{2}$
Increasing on $(-\infty, -1)$ and $\left(-1, \frac{1}{4}\right)$
Decreasing on $\left(\frac{1}{4}, \frac{3}{2}\right)$ and $\left(\frac{3}{2}, \infty\right)$
Not symmetric
Unbounded
Local maximum at $\left(\frac{1}{4}, -\frac{16}{25}\right)$
Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -1$ and $x = 3/2$
End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0$

**38.** For $g(x) = 2/(x^2 + 4x + 3)$, the numerator is never zero, and so $g(x)$ never equals zero and the graph has no $x$-intercepts. Because $g(0) = 2/3$, the $y$-intercept is $2/3$. The denominator factors as $x^2 + 4x + 3 = (x + 1)(x + 3)$, so there are vertical asymptotes at $x = -3$ and $x = -1$. And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that
$\lim_{x \to -3^-} g(x) = \infty$, $\lim_{x \to -3^+} g(x) = -\infty$, $\lim_{x \to -1^-} g(x) = -\infty$, and $\lim_{x \to -1^+} g(x) = \infty$.
The graph also shows a local maximum of $-2$ at $x = -2$.

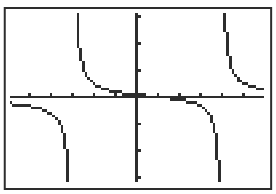


[–6.7, 2.7] by [–5, 5]

Intercept: $\left(0, \frac{2}{3}\right)$
Domain: $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$
Range: $(-\infty, -2] \cup (0, \infty)$
Continuity: All $x \neq -3, -1$
Increasing on $(-\infty, -3)$ and $(-3, -2]$
Decreasing on $[-2, -1)$ and $(-1, \infty)$
Symmetric about $x = -2$.
Unbounded

Local maximum at $(-2, -2)$
Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -3$ and $x = -1$
End behavior: $\lim_{x \to -\infty} g(x) = \lim_{x \to \infty} g(x) = 0$

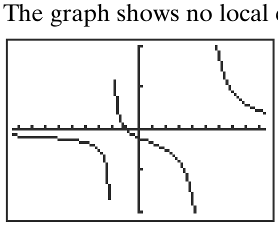
**39.** For $h(x) = (x - 1)/(x^2 - x - 12)$, the numerator is zero when $x = 1$, so the $x$-intercept of the graph is 1. Because $h(0) = 1/12$, the $y$-intercept is $1/12$.
The denominator factors as
$x^2 - x - 12 = (x + 3)(x - 4)$,
so there are vertical asymptotes at $x = -3$ and $x = 4$.
And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that
$\lim_{x \to -3^-} h(x) = -\infty$, $\lim_{x \to -3^+} h(x) = \infty$, $\lim_{x \to 4^-} h(x) = -\infty$, and $\lim_{x \to 4^+} h(x) = \infty$.
The graph shows no local extrema.


[–5.875, 5.875] by [–3.1, 3.1]

Intercepts: $\left(0, \frac{1}{12}\right)$, $(1, 0)$
Domain: $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$
Range: $(-\infty, \infty)$
Continuity: All $x \neq -3, 4$
Decreasing on $(-\infty, -3)$, $(-3, 4)$, and $(4, \infty)$
Not symmetric
Unbounded
No local extrema
Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -3$ and $x = 4$
End behavior: $\lim_{x \to -\infty} h(x) = \lim_{x \to \infty} h(x) = 0$

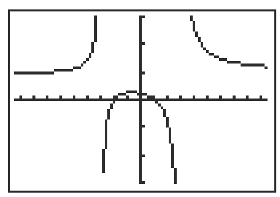
**40.** For $k(x) = (x + 1)/(x^2 - 3x - 10)$, the numerator is zero when $x = -1$, so the $x$-intercept of the graph is $-1$. Because $k(0) = -1/10$, the $y$-intercept is $-1/10$. The denominator factors as
$x^2 - 3x - 10 = (x + 2)(x - 5)$,
so there are vertical asymptotes at $x = -2$ and $x = 5$.

And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that
$\lim_{x \to -2^-} k(x) = -\infty$, $\lim_{x \to 2^+} k(x) = \infty$,
$\lim_{x \to 5^-} k(x) = -\infty$, and $\lim_{x \to 5^+} k(x) = \infty$.
The graph shows no local extrema.


[–9.4, 9.4] by [–1, 1]

Intercepts: $(-1, 0)$, $(0, -0.1)$
Domain: $(-\infty, -2) \cup (-2, 5) \cup (5, \infty)$
Range: $(-\infty, \infty)$
Continuity: All $x \neq -2, 5$
Decreasing on $(-\infty, -2)$, $(-2, 5)$, and $(5, \infty)$
Not symmetric
Unbounded
No local extrema
Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -2$ and $x = 5$
End behavior: $\lim_{x \to -\infty} k(x) = \lim_{x \to \infty} k(x) = 0$

**41.** For $f(x) = (x^2 + x - 2)/(x^2 - 9)$, the numerator factors as
$x^2 + x - 2 = (x + 2)(x - 1)$,
so the $x$-intercepts of the graph are $-2$ and 1. Because $f(0) = 2/9$, the $y$-intercept is $2/9$. The denominator factors as
$x^2 - 9 = (x + 3)(x - 3)$,
so there are vertical asymptotes at $x = -3$ and $x = 3$. And because the degree of the numerator equals the degree of the denominator with a ratio of leading terms that equals 1, the horizontal asymptote is $y = 1$. The graph supports this information and allows us to conclude that
$\lim_{x \to -3^-} f(x) = \infty$, $\lim_{x \to -3^+} f(x) = -\infty$, $\lim_{x \to 3^-} f(x) = -\infty$, and $\lim_{x \to 3^+} f(x) = \infty$.
The graph also shows a local maximum of about 0.260 at about $x = -0.675$.

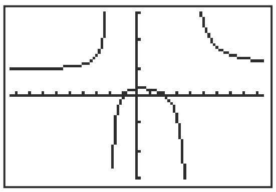

[–9.4, 9.4] by [–3, 3]

Intercepts: $(-2, 0)$, $(1, 0)$, $\left(0, \frac{2}{9}\right)$
Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
Range: $(-\infty, 0.260] \cup (1, \infty)$
Continuity: All $x \neq -3, 3$
Increasing on $(-\infty, -3)$ and $(-3, -0.675]$
Decreasing on $[-0.675, 3)$ and $(3, \infty)$
Not symmetric
Unbounded
Local maximum at about $(-0.675, 0.260)$
Horizontal asymptote: $y = 1$
Vertical asymptotes: $x = -3$ and $x = 3$
End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 1$

**42.** For $g(x) = (x^2 - x - 2)/(x^2 - 2x - 8)$, the numerator factors as
$x^2 - x - 2 = (x + 1)(x - 2)$,
so the $x$-intercepts of the graph are $-1$ and 2. Because $g(0) = 1/4$, the $y$-intercept is $1/4$. The denominator factors as
$x^2 - 2x - 8 = (x + 2)(x - 4)$,
so there are vertical asymptotes at $x = -2$ and $x = 4$. And because the degree of the numerator equals the degree of the denominator with a ratio of leading terms that equals 1,

the horizontal asymptote is $y = 1$. The graph supports this information and allows us to conclude that $\lim_{x\to -2^-} g(x) = \infty$, $\lim_{x\to -2^+} g(x) = -\infty$, $\lim_{x\to 4-} g(x) = -\infty$, and $\lim_{x\to 4^+} g(x) = \infty$.

The graph also shows a local maximum of about 0.260 at about $x = 0.324$.



[–9.4, 9.4] by [–3, 3]

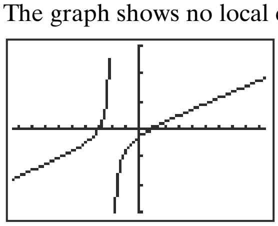
Intercepts: $(-1, 0)$, $(2, 0)$, $\left(0, \frac{1}{4}\right)$
Domain: $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$
Range: $(-\infty, 0.260] \cup (1, \infty)$
Continuity: All $x \neq -2, 4$
Increasing on $(-\infty, -2)$ and $(-2, 0.324]$
Decreasing on $[0.324, 4)$ and $(4, \infty)$
Not symmetric.
Unbounded.
Local maximum at about $(0.324, 0.260)$
Horizontal asymptote: $y = 1$
Vertical asymptotes: $x = -2$ and $x = 4$
End behavior: $\lim_{x\to -\infty} g(x) = \lim_{x\to\infty} g(x) = 1$

**43.** For $h(x) = (x^2 + 2x - 3)/(x + 2)$, the numerator factors as
$x^2 + 2x - 3 = (x + 3)(x - 1)$,
so the $x$-intercepts of the graph are $-3$ and 1. Because $h(0) = -3/2$, the $y$-intercept is $-3/2$. The denominator is zero when $x = -2$, so there is a vertical asymptote at $x = -2$. Using long division, we rewrite $h(x)$ as

$$h(x) = \frac{x^2 + 2x - 2}{x + 2} = x - \frac{2}{x + 2},$$

so the end-behavior asymptote of $h(x)$ is $y = x$. The graph supports this information and allows us to conclude that $\lim_{x\to -2^-} h(x) = \infty$ and $\lim_{x\to -2^+} h(x) = -\infty$.

The graph shows no local extrema.



[–9.4, 9.4] by [–15, 15]

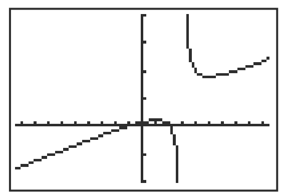
Intercepts: $(-3, 0)$, $(1, 0)$, $\left(0, -\frac{3}{2}\right)$
Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, \infty)$
Continuity: All $x \neq -2$
Increasing on $(-\infty, -2)$ and $(-2, \infty)$
Not symmetric.
Unbounded.
No local extrema.
Horizontal asymptote: None
Vertical asymptote: $x = -2$
Slant asymptote: $y = x$
End behavior: $\lim_{x\to -\infty} h(x) = -\infty$ and $\lim_{x\to\infty} h(x) = \infty$.

**44.** For $k(x) = (x^2 - x - 2)/(x - 3)$, the numerator factors as $x^2 - x - 2 = (x + 1)(x - 2)$, so the $x$-intercepts of the graph are $-1$ and 2. Because $k(0) = 2/3$, the $y$-intercept is $2/3$. The denominator is zero when $x = 3$, so there is a vertical asymptote at $x = 3$. Using long division, we rewrite $k(x)$ as

$$h(x) = \frac{x^2 - x - 2}{x - 3} = x + 2 + \frac{4}{x - 3},$$

so the end-behavior asymptote of $k(x)$ is $y = x + 2$. The graph supports this information and allows us to conclude that $\lim_{x\to 3^-} k(x) = -\infty$ and $\lim_{x\to 3^+} k(x) = \infty$.

The graph shows a local maximum of 1 at $x = 1$ and a local minimum of 9 at $x = 5$.



[–9.4, 9.4] by [–10, 20]

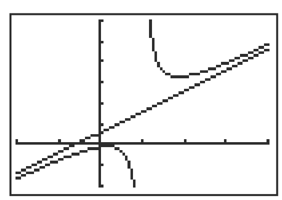
Intercepts: $(-1, 0)$, $(2, 0)$, $\left(0, \frac{2}{3}\right)$
Domain: $(-\infty, 3) \cup (3, \infty)$
Range: $(-\infty, 1] \cup [9, \infty)$
Continuity: All $x \neq 3$
Increasing on $(-\infty, 1]$ and $[5, \infty)$
Decreasing on $[1, 3)$ and $(3, 5]$
Not symmetric.
Unbounded.
Local maximum at $(1, 1)$; local minimum at $(5, 9)$
Horizontal asymptote: None
Vertical asymptote: $x = 3$
Slant asymptote: $y = x + 2$
End behavior: $\lim_{x\to -\infty} k(x) = -\infty$ and $\lim_{x\to\infty} k(x) = \infty$.

**45.** Divide $x^2 - 2x - 3$ by $x - 5$ to show that

$$f(x) = \frac{x^2 - 2x + 3}{x - 5} = x + 3 + \frac{18}{x - 5}.$$

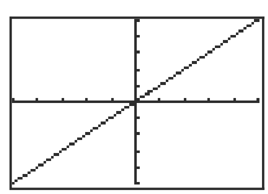
The end-behavior asymptote of $f(x)$ is $y = x + 3$.

**(a)**



[–10, 20] by [–10, 30]

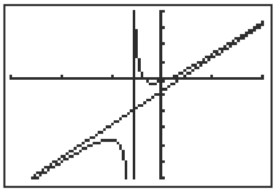
**(b)**



[–500, 500] by [–500, 500]

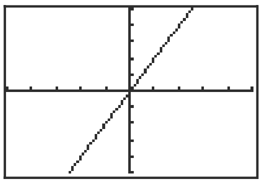
**46.** Divide $2x^2 + 2x - 3$ by $x + 3$ to show that
$f(x) = \dfrac{2x^2 + 2x - 3}{x + 3} = 2x - 4 + \dfrac{9}{x + 3}$.
The end-behavior asymptote of $f(x)$ is $y = 2x - 4$.

**(a)**



[–15, 10] by [–30, 20]

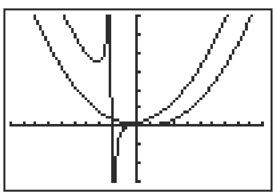
**(b)**



[–500, 500] by [–500, 500]

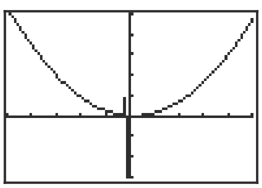
**47.** Divide $x^3 - x^2 + 1$ by $x + 2$ to show that
$f(x) = \dfrac{x^3 - x^2 + 1}{x + 2} = x^2 - 3x + 6 - \dfrac{11}{x + 2}$.
The end-behavior asymptote of $f(x)$ is $y = x^2 - 3x + 6$.

**(a)**



[–10, 10] by [–30, 60]

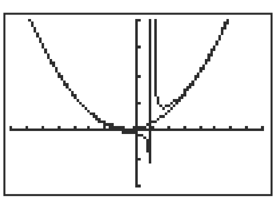
**(b)**



[–50, 50] by [–1500, 2500]

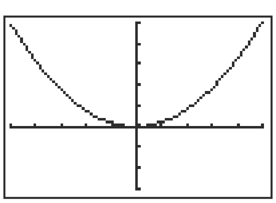
**48.** Divide $x^3 + 1$ by $x - 1$ to show that
$f(x) = \dfrac{x^3 + 1}{x - 1} = x^2 + x + 1 + \dfrac{2}{x - 1}$.
The end-behavior asymptote of $f(x)$ is $y = x^2 + x + 1$.

**(a)**



[–8, 8] by [–20, 40]

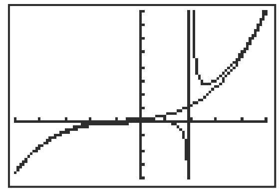
**(b)**



[–50, 50] by [–1500, 2500]

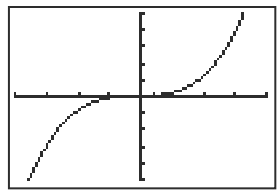
**49.** Divide $x^4 - 2x + 1$ by $x - 2$ to show that
$f(x) = \dfrac{x^4 - 2x + 1}{x - 2} = x^3 + 2x^2 + 4x + 6 + \dfrac{13}{x - 2}$.
The end-behavior asymptote of $f(x)$ is
$y = x^3 + 2x^2 + 4x + 6$.

**(a)**



[–5, 5] by [–100, 200]

**(b)**

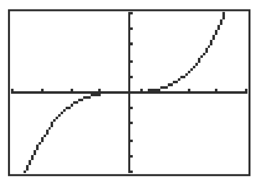


[–20, 20] by [–5000, 5000]

**50.** Divide $x^5 + 1$ by $x^2 + 1$ to show that
$f(x) = \dfrac{x^5 + 1}{x^2 + 1} = x^3 - x + \dfrac{x + 1}{x^2 + 1}$.
The end-behavior asymptote of $f(x)$ is $y = x^3 - x$.

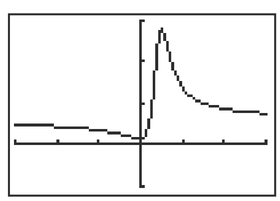
**(a)** There are no vertical asymptotes, since the denominator $x^2 + 1$ is never zero.

**(b)**



[–20, 20] by [–5000, 5000]

**51.** For $f(x) = (3x^2 - 2x + 4)/(x^2 - 4x + 5)$, the numerator is never zero, and so $f(x)$ never equals zero and the graph has no $x$-intercepts. Because $f(0) = 4/5$, the $y$-intercept is $4/5$. The denominator is never zero, and so there are no vertical asymptotes. And because the degree of the numerator equals the degree of the denominator with a ratio of leading terms that equals 3, the horizontal asymptote is $y = 3$. The graph supports this information. The graph also shows a local maximum of about 14.227 at about $x = 2.445$ and a local minimum of about 0.773 at about $x = -0.245$.

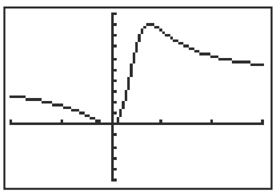


[–15, 15] by [–5, 15]

Intercept: $\left(0, \dfrac{4}{5}\right)$
Domain: $(-\infty, \infty)$
Range: $[0.773, 14.227]$
Continuity: $(-\infty, \infty)$
Increasing on $[-0.245, 2.445]$

Decreasing on $(-\infty, -0.245]$, $[2.445, \infty)$
Not symmetric.
Bounded.
Local maximum at $(2.445, 14.227)$; local minimum at $(-0.245, 0.773)$
Horizontal asymptote: $y = 3$
No vertical asymptotes.
End behavior: $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 3$

**52.** For $g(x) = (4x^2 + 2x)/(x^2 - 4x + 8)$, the numerator factors as
$4x^2 + 2x = 2x(2x + 1)$,
so the $x$-intercepts of the graph are $-1/2$ and 0. Because $g(0) = 0$, the $y$-intercept is 0. The denominator is never zero, and so there are no vertical asymptotes. And because the degree of the numerator equals the degree of the denominator with a ratio of leading terms that equals 4, the horizontal asymptote is $y = 4$. The graph supports this information. The graph also shows a local maximum of about 9.028 at about $x = 3.790$ and a local minimum of about $-0.028$ at about $x = -0.235$.



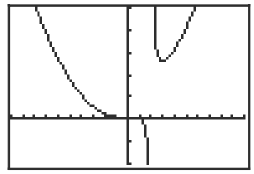
[–10, 15] by [–5, 10]

Intercepts: $\left(-\frac{1}{2}, 0\right)$, $(0, 0)$
Domain: $(-\infty, \infty)$
Range: $[-0.028, 9.028]$
Continuity: $(-\infty, \infty)$
Increasing on $[-0.235, 3.790]$
Decreasing on $(-\infty, -0.235]$, $[3.790, \infty)$.
Not symmetric.
Bounded.
Local maximum at $(3.790, 9.028)$; local minimum at $(-0.235, -0.028)$
Horizontal asymptote: $y = 4$
No vertical asymptotes.
End behavior: $\lim_{x\to-\infty} g(x) = \lim_{x\to\infty} g(x) = 4$

**53.** For $h(x) = (x^3 - 1)/(x - 2)$, the numerator factors as
$x^3 - 1 = (x - 1)(x^2 + x + 1)$,
so the $x$-intercept of the graph is 1. The $y$-intercept is $h(0) = 1/2$. The denominator is zero when $x = 2$, so the vertical asymptote is $x = 2$. Because we can rewrite $h(x)$ as

$$h(x) = \frac{x^3 - 1}{x - 2} = x^2 + 2x + 4 + \frac{7}{x - 2},$$

we know that the end-behavior asymptote is $y = x^2 + 2x + 4$. The graph supports this information and allows us to conclude that
$\lim_{x\to2^-} h(x) = -\infty$, $\lim_{x\to2^+} h(x) = \infty$.
The graph also shows a local maximum of about 0.586 at about $x = 0.442$, a local minimum of about 0.443 at about $x = -0.384$, and another local minimum of about 25.970 at about $x = 2.942$.



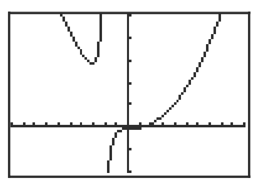
[–10, 10] by [–20, 50]

Intercepts: $(1, 0)$, $\left(0, \frac{1}{2}\right)$
Domain: $(-\infty, 2) \cup (2, \infty)$
Range: $(-\infty, \infty)$
Continuity: All real $x \neq 2$
Increasing on $[-0.384, 0.442]$, $[2.942, \infty)$
Decreasing on $(-\infty, -0.384]$, $[0.442, 2)$, $(2, 2.942]$
Not symmetric.
Unbounded.
Local maximum at $(0.442, 0.586)$; local minimum at $(-0.384, 0.443)$ and $(2.942, 25.970)$
No horizontal asymptote. End-behavior asymptote: $y = x^2 + 2x + 4$
Vertical asymptote: $x = 2$
End behavior: $\lim_{x\to-\infty} h(x) = \lim_{x\to\infty} h(x) = \infty$

**54.** For $k(x) = (x^3 - 2)/(x + 2)$, the numerator is zero when $x = \sqrt[3]{2}$, so the $x$-intercept of the graph is $\sqrt[3]{2}$. The $y$-intercept is $k(0) = -1$. The denominator is zero when $x = -2$, so the vertical asymptote is $x = -2$. Because we can rewrite $k(x)$ as

$$k(x) = \frac{x^3 - 2}{x + 2} = x^2 - 2x + 4 - \frac{10}{x - 2},$$

we know that the end-behavior asymptote is $y = x^2 - 2x + 4$. The graph supports this information and allows us to conclude that
$\lim_{x\to-2^-} k(x) = \infty$, $\lim_{x\to-2^+} k(x) = -\infty$.
The graph also shows a local minimum of about 28.901 at about $x = -3.104$.



[–10, 10] by [–20, 50]

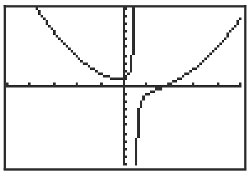
Intercepts: $(\sqrt[3]{2}, 0)$, $(0, -1)$
Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, \infty)$
Continuity: All real $x \neq -2$
Increasing on $[-3.104, -2)$, $(-2, \infty)$
Decreasing on $(-\infty, -3.104]$
Not symmetric.
Unbounded.
Local minimum at $(-3.104, 28.901)$
No horizontal asymptote. End-behavior asymptote: $y = x^2 - 2x + 4$
Vertical asymptote: $x = -2$
End behavior: $\lim_{x\to-\infty} k(x) = \lim_{x\to\infty} k(x) = \infty$

**55.** $f(x) = (x^3 - 2x^2 + x - 1)/(2x - 1)$ has only one $x$-intercept, and we can use the graph to show that it is about 1.755. The $y$-intercept is $f(0) = 1$. The denominator is zero when $x = 1/2$, so the vertical asymptote is $x = 1/2$. Because we can rewrite $f(x)$ as

$$f(x) = \frac{x^3 - 2x^2 + x - 1}{2x - 1} = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8} - \frac{7}{16(2x - 1)},$$

we know that the end-behavior asymptote is $y = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8}$. The graph supports this information and allows us to conclude that $\lim_{x\to 1/2^-} f(x) = \infty$, $\lim_{x\to 1/2^+} f(x) = -\infty$.

The graph also shows a local minimum of about 0.920 at about $x = -0.184$.



[–5, 5] by [–10, 10]

Intercepts: $(1.755, 0)$, $(0, 1)$

Domain: All $x \neq \frac{1}{2}$

Range: $(-\infty, \infty)$

Continuity: All $x \neq \frac{1}{2}$

Increasing on $[-0.184, 0.5)$, $(0.5, \infty)$

Decreasing on $(-\infty, -0.184]$

Not symmetric.

Unbounded.

Local minimum at $(-0.184, 0.920)$

No horizontal asymptote. End-behavior asymptote: $y = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8}$

Vertical asymptote: $x = \frac{1}{2}$

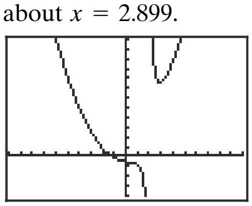
End behavior: $\lim_{x\to -\infty} f(x) = \lim_{x\to\infty} f(x) = \infty$

**56.** $g(x) = (2x^3 - 2x^2 - x + 5)/(x - 2)$ has only one $x$-intercept, and we can use the graph to show that it is about $-1.189$. The $y$-intercept is $g(0) = -5/2$. The denominator is zero when $x = 2$, so the vertical asymptote is $x = 2$. Because we can rewrite $g(x)$ as

$$g(x) = \frac{2x^3 - 2x^2 - x + 5}{x - 2} = 2x^2 + 2x + 3 + \frac{11}{x - 2},$$

we know that the end-behavior asymptote is $y = 2x^2 + 2x + 3$. The graph supports this information and allows us to conclude that $\lim_{x\to 2^-} g(x) = -\infty$, $\lim_{x\to 2^+} g(x) = \infty$.

The graph also shows a local minimum of about 37.842 at about $x = 2.899$.



[–10, 10] by [–20, 60]

Intercepts: $(-1.189, 0)$, $(0, -2.5)$

Domain: All $x \neq 2$

Range: $(-\infty, \infty)$

Continuity: All $x \neq 2$

Increasing on $[2.899, \infty)$

Decreasing on $(-\infty, 2)$, $(2, 2.899]$

Not symmetric.

Unbounded.

Local minimum at $(2.899, 37.842)$

No horizontal asymptote. End-behavior asymptote: $y = 2x^2 + 2x + 3$

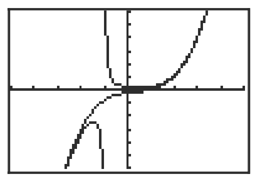
Vertical asymptote: $x = 2$

End behavior: $\lim_{x\to -\infty} g(x) = \lim_{x\to\infty} g(x) = \infty$

**57.** For $h(x) = (x^4 + 1)/(x + 1)$, the numerator is never zero, and so $h(x)$ never equals zero and the graph has no $x$-intercepts. Because $h(0) = 1$, the $y$-intercept is 1. So the one intercept is the point $(0, 1)$. The denominator is zero when $x = -1$, so $x = -1$ is a vertical asymptote. Divide $x^4 + 1$ by $x + 1$ to show that

$$h(x) = \frac{x^4 + 1}{x + 1} = x^3 - x^2 + x - 1 + \frac{2}{x + 1}.$$

The end-behavior asymptote of $h(x)$ is $y = x^3 - x^2 + x - 1$.

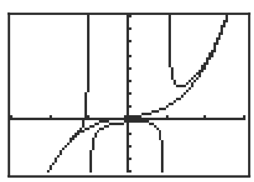


[–5, 5] by [–30, 30]

**58.** $k(x) = (2x^5 + x^2 - x + 1)/(x^2 - 1)$ has only one $x$-intercept, and we can use the graph to show that it is about $-1.108$. Because $k(0) = -1$, the $y$-intercept is $-1$. So the intercepts are $(-1.108, 0)$ and $(0, -1)$. The denominator is zero when $x = \pm 1$, so $x = -1$ and $x = 1$ are vertical asymptotes. Divide $2x^5 + x^2 - x + 1$ by $x^2 - 1$ to show that

$$k(x) = \frac{2x^5 + x^2 - x + 1}{x^2 - 1} = 2x^3 + 2x + 1 + \frac{x + 2}{x^2 - 1}.$$

The end-behavior asymptote of $k(x)$ is $y = 2x^3 + 2x + 1$.

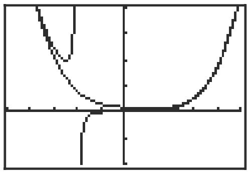


[–3, 3] by [–20, 40]

**59.** For $f(x) = (x^5 - 1)/(x + 2)$, the numerator factors as $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$, and since the second factor is never zero (as can be verified by Descartes' Rule of Signs or by graphing), the $x$-intercept of the graph is 1. Because $f(0) = -1/2$, the $y$-intercept is $-1/2$. So the intercepts are $(1, 0)$ and $(0, -1/2)$.
The denominator is zero when $x = -2$, so $x = -2$ is a vertical asymptote. Divide $x^5 - 1$ by $x + 2$ to show that

$$f(x) = \frac{x^5 - 1}{x + 2} = x^4 - 2x^3 + 4x^2 - 8x + 16 - \frac{33}{x + 2}.$$

The end-behavior asymptote of $f(x)$ is $y = x^4 - 2x^3 + 4x^2 - 8x + 16$.

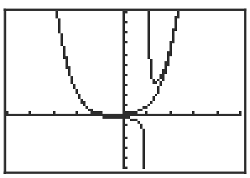


[–5, 5] by [–200, 400]

**60.** For $g(x) = (x^5 + 1)/(x - 1)$, the numerator factors as $x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$, and since the second factor is never zero (as can be verified by graphing), the $x$-intercept of the graph is $-1$. Because $g(0) = -1$, the $y$-intercept is $-1$. So the intercepts are $(-1, 0)$ and $(0, -1)$. The denominator is zero when $x = 1$, so $x = 1$ is a vertical asymptote. Divide $x^5 + 1$ by $x - 1$ to show that

$$g(x) = \frac{x^5 + 1}{x - 1} = x^4 + x^3 + x^2 + x + 1 + \frac{2}{x - 1}.$$

The end-behavior asymptote of $g(x)$ is $y = x^4 + x^3 + x^2 + x + 1$.

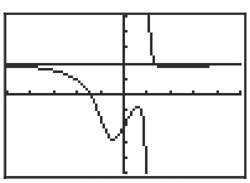


[–5, 5] by [–25, 50]

**61.** $h(x) = (2x^3 - 3x + 2)/(x^3 - 1)$ has only one $x$-intercept, and we can use the graph to show that it is about $-1.476$. Because $h(0) = -2$, the $y$-intercept is $-2$. So the intercepts are $(-1.476, 0)$ and $(0, -2)$. The denominator is zero when $x = 1$, so $x = 1$ is a vertical asymptote. Divide $2x^3 - 3x + 2$ by $x^3 - 1$ to show that

$$h(x) = \frac{2x^3 - 3x + 2}{x^3 - 1} = 2 - \frac{3x - 4}{x^3 - 1}.$$

The end-behavior asymptote of $h(x)$ is $y = 2$, a horizontal line.

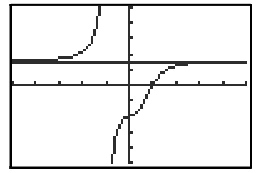


[–5, 5] by [–5, 5]

**62.** For $k(x) = (3x^3 + x - 4)/(x^3 + 1)$, the numerator factors as $3x^3 + x - 4 = (x - 1)(3x^2 + 3x + 4)$, and since the second factor is never zero (as can be verified by Descartes' Rule of Signs or by graphing), the $x$-intercept of the graph is 1. Because $k(0) = -4$, the $y$-intercept is $-4$. So the intercepts are $(1, 0)$ and $(0, -4)$. The denominator is zero when $x = -1$, so $x = -1$ is a vertical asymptote. Divide $3x^3 + x - 4$ by $x^3 + 1$ to show that

$$k(x) = \frac{3x^3 + x - 4}{x^3 + 1} = 3 + \frac{x - 7}{x^3 + 1}.$$

The end-behavior asymptote of $k(x)$ is $y = 3$, a horizontal line.



[–5, 5] by [–10, 10]

**63.** False. If the denominator is never zero, there will be no vertical asymptote. For example, $f(x) = 1/(x^2 + 1)$ is a rational function and has no vertical asymptotes.

**64.** False. A rational function is the quotient of two polynomials, and $\sqrt{x^2 + 4}$ is not a polynomial.

**65.** The excluded values are those for which $x^3 + 3x = 0$, namely 0 and $-3$. The answer is E.

**66.** $g(x)$ results from $f(x)$ by replacing $x$ with $x + 3$, which represents a shift of 3 units to the left. The answer is A.

**67.** Since $x + 5 = 0$ when $x = -5$, there is a vertical asymptote. And because $x^2/(x + 5) = x - 5 + 25/(x + 5)$, the end behavior is characterized by the slant asymptote $y = x - 5$. The answer is D.

**68.** The quotient of the leading terms is $x^4$, so the answer is E.

**69.** **(a)** No: the domain of $f$ is $(-\infty, 3) \cup (3, \infty)$; the domain of $g$ is all real numbers.

**(b)** No: while it is not defined at 3, it does not tend toward $\pm\infty$ on either side.

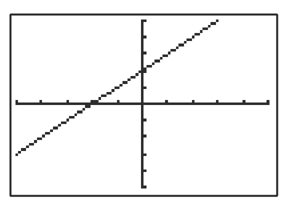
**(c)** Most grapher viewing windows do not reveal that $f$ is undefined at 3.

**(d)** Almost—but not quite; they are equal for all $x \neq 3$.

**70.** **(a)** $f(x) = \dfrac{x^2 + x - 2}{x - 1} = \dfrac{(x + 2)(x - 1)}{x - 1} = x + 2$
$= g(x)$ when $x \neq 1$

| | $f$ | $g$ |
|---|---|---|
| Asymptotes | $x = 1$ | none |
| Intercepts | $(0, 2)$ $(-2, 0)$ | $(0, 2)$ $(-2, 0)$ |
| Domain | $(-\infty, 1) \cup (1, \infty)$ | $(-\infty, \infty)$ |

The functions are identical at all points except $x = 1$, where $f$ has a discontinuity.

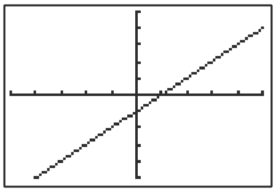


[−5, 5] by [−5, 5]

**(b)** $f(x) = \dfrac{x^2 - 1}{x + 1} = \dfrac{(x + 1)(x - 1)}{x + 1} = x - 1$
$= g(x)$ when $x \neq -1$

| | $f$ | $g$ |
|---|---|---|
| Asymptotes | $x = -1$ | none |
| Intercepts | $(0, -1)$ $(1, 0)$ | $(0, -1)$ $(1, 0)$ |
| Domain | $(-\infty, -1) \cup (-1, \infty)$ | $(-\infty, \infty)$ |

The functions are identical except at $x = -1$, where $f$ has a discontinuity.



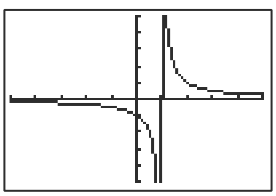
$[-5, 5]$ by $[-5, 5]$

**(c)** $f(x) = \dfrac{x^2 - 1}{x^3 - x^2 - x + 1} = \dfrac{x^2 - 1}{(x^2 - 1)(x - 1)}$

$= \dfrac{1}{x - 1} = g(x)$ when $x \neq -1$

| | $f$ | $g$ |
|---|---|---|
| Asymptotes | $x = 1, x = -1$ | $x = 1$ |
| Intercepts | $(0, -1)$ | $(0, -1)$ |
| Domain | $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ | $(-\infty, 1) \cup (1, \infty)$ |

The functions are identical except at $x = -1$, where $f(x)$ has a discontinuity.



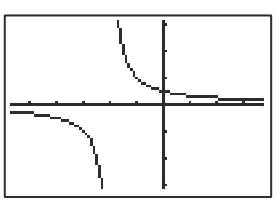
$[-5, 5]$ by $[-5, 5]$

**(d)** $f(x) = \dfrac{x - 1}{x^2 + x - 2} = \dfrac{x - 1}{(x + 2)(x - 1)} = \dfrac{1}{x + 2}$

$= g(x)$ when, $x \neq -2$

| | $f$ | $g$ |
|---|---|---|
| Asymptotes | $x = 1, x = -2$ | $x = -2$ |
| Intercepts | $\left(0, \frac{1}{2}\right)$ | $\left(0, \frac{1}{2}\right)$ |
| Domain | $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$ | $(-\infty, -2) \cup (-2, \infty)$ |

Except at $x = 1$, where $f$ has a discontinuity, the functions are identical.



$[-5.7, 3.7]$ by $[-3.1, 3.1]$

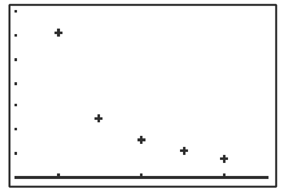
**71. (a)** The volume is $f(x) = k/x$, where $x$ is pressure and $k$ is a constant. $f(x)$ is a quotient of polynomials and hence is rational, but $f(x) = k \cdot x^{-1}$, so it is a power function with constant of variation $k$ and pressure $-1$.

**(b)** If $f(x) = kx^a$, where $a$ is a negative integer, then the power function $f$ is also a rational function.

**(c)** $V = \dfrac{k}{P}$, so $k = (2.59)(0.866) = 2.24294$.

If $P = 0.532$, then $V = \dfrac{2.24294}{0.532} \approx 4.22$ L.

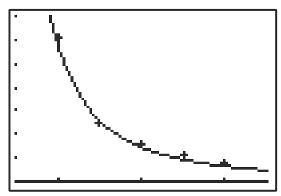
**72. (a)**



$[0.5, 3.5]$ by $[0, 7]$

**(b)** One method for determining $k$ is to find the power regression for the data points using a calculator, discussed in previous sections. By this method, we find that a good approximation of the data points is given by the curve $y \approx 5.81 \cdot x^{-1.88}$. Since $-1.88$ is very close to $-2$, we graph the curve to see if $k = 5.81$ is reasonable.

**(c)**



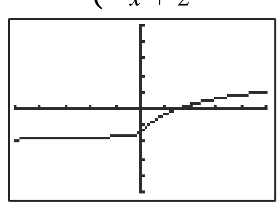
$[0.5, 3.5]$ by $[0, 7]$

**(d)** At 2.2 m, the light intensity is approximately 1.20 W/m$^2$.
At 4.4 m, the light intensity is approximately 0.30 W/m$^2$.

**73.** Horizontal asymptotes: $y = -2$ and $y = 2$.

Intercepts: $\left(0, -\frac{3}{2}\right), \left(\frac{3}{2}, 0\right)$

$$h(x) = \begin{cases} \dfrac{2x - 3}{x + 2} & x \geq 0 \\ \dfrac{2x - 3}{-x + 2} & x < 0 \end{cases}$$

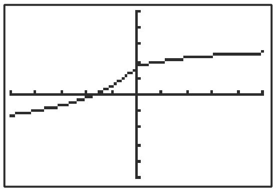


$[-5, 5]$ by $[-5, 5]$

**74.** Horizontal asymptotes: $y = \pm 3$.

Intercepts: $\left(0, \frac{5}{3}\right), \left(-\frac{5}{3}, 0\right)$

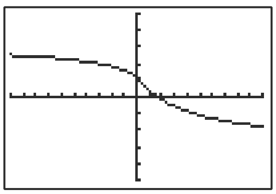
$$h(x) = \begin{cases} \dfrac{3x + 5}{x + 3} & x \geq 0 \\ \dfrac{3x + 5}{-x + 3} & x < 0 \end{cases}$$



$[-5, 5]$ by $[-5, 5]$

**75.** Horizontal asymptotes: $y = \pm 3$.
Intercepts: $\left(0, \frac{5}{4}\right), \left(\frac{5}{3}, 0\right)$

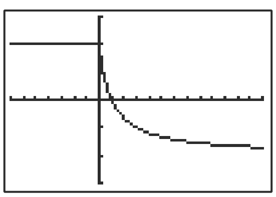
$$f(x) = \begin{cases} \dfrac{5-3x}{x+4} & x \geq 0 \\ \dfrac{5-3x}{-x+4} & x < 0 \end{cases}$$



$[-10, 10]$ by $[-5, 5]$

**76.** Horizontal asymptotes: $y = \pm 2$.
Intercepts: $(0, 2), (1, 0)$

$$f(x) = \begin{cases} \dfrac{2-2x}{x+1} & x \geq 0 \\ 2 & x < 0 \end{cases}$$



$[-7, 13]$ by $[-3, 3]$

**77.** The graph of $f$ is the graph of $y = \frac{1}{x}$ shifted horizontally $-d/c$ units, stretched vertically by a factor of $|bc - ad|/c^2$, reflected across the $x$-axis if and only if $bc - ad < 0$, and then shifted vertically by $a/c$.

**78.** Yes, domain $= (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$; range $= (-\infty, 1) \cup (1, \infty)$; continuous and decreasing on each interval within their common domain; $x$-intercepts $= (-1 \pm \sqrt{5})/2$; no $y$-intercepts; hole at $(0, 1)$; horizontal asymptote of $y = 1$; vertical asymptotes of $x = \pm 1$; neither even nor odd; unbounded; no local extrema; end behavior: $\lim_{|x|\to\infty} f(x) = \lim_{|x|\to\infty} g(x) = 1$.

## ■ Section 2.7 Solving Equations in One Variable

### Quick Review 2.7

**1.** The denominator is $x^2 + x - 12 = (x-3)(x+4)$, so the new numerator is $2x(x+4) = 2x^2 + 8x$.

**2.** The numerator is $x^2 - 1 = (x-1)(x+1)$, so the new denominator is $(x+1)(x+1) = x^2 + 2x + 1$.

**3.** The LCD is the LCM of 12, 18, and 6, namely 36.

$$\frac{5}{12} + \frac{7}{18} - \frac{5}{6} = \frac{15}{36} + \frac{14}{36} - \frac{30}{36} = -\frac{1}{36}$$

**4.** The LCD is $x(x-1)$.

$$\frac{3}{x-1} - \frac{1}{x} = \frac{3x}{x(x-1)} - \frac{x-1}{x(x-1)} = \frac{3x - x + 1}{x(x-1)} = \frac{2x+1}{x^2 - x}$$

**5.** The LCD is $(2x+1)(x-3)$.

$$\frac{x}{2x+1} - \frac{2}{x-3} = \frac{x(x-3)}{(2x+1)(x-3)} - \frac{2(2x+1)}{(2x+1)(x-3)} = \frac{x^2 - 3x - 4x - 2}{(2x+1)(x-3)} = \frac{x^2 - 7x - 2}{(2x+1)(x-3)}$$

**6.** $x^2 - 5x + 6 = (x-2)(x-3)$ and $x^2 - x - 6 = (x+2)(x-3)$, so the LCD is $(x-2)(x-3)(x+2)$.

$$\frac{x+1}{x^2 - 5x + 6} - \frac{3x + 11}{x^2 - x - 6}$$
$$= \frac{(x+1)(x+2)}{(x-2)(x-3)(x+2)} - \frac{(3x+11)(x-2)}{(x-2)(x-3)(x+2)}$$
$$= \frac{x^2 + 3x + 2 - 3x^2 - 5x + 22}{(x-2)(x-3)(x+2)}$$
$$= \frac{-2x^2 - 2x + 24}{(x-2)(x-3)(x+2)}$$
$$= \frac{-2(x-3)(x+4)}{(x-2)(x-3)(x+2)}$$
$$= \frac{-2x - 8}{(x-2)(x+2)}, x \neq 3$$

**7.** For $2x^2 - 3x - 1 = 0$: $a = 2, b = -3$, and $c = -1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)} = \frac{3 \pm \sqrt{9 - (-8)}}{4} = \frac{3 \pm \sqrt{17}}{4}$$

**8.** For $2x^2 - 5x - 1 = 0$: $a = 2, b = -5$, and $c = -1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)} = \frac{5 \pm \sqrt{25 - (-8)}}{4} = \frac{5 \pm \sqrt{33}}{4}$$

**9.** For $3x^2 + 2x - 2 = 0$: $a = 3, b = 2$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-2)}}{2(3)}$$
$$= \frac{-2 \pm \sqrt{4 - (-24)}}{6} = \frac{-2 \pm \sqrt{28}}{6}$$
$$= \frac{-2 \pm 2\sqrt{7}}{6} = \frac{-1 \pm \sqrt{7}}{3}$$

**10.** For $x^2 - 3x - 9 = 0$: $a = 1, b = -3,$ and $c = -9$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{9 - (-36)}}{2} = \frac{3 \pm \sqrt{45}}{2}$$
$$= \frac{3 \pm 3\sqrt{5}}{2}$$

## Section 2.7 Exercises

**1.** Algebraically: $\frac{x-2}{3} + \frac{x+5}{3} = \frac{1}{3}$

$(x-2) + (x+5) = 1$
$2x + 3 = 1$
$2x = -2$
$x = -1$

Numerically: For $x = -1$,

$\frac{x-2}{3} + \frac{x+5}{3} = \frac{-1-2}{3} + \frac{-1+5}{3}$
$= \frac{-3}{3} + \frac{4}{3}$
$= \frac{1}{3}$.

**2.** Algebraically: $x + 2 = \frac{15}{x}$

$x^2 + 2x = 15 \ (x \neq 0)$
$x^2 + 2x - 15 = 0$
$(x-3)(x+5) = 0$
$x - 3 = 0$ or $x + 5 = 0$
$x = 3$ or $x = -5$

Numerically: For $x = 3$,
$x + 2 = 3 + 2 = 5$ and
$\frac{15}{x} = \frac{15}{3} = 5$.
For $x = -5$,
$x + 2 = -5 + 2 = -3$ and
$\frac{15}{x} = \frac{15}{-5} = -3$.

**3.** Algebraically: $x + 5 = \frac{14}{x}$

$x^2 + 5x = 14 \quad (x \neq 0)$
$x^2 + 5x - 14 = 0$
$(x-2)(x+7) = 0$
$x - 2 = 0$ or $x + 7 = 0$
$x = 2$ or $x = -7$

Numerically: For $x = 2$,
$x + 5 = 2 + 5 = 7$ and
$\frac{14}{x} = \frac{14}{2} = 7$.
For $x = -7$,
$x + 5 = -7 + 5 = -2$ and
$\frac{14}{x} = \frac{14}{-7} = -2$.

**4.** Algebraically: $\frac{1}{x} - \frac{2}{x-3} = 4$

$(x-3) - 2x = 4x(x-3) \ (x \neq 0, 3)$
$-x - 3 = 4x^2 - 12x$
$-4x^2 + 11x - 3 = 0$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(-4)(-3)}}{2(-4)}$$
$$= \frac{-11 \pm \sqrt{73}}{-8}$$
$$x = \frac{11 + \sqrt{73}}{8} \approx 2.443 \text{ or } x = \frac{11 - \sqrt{73}}{8} \approx 0.307$$

Numerically: Use a graphing calculator to support your answers numerically.

**5.** Algebraically: $x + \frac{4x}{x-3} = \frac{12}{x-3}$

$x(x-3) + 4x = 12 \ (x \neq 3)$
$x^2 - 3x + 4x = 12$
$x^2 + x - 12 = 0$
$(x+4)(x-3) = 0$
$x + 4 = 0$ or $x - 3 = 0$
$x = -4$ or $x = 3$ — but $x = 3$ is extraneous.

Numerically: For $x = -4$,
$x + \frac{4x}{x-3} = -4 + \frac{4(-4)}{-4-3} = -4 + \frac{16}{7} = -\frac{12}{7}$ and
$\frac{12}{x-3} = \frac{12}{-4-3} = -\frac{12}{7}$.

**6.** Algebraically: $\frac{3}{x-1} + \frac{2}{x} = 8$

$3x + 2(x-1) = 8x(x-1) \quad (x \neq 0, 1)$
$5x - 2 = 8x^2 - 8x$
$-8x^2 + 13x - 2 = 0$

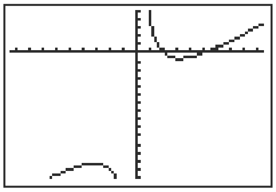
$$x = \frac{-13 \pm \sqrt{13^2 - 4(-8)(-2)}}{2(-8)}$$
$$= \frac{-13 \pm \sqrt{105}}{-16}$$
$$x = \frac{13 + \sqrt{105}}{16} \approx 1.453 \text{ or } x = \frac{13 - \sqrt{105}}{16} \approx 0.172$$

Numerically: Use a graphing calculator to support your answers numerically.

**7.** Algebraically: $x + \frac{10}{x} = 7$

$x^2 + 10 = 7x \quad (x \neq 0)$
$x^2 - 7x + 10 = 0$
$(x-2)(x-5) = 0$
$x - 2 = 0$ or $x - 5 = 0$
$x = 2$ or $x = 5$

Graphically: The graph of $f(x) = x + \frac{10}{x} - 7$ suggests that the $x$-intercepts are 2 and 5.



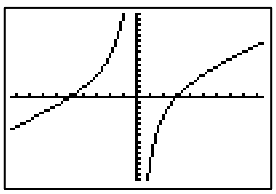
[–9.4, 9.4] by [–15, 5]

Then the solutions are $x = 2$ and $x = 5$.

**8.** Algebraically: $x + 2 = \dfrac{15}{x}$

$$x^2 + 2x = 15 \quad (x \neq 0)$$
$$x^2 + 2x - 15 = 0$$
$$(x + 5)(x - 3) = 0$$
$$x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -5 \quad \text{or} \quad x = 3$$

Graphically: The graph of $f(x) = x + 2 - \dfrac{15}{x}$ suggests that the $x$-intercepts are $-5$ and 3.



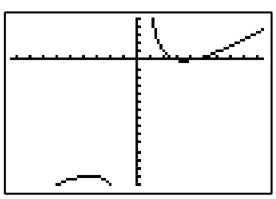
[–9.4, 9.4] by [–15, 15]

Then the solutions are $x = -5$ and $x = 3$.

**9.** Algebraically: $x + \dfrac{12}{x} = 7$

$$x^2 + 12 = 7x \quad (x \neq 0)$$
$$x^2 - 7x + 12 = 0$$
$$(x - 3)(x - 4) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = 3 \quad \text{or} \quad x = 4$$

Graphically: The graph of $f(x) = x + \dfrac{12}{x} - 7$ suggests that the $x$-intercepts are 3 and 4.



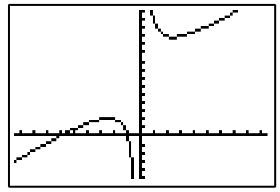
[–9.4, 9.4] by [–15, 5]

Then the solutions are $x = 3$ and $x = 4$.

**10.** Algebraically: $x + \dfrac{6}{x} = -7$

$$x^2 + 6 = -7x \quad (x \neq 0)$$
$$x^2 + 7x + 6 = 0$$
$$(x + 6)(x + 1) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -6 \quad \text{or} \quad x = -1$$

Graphically: The graph of $f(x) = x + \dfrac{6}{x} + 7$ suggests that the $x$-intercepts are $-6$ and $-1$.



[–9.4, 9.4] by [–5, 15]

Then the solutions are $x = -6$ and $x = -1$.

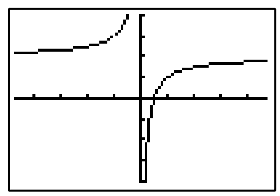
**11.** Algebraically: $2 - \dfrac{1}{x + 1} = \dfrac{1}{x^2 + x}$

$[\text{and } x^2 + x = x(x + 1)]$

$$2(x^2 + x) - x = 1 \quad (x \neq 0, -1)$$
$$2x^2 + x - 1 = 0$$
$$(2x - 1)(x + 1) = 0$$
$$2x - 1 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = \frac{1}{2} \quad \text{or} \quad x = -1$$

— but $x = -1$ is extraneous.

Graphically: The graph of $f(x) = 2 - \dfrac{1}{x + 1} - \dfrac{1}{x^2 + x}$ suggests that the $x$-intercept is $\dfrac{1}{2}$. There is a hole at $x = -1$.



[–4.7, 4.7] by [–4, 4]

Then the solution is $x = \dfrac{1}{2}$.

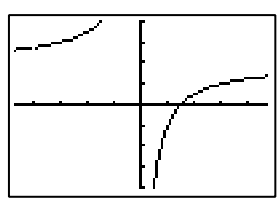
**12.** Algebraically: $2 - \dfrac{3}{x + 4} = \dfrac{12}{x^2 + 4x}$

$[\text{and } x^2 + 4x = x(x + 4)]$

$$2(x^2 + 4x) - 3x = 12 \quad (x \neq 0, -4)$$
$$2x^2 + 5x - 12 = 0$$
$$(2x - 3)(x + 4) = 0$$
$$2x - 3 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = \frac{3}{2} \quad \text{or} \quad x = -4$$

— but $x = -4$ is extraneous.

Graphically: The graph of $f(x) = 2 - \dfrac{3}{x + 4} - \dfrac{12}{x^2 + 4x}$ suggests that the $x$-intercept is $\dfrac{3}{2}$. There is a hole at $x = -4$.



[–4.7, 4.7] by [–4, 4]

Then the solution is $x = \dfrac{3}{2}$.

**13.** Algebraically: $\frac{3x}{x+5} + \frac{1}{x-2} = \frac{7}{x^2+3x-10}$

$[\text{and } x^2 + 3x - 10 = (x+5)(x-2)]$

$3x(x-2) + (x+5) = 7 \quad (x \neq -5, 2)$

$3x^2 - 5x - 2 = 0$

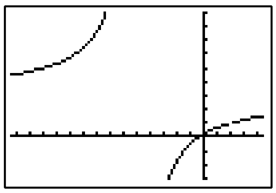
$(3x+1)(x-2) = 0$

$3x + 1 = 0 \quad \text{or} \quad x - 2 = 0$

$x = -\frac{1}{3} \quad \text{or} \quad x = 2$

— but $x = 2$ is extraneous.

Graphically: The graph of $f(x) = \frac{3x}{x+5} + \frac{1}{x-2} - \frac{7}{x^2+3x-10}$ suggests that the $x$-intercept is $-\frac{1}{3}$. There is a hole at $x = 2$.



[–14.4, 4.4] by [–3, 9]

Then the solution is $x = -\frac{1}{3}$.

**14.** Algebraically: $\frac{4x}{x+4} + \frac{3}{x-1} = \frac{15}{x^2+3x-4}$

$[\text{and } x^2 + 3x - 4 = (x+4)(x-1)]$

$4x(x-1) + 3(x+4) = 15 \ (x \neq -4, 1)$

$4x^2 - x - 3 = 0$

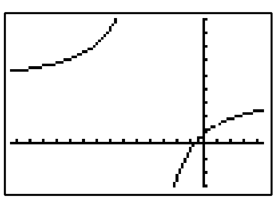
$(4x+3)(x-1) = 0$

$4x + 3 = 0 \quad \text{or} \quad x - 1 = 0$

$x = -\frac{3}{4} \quad \text{or} \quad x = 1$

— but $x = 1$ is extraneous.

Graphically: The graph of $f(x) = \frac{4x}{x+4} + \frac{3}{x-1} - \frac{15}{x^2+3x-4}$ suggests that the $x$-intercept is $-\frac{3}{4}$. There is a hole at $x = 1$.



[–12.4, 6.4] by [–5, 10]

Then the solution is $x = -\frac{3}{4}$.

**15.** Algebraically: $\frac{x-3}{x} - \frac{3}{x+1} + \frac{3}{x^2+x} = 0$

$[\text{and } x^2 + x = x(x+1)]$

$(x-3)(x+1) - 3x + 3 = 0 \quad (x \neq 0, -1)$

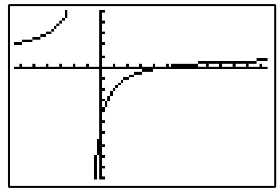
$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0 \quad \text{or} \quad x - 5 = 0$

$x = 0 \quad \text{or} \quad x = 5$ — but $x = 0$ is extraneous.

Graphically: The graph of $f(x) = \frac{x-3}{x} - \frac{3}{x+1} + \frac{3}{x^2+x}$ suggests that the $x$-intercept is 5. The $x$-axis hides a hole at $x = 0$.



[–6.4, 12.4] by [–10, 5]

Then the solution is $x = 5$.

**16.** Algebraically: $\frac{x+2}{x} - \frac{4}{x-1} + \frac{2}{x^2-x} = 0$

$[\text{and } x^2 - x = x(x-1)]$

$(x+2)(x-1) - 4x + 2 = 0 \quad (x \neq 0, 1)$

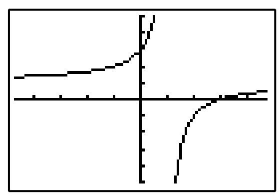
$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \quad \text{or} \quad x - 3 = 0$

$x = 0 \quad \text{or} \quad x = 3$ — but $x = 0$ is extraneous.

Graphically: The graph of $f(x) = \frac{x+2}{x} - \frac{4}{x-1} + \frac{2}{x^2-x}$ suggests that the $x$-intercept is 3. The $x$-axis hides a hole at $x = 0$.



[–4.7, 4.7] by [–5, 5]

Then the solution is $x = 3$.

**17.** Algebraically: $\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}$

$[\text{and } x^2 + 2x = x(x+2)]$

$3x + 6 = (3-x)(x+2) \quad (x \neq -2, 0)$

$3x + 6 = -x^2 + x + 6$

$x^2 + 2x = 0$

$x(x+2) = 0$

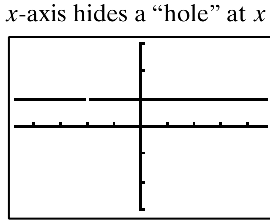
$x = 0 \quad \text{or} \quad x + 2 = 0$

$x = 0 \quad \text{or} \quad x = -2$

— but both solutions are extraneous.

No real solutions.

Graphically: The graph of $f(x) = \frac{3}{x+2} + \frac{6}{x^2+2x} - \frac{3-x}{x}$ suggests that there are no $x$-intercepts. There is a hole at $x = -2$, and the $x$-axis hides a "hole" at $x = 0$.



[–4.7, 4.7] by [–3, 3]

Then there are no real solutions.

**18.** Algebraically: $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$

$[\text{and } x^2 + 3x = x(x+3)]$

$(x+3)^2 - 2x = 6 \quad (x \neq -3, 0)$

$x^2 + 4x + 3 = 0$

$(x+1)(x+3) = 0$

$x + 1 = 0 \quad \text{or} \quad x + 3 = 0$

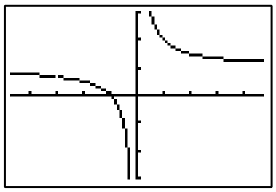
$x = -1 \quad \text{or} \quad x = -3$

— but $x = -3$ is extraneous.

Graphically: The graph of

$f(x) = \frac{x+3}{x} - \frac{2}{x+3} - \frac{6}{x^2+3x}$

suggests that the $x$-intercept is $-1$. There is a hole at $x = -3$.



[–4.7, 4.7] by [–3, 3]

Then the solution is $x = -3$.

**19.** There is no $x$-intercept at $x = -2$. That is the extraneous solution.

**20.** There is no $x$-intercept at $x = 3$. That is the extraneous solution.

**21.** Neither possible solution corresponds to an $x$-intercept of the graph. Both are extraneous.

**22.** There is no $x$-intercept at $x = 3$. That is the extraneous solution.

**23.** $\frac{2}{x-1} + x = 5$

$2 + x(x-1) = 5(x-1) \quad (x \neq 1)$

$x^2 - x + 2 = 5x - 5$

$x^2 - 6x + 7 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{6 \pm \sqrt{8}}{2} = 3 \pm \sqrt{2}$

$x = 3 + \sqrt{2} \approx 4.414$ or

$x = 3 - \sqrt{2} \approx 1.586$

**24.** $\frac{x^2 - 6x + 5}{x^2 - 2} = 3$

$x^2 - 6x + 5 = 3(x^2 - 2) \quad (x \neq \pm\sqrt{2})$

$-2x^2 - 6x + 11 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-2)(11)}}{2(-2)}$

$x = \frac{6 \pm \sqrt{124}}{-4} = \frac{-3 \pm \sqrt{31}}{2}$

$x = \frac{-3 + \sqrt{31}}{2} \approx 1.284$ or

$x = \frac{-3 - \sqrt{31}}{2} \approx -4.284$

**25.** $\frac{x^2 - 2x + 1}{x + 5} = 0$

$x^2 - 2x + 1 = 0 \quad (x \neq 5)$

$(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

**26.** $\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{x^2 + x - 2}$

$[\text{and } x^2 + x - 2 = (x+2)(x-1)]$

$3x(x-1) + 2(x+2) = 5 \quad (x \neq -2, 1)$

$3x^2 - x - 1 = 0$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{1 \pm \sqrt{13}}{6}$

$x = \frac{1 + \sqrt{13}}{6} \approx 0.768$ or

$x = \frac{1 - \sqrt{13}}{6} \approx -0.434$

**27.** $\frac{4x}{x+4} + \frac{5}{x-1} = \frac{15}{x^2 + 3x - 4}$

$[\text{and } x^2 + 3x - 4 = (x+4)(x-1)]$

$4x(x-1) + 5(x+4) = 15 \quad (x \neq -4, 1)$

$4x^2 + x + 5 = 0$

The discriminant is $b^2 - 4ac = 1^2 - 4(4)(5) = -79 < 0$. There are no real solutions.

**28.** $\frac{3x}{x+1} + \frac{5}{x-2} = \frac{15}{x^2 - x - 2}$

$[\text{and } x^2 - x - 2 = (x+1)(x-2)]$

$3x(x-2) + 5(x+1) = 15 \quad (x \neq -1, 2)$

$3x^2 - x - 10 = 0$

$(3x + 5)(x - 2) = 0$

$3x + 5 = 0 \quad \text{or} \quad x - 2 = 0$

$x = -\frac{5}{3} \quad \text{or} \quad x = 2$

— but $x = 2$ is extraneous.

The solution is $x = -\frac{5}{3}$.

**29.** $x^2 + \frac{5}{x} = 8$

$x^3 + 5 = 8x \quad (x \neq 0)$

Using a graphing calculator to find the $x$-intercepts of $f(x) = x^3 - 8x + 5$ yields the solutions $x \approx -3.100$, $x \approx 0.661$, and $x \approx 2.439$.

**30.** $x^2 - \frac{3}{x} = 7$

$x^3 - 3 = 7x \quad (x \neq 0)$

Using a graphing calculator to find the $x$-intercepts of $f(x) = x^3 - 7x - 3$ yields the solutions $x \approx -2.398$, $x \approx -0.441$, and $x \approx 2.838$.

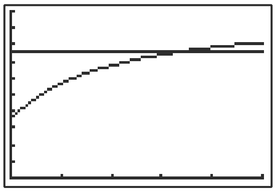
**31.** **(a)** The total amount of solution is $(125 + x)$ mL; of this, the amount of acid is $x$ plus 60% of the original amount, or $x + 0.6(125)$.

**(b)** $y = 0.83$

**(c)** $C(x) = \dfrac{x + 75}{x + 125} = 0.83$. Multiply both sides by $x + 125$, then rearrange to get $0.17x = 28.75$, so that $x \approx 169.12$ mL.

**32. (a)** $C(x) = \dfrac{x + 0.35(100)}{x + 100} = \dfrac{x + 35}{x + 100}$

**(b)** Graph $C(x)$ along with $y = 0.75$; observe where the first graph intersects the second.



[0, 250] by [0, 1]

For $x = 160$, $C(x) = 0.75$. Use 160 mL.

**(c)** Starting from $\dfrac{x + 35}{x + 100} = 0.75$, multiply by $x + 100$ and rearrange to get $0.25x = 40$, so that $x = 160$ mL. That is how much pure acid must be added.

**33. (a)** $C(x) = \dfrac{3000 + 2.12x}{x}$

**(b)** A profit is realized if $C(x) < 2.75$, or $3000 + 2.12x < 2.75x$. Then $3000 < 0.63x$, so that $x > 4761.9$—4762 hats per week.

**(c)** They must have $2.75x - (3000 + 2.12x) > 1000$ or $0.63x > 4000$: 6350 hats per week.

**34. (a)** $P(10) = 200$, $P(40) = 350$, $P(100) = 425$

**(b)** As $t \to \infty$, $P(t) \to 500$. So, yes. The horizontal asymptote is $y = 500$.

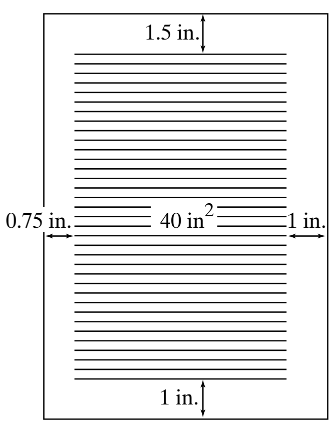
**(c)** $\lim_{t\to\infty} P(t) = \lim_{t\to\infty}\left(500 - \dfrac{9000}{t + 20}\right) = 500$, so the bear population will never exceed 500.

**35. (a)** If $x$ is the length, then $182/x$ is the width.

$$P(x) = 2x + 2\left(\frac{182}{x}\right) = 2x + \frac{364}{x}$$

**(b)** The graph of $P(x) = 2x + 364/x$ has a minimum when $x \approx 13.49$, so that the rectangle is square. Then $P(13.49) = 2(13.49) + 364/13.49 \approx 53.96$ ft.

**36. (a)**




The height of the print material is $40/x$. The total area is

$$A(x) = (x + 0.75 + 1)\left(\frac{40}{x} + 1.5 + 1\right)$$
$$= (x + 1.75)\left(\frac{40}{x} + 2.5\right).$$

**(b)** The graph of $A(x) = (x + 1.75)\left(\dfrac{40}{x} + 2.5\right)$ has a minimum when $x \approx 5.29$, so the dimensions are about $5.29 + 1.75 = 7.04$ in. wide by $40/5.29 + 2.5 \approx 10.06$ in. high. And $A(5.29) \approx 70.8325\text{ in}^2$.

**37. (a)** Since $V = \pi r^2 h$, the height here is $V/(\pi r^2)$. And since in general, $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2V/r$, here $S(x) = 2\pi x^2 + 1000/x$ ($0.5\ L = 500\text{ cm}^3$).

**(b)** Solving $2\pi x^2 + 1000/x = 900$ graphically by finding the zeros of $f(x) = 2\pi x^2 + 1000/x - 900$ yields two solutions: either $x \approx 1.12$ cm, in which case $h \approx 126.88$ cm, or $x \approx 11.37$ cm, in which case $h \approx 1.23$ cm.

**38. (a)** If $x$ is the length, then $1000/x$ is the width. The total area is $A(x) = (x + 4)(1000/x + 4)$.

**(b)** The least area comes when the pool is square, so that $x = \sqrt{1000} \approx 31.62$ ft. With dimensions 35.62 ft × 35.62 ft for the plot of land, $A(31.62) \approx 1268.98\text{ ft}^2$.

**39. (a)**

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\frac{1}{R} = \frac{1}{2.3} + \frac{1}{x}$$
$$2.3x = xR + 2.3R$$
$$R(x) = \frac{2.3x}{x + 2.3}$$

**(b)**

$$2.3x = xR + 2.3R$$
$$x = \frac{2.3R}{2.3 - R}$$

For $R = 1.7$, $x \approx 6.52$ ohms.

**40. (a)** If $x$ is the length, then $200/x$ is the width.

$$P(x) = 2x + 2\left(\frac{200}{x}\right) = 2x + \frac{400}{x}$$

**(b)** $70 = 2x + \dfrac{400}{x}$

$70x = 2x^2 + 400$

$2x^2 - 70x + 400 = 0$

The quadratic formula gives $x \approx 7.1922$ or $x \approx 27.8078$.

When one of those values is considered as the length, the other is the width. The dimensions are 7.1922 m × 27.8078 m.

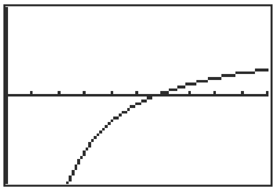
**41. (a)** Drain A can drain 1/4.75 of the pool per hour, while drain B can drain $1/t$ of the pool per hour. Together, they can drain a fraction

$$D(t) = \frac{1}{4.75} + \frac{1}{t} = \frac{t + 4.75}{4.75t}$$

of the pool in 1 hour.

**(b)** The information implies that $D(t) = 1/2.6$, so we solve $\dfrac{1}{2.6} = \dfrac{1}{4.75} + \dfrac{1}{t}$.

Graphically: The function $f(t) = \frac{1}{2.6} - \frac{1}{4.75} - \frac{1}{t}$ has a zero at $t \approx 5.74$ hr, so that is the solution.



[0, 10] by [−0.25, 0.25]

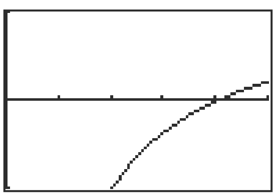
Algebraically: $\frac{1}{2.6} = \frac{1}{4.75} + \frac{1}{t}$

$$4.75t = 2.6t + 2.6(4.75)$$
$$t = \frac{2.6(4.75)}{4.75 - 2.6} \approx 5.74$$

**42. (a)** With $x$ as the bike speed, $x + 43$ is the car speed. Biking time $= 17/x$ and driving time $= 53/(x + 43)$, so

$$T = \frac{17}{x} + \frac{53}{x + 43}.$$

**(b)** Graphically: The function $f(x) = \frac{5}{3} - \frac{17}{x} - \frac{53}{x + 43}$ has a zero at $x \approx 20.45$.



[0, 25] by [−1, 1]

Algebraically: 1 h 40 min $= 1\frac{2}{3}$ h $= \frac{5}{3}$ h

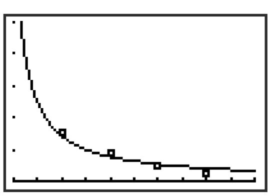
$$\frac{5}{3} = \frac{17}{x} + \frac{53}{x + 43}$$
$$5x(x + 43) = 51(x + 43) + 159x$$
$$5x^2 + 215x = 51x + 2193 + 159x$$
$$5x^2 + 5x - 2193 = 0$$

Using the quadratic formula and selecting the positive solution yields

$$x = \frac{-5 + \sqrt{43{,}885}}{10} \approx 20.45.$$

The rate of the bike was about 20.45 mph.

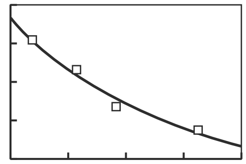
**43. (a)**



[60, 110] by [0, 50]

**(b)** When $a = 74$, $E(74) = 170/(74 - 58) \approx 10.6$. On average, a U.S. 74-year-old will live 10.6 more years.

**44. (a)**



[2000, 4000] by [15, 35]

**(b)** When $x = 3200$, $y = \frac{133{,}000}{2(3200) + 1} \approx 20.778$ mpg.

When $x = 3100$, $y = \frac{133{,}000}{2(3100) + 1} \approx 21.448$ mpg.

This leads to an increase of 21.448 mpg − 20.778 mpg = 0.67 mpg.

**45.** False. An extraneous solution is a value that, though generated by the solution-finding process, does not work in the original equation. In an equation containing rational expressions, an extraneous solution is typically a solution to the version of the equation that has been cleared of fractions but not to the original version.

**46.** True. For a fraction to equal zero, the numerator has to be zero, and 1 is not zero.

**47.** $x - \frac{3x}{x + 2} = \frac{6}{x + 2}$

$$x(x + 2) - 3x = 6 \quad (x \neq -2)$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$

$x = 3$ or $x = -2$ — but $x = -2$ is extraneous.

The answer is D.

**48.** $1 - \frac{3}{x} = \frac{6}{x^2 + 2x}$ [and $x^2 + 2x = x(x + 2)$]

$$x^2 + 2x - 3(x + 2) = 6 \quad (x \neq -2, 0)$$
$$x^2 - x - 12 = 0$$
$$(x + 3)(x - 4) = 0$$

$x = -3$ or $x = 4$

The answer is C.

**49.** $\frac{x}{x + 2} + \frac{2}{x - 5} = \frac{14}{x^2 - 3x - 10}$

[and $x^2 - 3x - 10 = (x + 2)(x - 5)$]

$$x(x - 5) + 2(x + 2) = 14 \quad (x \neq -2, 5)$$
$$x^2 - 3x - 10 = 0$$
$$(x + 2)(x - 5) = 0$$

$x = -2$ or $x = 5$ — but both solutions are extraneous.

The answer is E.

**50.** $0.2 \times 10$ or $2 =$ liters of pure acid in 20% solution

$0.30 \times 30$ or $9 =$ liters of pure acid in 30% solution

$$\frac{\text{L of pure acid}}{\text{L of mixture}} = \text{concentration of acid}$$
$$\frac{2 + 9}{10 + 30} = \frac{11}{40} = 0.275 = 27.5\%$$

The answer is D.

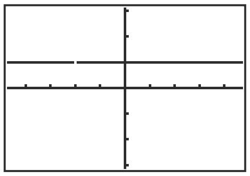
**51. (a)** The LCD is $x^2 + 2x = x(x + 2)$.

$$f(x) = \frac{x - 3}{x} + \frac{3}{x + 2} + \frac{6}{x^2 + 2x}$$
$$= \frac{(x - 3)(x + 2)}{x^2 + 2x} + \frac{3x}{x^2 + 2x} + \frac{6}{x^2 + 2x}$$
$$= \frac{x^2 - x - 6 + 3x + 6}{x^2 + 2x}$$
$$= \frac{x^2 + 2x}{x^2 + 2x}$$

**(b)** All $x \neq 0, -2$

**(c)** $f(x) = \begin{cases} 1 & x \neq -2, 0 \\ \text{undefined} & x = -2 \text{ or } x = 0 \end{cases}$

**(d)** The graph appears to be the horizontal line $y = 1$ with holes at $x = -2$ and $x = 0$.



[−4.7, 4.7] by [−3.1, 3.1]

This matches the definition in part (c).

**52.** $y = 1 + \dfrac{1}{1+x}$

$y(1+x) = (1+x) + 1$

$y + xy = x + 2$

$xy - x = 2 - y$

$x = \dfrac{2-y}{y-1}$

**53.** $y = 1 - \dfrac{1}{1-x}$

$y(1-x) = (1-x) - 1$

$y - xy = -x$

$y = xy - x$

$x = \dfrac{y}{y-1}$

**54.** $y = 1 + \dfrac{1}{1 + \dfrac{1}{x}}$

$y = 1 + \dfrac{x}{1+x}$

$y(1+x) = (1+x) + x$

$y + xy = 2x + 1$

$xy - 2x = 1 - y$

$x = \dfrac{1-y}{y-2}$

**55.** $y = 1 + \dfrac{1}{1 + \dfrac{1}{1-x}}$

$y = 1 + \dfrac{1-x}{1-x+1}$

$y = 1 + \dfrac{1-x}{2-x}$

$y(2-x) = (2-x) + (1-x)$

$2y - xy = 3 - 2x$

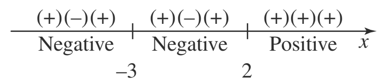
$2y - 3 = xy - 2x$

$x = \dfrac{2y-3}{y-2}$

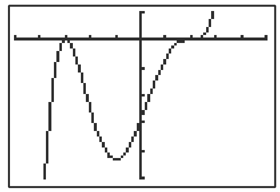
## ■ Section 2.8 Solving Inequalities in One Variable

### Exploration 1

**1. (a)**

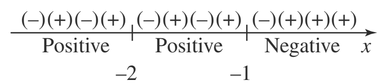



**(b)**

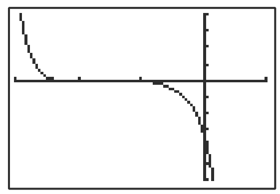


[−5, 5] by [−250, 50]

**2. (a)**

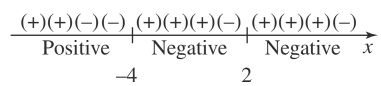



**(b)**

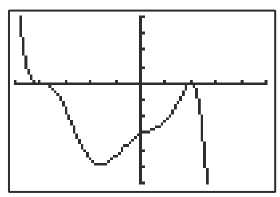


[−3, 1] by [−30, 20]

**3. (a)**




**(b)**



[−5, 5] by [−3000, 2000]

### Quick Review 2.8

**1.** $\lim_{x\to\infty} f(x) = \infty,\ \lim_{x\to-\infty} f(x) = -\infty$

**2.** $\lim_{x\to\infty} f(x) = -\infty,\ \lim_{x\to-\infty} f(x) = -\infty$

**3.** $\lim_{x\to\infty} g(x) = \infty,\ \lim_{x\to-\infty} g(x) = \infty$

**4.** $\lim_{x\to\infty} g(x) = \infty,\ \lim_{x\to-\infty} g(x) = -\infty$

**5.** $\dfrac{x^3+5}{x}$

**6.** $\dfrac{x^3-3}{x}$

**7.** $\dfrac{x(x-3) - 2(2x+1)}{(2x+1)(x-3)} = \dfrac{x^2 - 3x - 4x - 2}{(2x+1)(x-3)}$

$= \dfrac{x^2 - 7x - 2}{(2x+1)(x-3)} = \dfrac{x^2 - 7x - 2}{2x^2 - 5x - 3}$

**8.** $\dfrac{x(3x-4)+(x+1)(x-1)}{(x-1)(3x-4)} = \dfrac{3x^2-4x+x^2-1}{(x-1)(3x-4)}$
$= \dfrac{4x^2-4x-1}{(x-1)(3x-4)} = \dfrac{4x^2-4x-1}{3x^2-7x+4}$

**9. (a)** $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2}$ or $\pm 1, \pm\frac{1}{2}, \pm 3, \pm\frac{3}{2}$

**(b)** A graph suggests that $-1$ and $\frac{3}{2}$ are good candidates for zeros.

| | | | | |
|---|---|---|---|---|
| $-1$ | 2 | 1 | $-4$ | $-3$ |
| | | $-2$ | 1 | 3 |
| $3/2$ | 2 | $-1$ | $-3$ | 0 |
| | | 3 | 3 | |
| | 2 | 2 | 0 | |

$2x^3+x^2-4x-3 = (x+1)\left(x-\frac{3}{2}\right)(2x+2)$
$= (x+1)(2x-3)(x+1)$

**10. (a)** $\dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3}$
or $\pm 1, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}, \pm 4, \pm\frac{4}{3}, \pm 8, \pm\frac{8}{3}$

**(b)** A graph suggests that $-2$ and 1 are good candidates for zeros.

| | | | | |
|---|---|---|---|---|
| $-2$ | 3 | $-1$ | $-10$ | 8 |
| | | $-6$ | 14 | $-8$ |
| 1 | 3 | $-7$ | 4 | 0 |
| | | 3 | $-4$ | |
| | 3 | $-4$ | 0 | |

$3x^3-x^2-10x+8 = (x+2)(x-1)(3x-4)$

## Section 2.8 Exercises

**1. (a)** $f(x) = 0$ when $x = -2, -1, 5$
**(b)** $f(x) > 0$ when $-2 < x < -1$ or $x > 5$
**(c)** $f(x) < 0$ when $x < -2$ or $-1 < x < 5$

(–)(–)(–) | (+)(–)(–) | (+)(+)(–) | (+)(+)(+)
Negative | Positive | Negative | Positive   $x$
$-2$, $-1$, $5$

**2. (a)** $f(x) = 0$ when $x = 7, -\frac{1}{3}, -4$
**(b)** $f(x) > 0$ when $-4 < x < -\frac{1}{3}$ or $x > 7$
**(c)** $f(x) < 0$ when $x < -4$ or $-\frac{1}{3} < x < 7$

(–)(–)(–) | (–)(–)(+) | (–)(+)(+) | (+)(+)(+)
Negative | Positive | Negative | Positive   $x$
$-4$, $-\frac{1}{3}$, $7$

**3. (a)** $f(x) = 0$ when $x = -7, -4, 6$
**(b)** $f(x) > 0$ when $x < -7$ or $-4 < x < 6$ or $x > 6$
**(c)** $f(x) < 0$ when $-7 < x < -4$

$(-)(-)(-)^2$ | $(+)(-)(-)^2$ | $(+)(+)(-)^2$ | $(+)(+)(+)^2$
Positive | Negative | Positive | Positive   $x$
$-7$, $-4$, $6$

**4. (a)** $f(x) = 0$ when $x = -\frac{3}{5}, 1$
**(b)** $f(x) > 0$ when $x < -\frac{3}{5}$ or $x > 1$
**(c)** $f(x) < 0$ when $-\frac{3}{5} < x < 1$

(–)(+)(–) | (+)(+)(–) | (+)(+)(+)
Positive | Negative | Positive   $x$
$-\frac{3}{5}$, $1$

**5. (a)** $f(x) = 0$ when $x = 8, -1$
**(b)** $f(x) > 0$ when $-1 < x < 8$ or $x > 8$
**(c)** $f(x) < 0$ when $x < -1$

$(+)(-)^2(-)^3$ | $(+)(-)^2(+)^3$ | $(+)(+)^2(+)^3$
Negative | Positive | Positive   $x$
$-1$, $8$

**6. (a)** $f(x) = 0$ when $x = -2, 9$
**(b)** $f(x) > 0$ when $-2 < x < 9$ or $x > 9$
**(c)** $f(x) < 0$ when $x < -2$

$(-)^3(+)(-)^4$ | $(+)^3(+)(-)^4$ | $(+)^3(+)(+)^4$
Negative | Positive | Positive   $x$
$-2$, $9$

**7.** $(x+1)(x-3)^2 = 0$ when $x = -1, 3$

$(-)(-)^2$ | $(+)(-)^2$ | $(+)(+)^2$
Negative | Positive | Positive   $x$
$-1$, $3$

By the sign chart, the solution of $(x+1)(x-3)^2 > 0$ is $(-1, 3) \cup (3, \infty)$.

**8.** $(2x+1)(x-2)(3x-4) = 0$ when $x = -\frac{1}{2}, 2, \frac{4}{3}$

(–)(–)(–) | (+)(–)(–) | (+)(–)(+) | (+)(+)(+)
Negative | Positive | Negative | Positive   $x$
$-\frac{1}{2}$, $\frac{4}{3}$, $2$

By the sign chart, the solution of $(2x+1)(x-2)(3x-4) \le 0$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{4}{3}, 2\right]$.

**9.** $(x+1)(x^2-3x+2) = (x+1)(x-1)(x-2) = 0$ when $x = -1, 1, 2$

(–)(–)(–) | (+)(–)(–) | (+)(+)(–) | (+)(+)(+)
Negative | Positive | Negative | Positive   $x$
$-1$, $1$, $2$

By the sign chart, the solution of $(x+1)(x-1)(x-2) < 0$ is $(-\infty, -1) \cup (1, 2)$.

**10.** $(2x-7)(x^2-4x+4) = (2x-7)(x-2)^2 = 0$ when $x = \frac{7}{2}, 2$

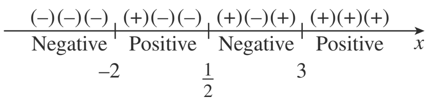
$(-)(-)^2$ | $(-)(+)^2$ | $(+)(+)^2$
Negative | Negative | Positive   $x$
$2$, $\frac{7}{2}$

By the sign chart, the solution of $(2x-7)(x-2)^2 > 0$ is $\left(\frac{7}{2}, \infty\right)$.

**11.** By the Rational Zeros Theorem, the possible rational zeros are $\pm 1,\ \pm\frac{1}{2},\ \pm 2,\ \pm 3,\ \pm\frac{3}{2}, \pm 6$. A graph suggests that $-2, \frac{1}{2}$, and 3 are good candidates to be zeros.

| | | | | |
|---|---|---|---|---|
| $-2\rfloor$ | 2 | −3 | −11 | 6 |
| | | −4 | 14 | −6 |
| $3\rfloor$ | 2 | −7 | 3 | 0 |
| | | 6 | −3 | |
| | 2 | −1 | 0 | |

$2x^3 - 3x^2 - 11x + 6 = (x + 2)(x - 3)(2x - 1) = 0$ when $x = -2, 3, \frac{1}{2}$

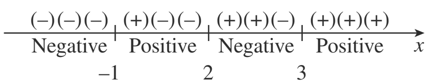



By the sign chart, the solution of $(x + 2)(x - 3)(2x - 1) \geq 0$ is $\left[-2, \frac{1}{2}\right] \cup [3, \infty)$.

**12.** By the Rational Zeros Theorem, the possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 6$. A graph suggests that −1, 2, and 3 are good candidates to be zeros.

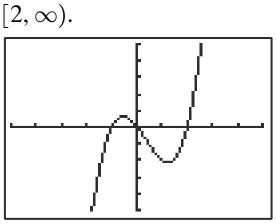
| | | | | |
|---|---|---|---|---|
| $-1\rfloor$ | 1 | −4 | 1 | 6 |
| | | −1 | 5 | −6 |
| $2\rfloor$ | 1 | −5 | 6 | 0 |
| | | 2 | −6 | |
| | 1 | −3 | 0 | |

$x^3 - 4x^2 + x + 6 = (x + 1)(x - 2)(x - 3) = 0$ when $x = -1, 2, 3$.



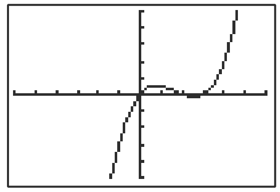

By the sign chart, the solution of $(x + 1)(x - 2)(x - 3) \leq 0$ is $(-\infty, -1] \cup [2, 3]$.

**13.** The zeros of $f(x) = x^3 - x^2 - 2x$ appear to be −1, 0, and 2. Substituting these values into $f$ confirms this. The graph shows that the solution of $x^3 - x^2 - 2x \geq 0$ is $[-1, 0] \cup [2, \infty)$.



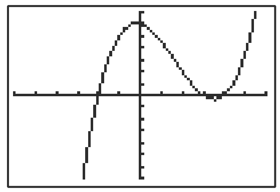
$[-5, 5]$ by $[-5, 5]$

**14.** The zeros of $f(x) = 2x^3 - 5x^2 + 3x$ appear to be 0, 1, and $\frac{3}{2}$. Substituting these values into $f$ confirms this. The graph shows that the solution of $2x^3 - 5x^2 + 3x < 0$ is $(-\infty, 0) \cup \left(1, \frac{3}{2}\right)$.



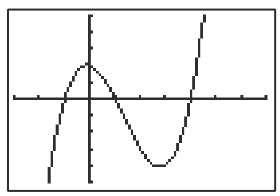
$[-3, 3]$ by $[-5, 5]$

**15.** The zeros of $f(x) = 2x^3 - 5x^2 - x + 6$ appear to be −1, $\frac{3}{2}$, and 2. Substituting these values into $f$ confirms this. The graph shows that the solution of $2x^3 - 5x^2 - x + 6 > 0$ is $\left(-1, \frac{3}{2}\right) \cup (2, \infty)$.



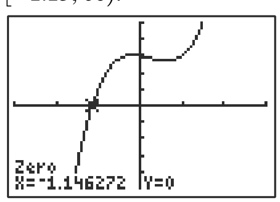
$[-3, 3]$ by $[-7, 7]$

**16.** The zeros of $f(x) = x^3 - 4x^2 - x + 4$ appear to be −1, 1, and 4. Substituting these values into $f$ confirms this. The graph shows that the solution of $x^3 - 4x^2 - x + 4 \leq 0$ is $(-\infty, -1] \cup [1, 4]$.



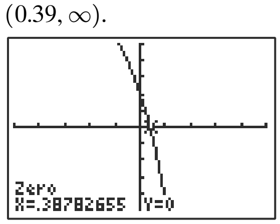
$[-3, 7]$ by $[-10, 10]$

**17.** The only zero of $f(x) = 3x^3 - 2x^2 - x + 6$ is found graphically to be $x \approx -1.15$. The graph shows that the solution of $3x^3 - 2x^2 - x + 6 \geq 0$ is approximately $[-1.15, \infty)$.




$[-3, 3]$ by $[-10, 10]$

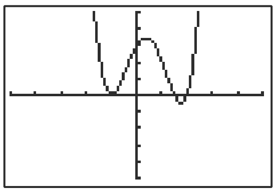
**18.** The only zero of $f(x) = -x^3 - 3x^2 - 9x + 4$ is found graphically to be $x \approx 0.39$. The graph shows that the solution of $-x^3 - 3x^2 - 9x + 4 < 0$ is approximately $(0.39, \infty)$.




$[-5, 5]$ by $[-10, 10]$

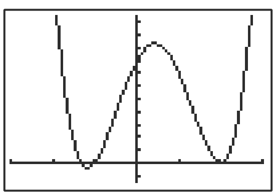
**19.** The zeros of $f(x) = 2x^4 - 3x^3 - 6x^2 + 5x + 6$ appear to be $-1, \frac{3}{2}$, and 2. Substituting these into $f$ confirms this.

The graph shows that the solution of $2x^4 - 3x^3 - 6x^2 + 5x + 6 < 0$ is $\left(\frac{3}{2}, 2\right)$.



$[-5, 5]$ by $[-10, 10]$

**20.** The zeros of $f(x) = 3x^4 - 5x^3 - 12x^2 + 12x + 16$ appear to be $-\frac{4}{3}$, $-1$, and 2. Substituting these into $f$ confirms this. The graph shows that the solution of $3x^4 - 5x^3 - 12x^2 + 12x + 16 \geq 0$ is $\left(-\infty, -\frac{4}{3}\right] \cup [-1, \infty)$.



$[-3, 3]$ by $[-3, 23]$

**21.** $f(x) = (x^2 + 4)(2x^2 + 3)$

**(a)** The solution is $(-\infty, \infty)$, because both factors of $f(x)$ are always positive.

**(b)** The solution is $(-\infty, \infty)$, for the same reason as in **(a)**.

**(c)** There are no solutions, because both factors of $f(x)$ are always positive.

**(d)** There are no solutions, for the same reason as in part (c).

**22.** $f(x) = (x^2 + 1)(-2 - 3x^2)$

**(a)** There are no solutions, because $x^2 + 1$ is always positive and $-2 - 3x^2$ is always negative.

**(b)** There are no solutions, for the same reason as in part (a).

**(c)** $(-\infty, \infty)$, because $x^2 + 1$ is always positive and $-2 - 3x^2$ is always negative.

**(d)** $(-\infty, \infty)$, for the same reason as in part (c).

**23.** $f(x) = (2x^2 - 2x + 5)(3x - 4)^2$
The first factor is always positive because the leading term has a positive coefficient and the discriminant $(-2)^2 - 4(2)(5) = -36$ is negative. The only zero is $x = 4/3$, with multiplicity two, since that is the solution for $3x - 4 = 0$.

**(a)** True for all $x \neq \frac{4}{3}$

**(b)** $(-\infty, \infty)$

**(c)** There are no solutions.

**(d)** $x = \frac{4}{3}$

**24.** $f(x) = (x^2 + 4)(3 - 2x)^2$
The first factor is always positive. The only zero is $x = 3/2$, with multiplicity two, since that is the solution for $3 - 2x = 0$.

**(a)** True for all $x \neq \frac{3}{2}$

**(b)** $(-\infty, \infty)$

**(c)** There are no solutions.

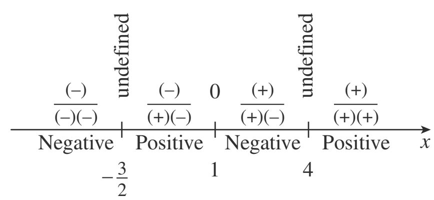
**(d)** $x = \frac{3}{2}$

**25. (a)** $f(x) = 0$ when $x = 1$

**(b)** $f(x)$ is undefined when $x = -\frac{3}{2}, 4$

**(c)** $f(x) > 0$ when $-\frac{3}{2} < x < 1$ or $x > 4$

**(d)** $f(x) < 0$ when $x < -\frac{3}{2}$ or $1 < x < 4$

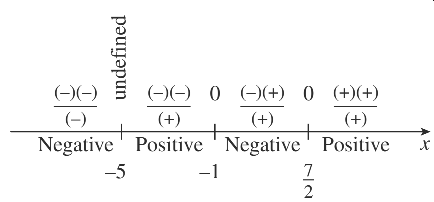



**26. (a)** $f(x) = 0$ when $x = \frac{7}{2}, -1$

**(b)** $f(x)$ is undefined when $x = -5$

**(c)** $f(x) > 0$ when $-5 < x < -1$ or $x > \frac{7}{2}$

**(d)** $f(x) < 0$ when $x < -5$ or $-1 < x < \frac{7}{2}$

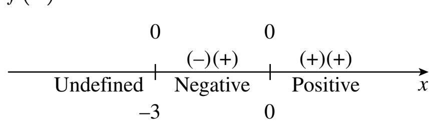



**27. (a)** $f(x) = 0$ when $x = 0, -3$

**(b)** $f(x)$ is undefined when $x < -3$

**(c)** $f(x) > 0$ when $x > 0$

**(d)** $f(x) < 0$ when $-3 < x < 0$

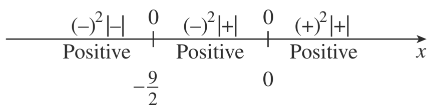



**28. (a)** $f(x) = 0$ when $x = 0, -\frac{9}{2}$

**(b)** None. $f(x)$ is never undefined.

**(c)** $f(x) > 0$ when $x \neq -\frac{9}{2}, 0$

**(d)** None. $f(x)$ is never negative.

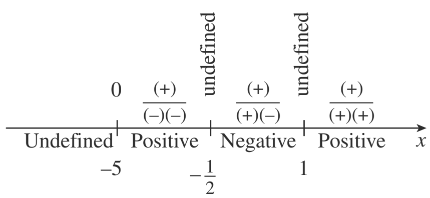



**29.** **(a)** $f(x) = 0$ when $x = -5$

**(b)** $f(x)$ is undefined when $x = -\frac{1}{2}, x = 1, x < -5$

**(c)** $f(x) > 0$ when $-5 < x < -\frac{1}{2}$ or $x > 1$

**(d)** $f(x) < 0$ when $-\frac{1}{2} < x < 1$

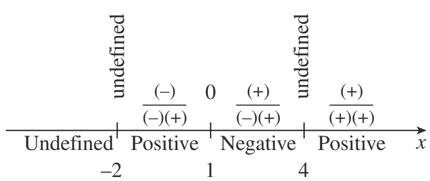



**30.** **(a)** $f(x) = 0$ when $x = 1$

**(b)** $f(x)$ is undefined when $x = 4, x \leq -2$

**(c)** $f(x) > 0$ when $-2 < x < 1$ or $x > 4$

**(d)** $f(x) < 0$ when $1 < x < 4$

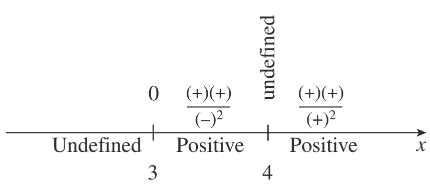



**31.** **(a)** $f(x) = 0$ when $x = 3$

**(b)** $f(x)$ is undefined when $x = 4, x < 3$

**(c)** $f(x) > 0$ when $3 < x < 4$ or $x > 4$

**(d)** None. $f(x)$ is never negative.

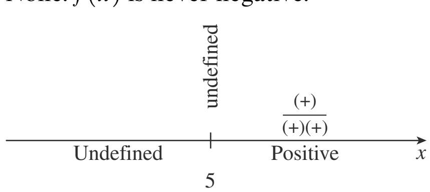



**32.** **(a)** None. $f(x)$is never 0.

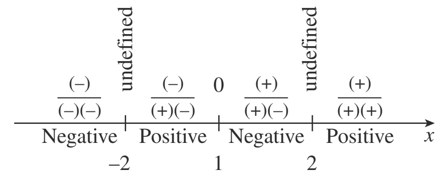
**(b)** $f(x)$ is undefined when $x \leq 5$

**(c)** $f(x) > 0$ when $5 < x < \infty$

**(d)** None. $f(x)$ is never negative.

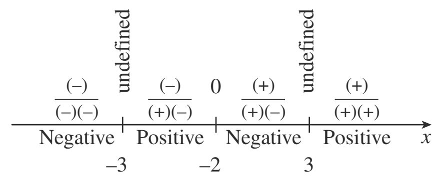



**33.** $f(x) = \dfrac{x - 1}{x^2 - 4} = \dfrac{x - 1}{(x + 2)(x - 2)}$ has points of potential sign change at $x = -2, 1, 2$.



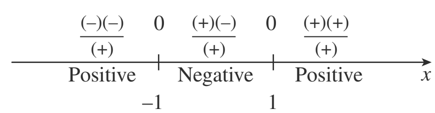

By the sign chart, the solution of $\dfrac{x - 1}{x^2 - 4} < 0$ is $(-\infty, -2) \cup (1, 2)$.

**34.** $f(x) = \dfrac{x + 2}{x^2 - 9} = \dfrac{x + 2}{(x + 3)(x - 3)}$ has points of potential sign change at $x = -3, -2, 3$.



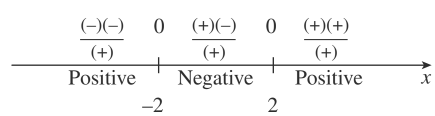

By the sign chart, the solution of $\dfrac{x + 2}{x^2 - 9} < 0$ is $(-\infty, -3) \cup (-2, 3)$.

**35.** $f(x) = \dfrac{x^2 - 1}{x^2 + 1} = \dfrac{(x + 1)(x - 1)}{(x^2 + 1)}$ has points of potential sign change at $x = -1, 1$.



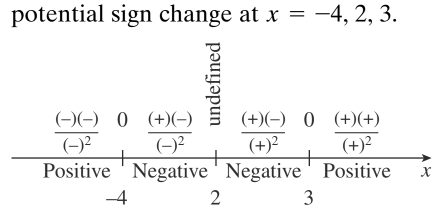

By the sign chart, the solution of $\dfrac{x^2 - 1}{x^2 + 1} \leq 0$ is $[-1, 1]$.

**36.** $f(x) = \dfrac{x^2 - 4}{x^2 + 4} = \dfrac{(x + 2)(x - 2)}{x^2 + 4}$ has points of potential sign change at $x = -2, 2$.



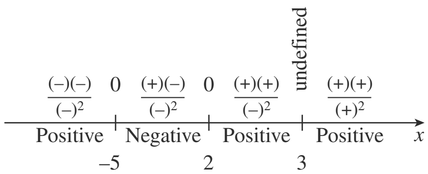

By the sign chart, the solution of $\dfrac{x^2 - 4}{x^2 + 4} > 0$ is $(-\infty, -2) \cup (2, \infty)$.

**37.** $f(x) = \dfrac{x^2 + x - 12}{x^2 - 4x + 4} = \dfrac{(x + 4)(x - 3)}{(x - 2)^2}$ has points of potential sign change at $x = -4, 2, 3$.



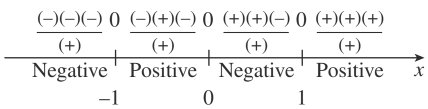

By the sign chart, the solution of $\dfrac{x^2 + x - 12}{x^2 - 4x + 4} > 0$ is $(-\infty, -4) \cup (3, \infty)$.

**38.** $f(x) = \dfrac{x^2 + 3x - 10}{x^2 - 6x + 9} = \dfrac{(x + 5)(x - 2)}{(x - 3)^2}$ has points of potential sign change at $x = -5, 2, 3$.



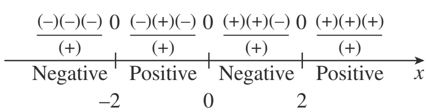

By the sign chart, the solution of $\dfrac{x^2 + 3x - 10}{x^2 - 6x + 9} < 0$ is $(-5, 2)$.

**39.** $f(x) = \dfrac{x^3 - x}{x^2 + 1} = \dfrac{x\,(x + 1)(x - 1)}{x^2 + 1}$ has points of potential sign change at $x = -1, 0, 1$.



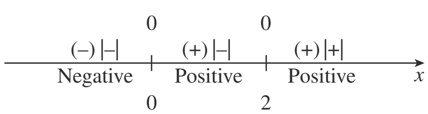

By the sign chart, the solution of $\dfrac{x^3 - x}{x^2 + 1} \geq 0$ is $[-1, 0] \cup [1, \infty)$.

**40.** $f(x) = \dfrac{x^3 - 4x}{x^2 + 2} = \dfrac{x(x + 2)(x - 2)}{x^2 + 2}$ has points of potential sign change at $x = -2, 0, 2$.



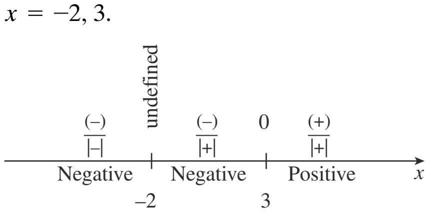

By the sign chart, the solution of $\dfrac{x^3 - 4x}{x^2 + 2} \leq 0$ is $(-\infty, -2] \cup [0, 2]$.

**41.** $f(x) = x|x - 2|$ has points of potential sign change at $x = 0, 2$.



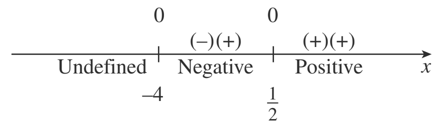

By the sign chart, the solution of $x|x - 2| > 0$ is $(0, 2) \cup (2, \infty)$.

**42.** $f(x) = \dfrac{x - 3}{|x + 2|}$ has points of potential sign change at $x = -2, 3$.



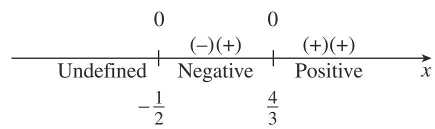

By the sign chart, the solution of $\dfrac{x - 3}{|x + 2|} < 0$ is $(-\infty, -2) \cup (-2, 3)$.

**43.** $f(x) = (2x - 1)\sqrt{x + 4}$ has a point of potential sign change at $x = \dfrac{1}{2}$. Note that the domain of $f$ is $[-4, \infty)$.



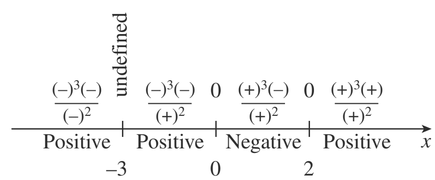

By the sign chart, the solution of $(2x - 1)\sqrt{x + 4} < 0$ is $\left(-4, \dfrac{1}{2}\right)$.

**44.** $f(x) = (3x - 4)\sqrt{2x + 1}$ has a point of potential sign change at $x = \dfrac{4}{3}$. Note that the domain of $f$ is $\left[-\dfrac{1}{2}, \infty\right)$.



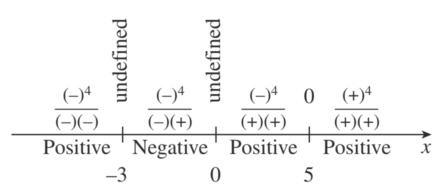

By the sign chart, the solution of $(3x - 4)\sqrt{2x + 1} \geq 0$ is $\left[\dfrac{4}{3}, \infty\right)$.

**45.** $f(x) = \dfrac{x^3(x - 2)}{(x + 3)^2}$ has points of potential sign change at $x = -3, 0, 2$.



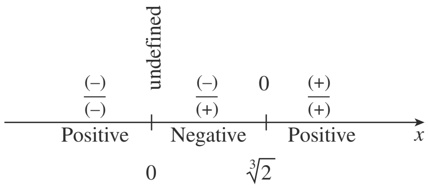

By the sign chart, the solution of $\dfrac{x^3(x - 2)}{(x + 3)^2} < 0$ is $(0, 2)$.

**46.** $f(x) = \dfrac{(x - 5)^4}{x(x + 3)} \geq 0$ has points of potential sign change at $x = -3, 0, 5$.



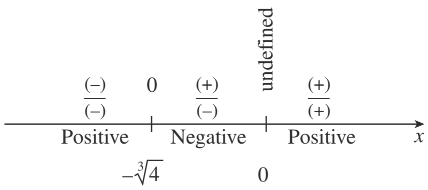

By the sign chart, the solution of $\dfrac{(x - 5)^4}{x\,(x + 3)} \geq 0$ is $(-\infty, -3) \cup (0, \infty)$.

**47.** $f(x) = x^2 - \dfrac{2}{x} = \dfrac{x^3 - 2}{x}$ has points of potential sign change at $x = 0, \sqrt[3]{2}$.



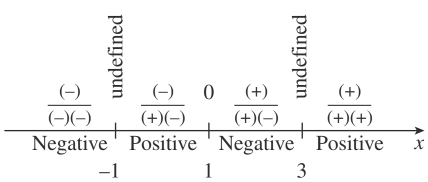

By the sign chart, the solution of $x^2 - \dfrac{2}{x} > 0$ is $(-\infty, 0) \cup (\sqrt[3]{2}, \infty)$.

**48.** $f(x) = x^2 + \dfrac{4}{x} = \dfrac{x^3 + 4}{x}$ has points of potential sign change at $x = 0, -\sqrt[3]{4}$.



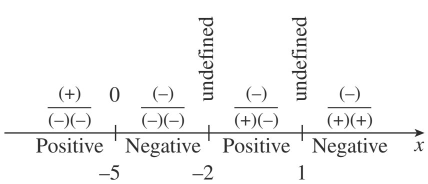

By the sign chart, the solution of $x^2 + \dfrac{4}{x} \geq 0$ is $(-\infty, -\sqrt[3]{4}] \cup (0, \infty)$.

**49.** $f(x) = \dfrac{1}{x + 1} + \dfrac{1}{x - 3} = \dfrac{2(x - 1)}{(x + 1)(x - 3)}$ has points of potential sign change at $x = -1, 1, 3$.



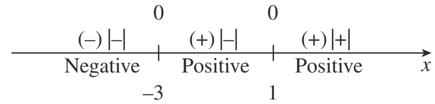

By the sign chart, the solution of $\dfrac{1}{x + 1} + \dfrac{1}{x - 3} \leq 0$ is $(-\infty, -1) \cup [1, 3)$.

**50.** $f(x) = \dfrac{1}{x + 2} - \dfrac{2}{x - 1} = \dfrac{-x - 5}{(x + 2)(x - 1)}$ has points of potential sign change at $x = -5, -2, 1$.




By the sign chart, the solution of $\dfrac{1}{x + 2} - \dfrac{2}{x - 1} > 0$ is $(-\infty, -5) \cup (-2, 1)$.

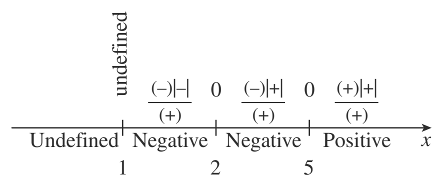
**51.** $f(x) = (x + 3)|x - 1|$ has points of potential sign change at $x = -3, 1$.




By the sign chart, the solution of $(x + 3)|x - 1| \geq 0$ is $[-3, \infty)$.

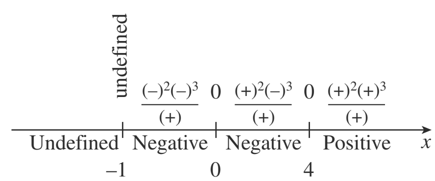
**52.** $f(x) = (3x + 5)^2|x - 2|$ is always 0 or positive since $(3x + 5)^2 \geq 0$ for all real $x$ and $|x - 2| \geq 0$ for all real $x$. Thus the inequality $(3x + 5)^2\,|x - 2| < 0$ has no solution.

**53.** $f(x) = \dfrac{(x - 5)|x - 2|}{\sqrt{2x - 2}}$ has points of potential sign change at $x = 2, 5$. Note that the domain of $f$ is $(1, \infty)$.




By the sign chart, the solution of $\dfrac{(x - 5)|x - 2|}{\sqrt{2x - 2}} \geq 0$ is $[5, \infty)$.

**54.** $f(x) = \dfrac{x^2(x - 4)^3}{\sqrt{x + 1}}$ has points of potential sign change at $x = 0, 4$. Note that the domain of $f$ is $(-1, \infty)$.




By the sign chart, the solution of $\dfrac{x^2(x - 4)^3}{\sqrt{x + 1}} < 0$ is $(-1, 0) \cup (0, 4)$.

**55.** One way to solve the inequality is to graph $y = 3(x - 1) + 2$ and $y = 5x + 6$ together, then find the interval along the $x$-axis where the first graph is below or intersects the second graph. Another way is to solve for $x$ algebraically. The solution is $[-3.5, \infty)$

**56.** Let $x$ be the number of hours worked. The repair charge is $25 + 18x$; this must be less than \$100. Starting with $25 + 18x < 100$, we have $18x < 75$, so $x < 4.166$. Therefore the electrician worked no more than 4 hours 7.5 minutes (which rounds to 4 hours).

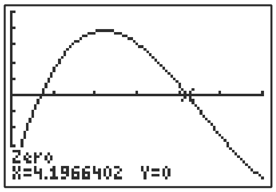
**57.** Let $x > 0$ be the width of a rectangle; then the length is $2x - 2$ and the perimeter is $P = 2[x + (2x - 2)]$. Solving $P < 200$ and $2x - 2 > 0$ (below) gives 1 in. $< x <$ 34 in.

$$\begin{aligned} 2[x + (2x - 2)] &< 200 \quad \text{and} \quad & 2x - 2 &> 0 \\ 2(3x - 2) &< 200 & 2x &> 2 \\ 6x - 4 &< 200 & x &> 1 \\ 6x &< 204 & & \\ x &< 34. & & \end{aligned}$$

**58.** Let $x$ be the number of candy bars made. Then the costs are $C = 0.13x + 2000$, and the income is $I = 0.35x$. Solving $C < I$ (below) gives $x > 9090.91$. The company will need to make and sell 9091 candy bars to make a profit.

$$\begin{aligned} 0.13x + 2000 &< 0.35x \\ 2000 &< 0.22x \\ x &> 9090.91 \end{aligned}$$

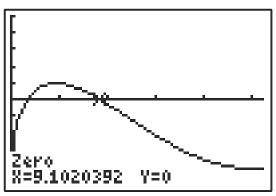
**59.** The lengths of the sides of the box are $x$, $12 - 2x$, and $15 - 2x$, so the volume is $x(12 - 2x)(15 - 2x)$. To solve $x(12 - 2x)(15 - 2x) \le 100$, graph $f(x) = x(12 - 2x)(15 - 2x) - 100$ and find the zeros: $x \approx 0.69$ and $x \approx 4.20$.




$[0, 6]$ by $[-100, 100]$

From the graph, the solution of $f(x) \le 0$ is approximately $[0, 0.69] \cup [4.20, 6]$. The squares should be such that either 0 in. $\le x \le 0.69$ in. or 4.20 in. $\le x \le 6$ in.

**60.** The circumference of the base of the cone is $8\pi - x$, $r = 4 - \dfrac{x}{2\pi}$, and $h = \sqrt{16 - \left(4 - \dfrac{x}{2\pi}\right)^2}$. The volume is $v = \dfrac{1}{3}\pi\left(4 - \dfrac{x}{2\pi}\right)^2 \sqrt{16 - \left(4 - \dfrac{x}{2\pi}\right)^2}$.
To solve $v \ge 21$, graph $v - 21$ and find the zeros: $x \approx 1.68$ in. or $x \approx 9.10$ in.




$[0, 26]$ by $[-25, 25]$

From the graph, the solution of $v - 21 \ge 0$ is approximately $[1.68, 9.10]$. The arc length should be in the range of 1.68 in. $\le x \le 9.10$ in.

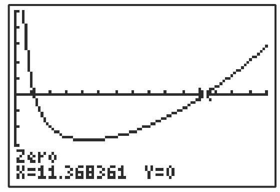
**61. (a)** $\dfrac{1}{2}L = 500 \text{ cm}^3$

$$V = \pi x^2 h = 500 \Rightarrow h = \frac{500}{\pi x^2}$$

$$S = 2\pi x h + 2\pi x^2 = 2\pi x\left(\frac{500}{\pi x^2}\right) + 2\pi x^2$$

$$= \frac{1000}{x} + 2\pi x^2 = \frac{1000 + 2\pi x^3}{x}$$

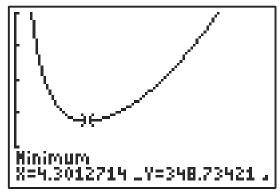
**(b)** Solve $S < 900$ by graphing $\dfrac{1000 + 2\pi x^3}{x} - 900$ and finding its zeros:
$x \approx 1.12$ and $x \approx 11.37$




$[0, 15]$ by $[-1000, 1000]$

From the graph, the solution of $S - 900 < 0$ is approximately $(1.12, 11.37)$. So the radius is between 1.12 cm and 11.37 cm. The corresponding height must be between 1.23 cm and 126.88 cm.

**(c)** Graph $S$ and find the minimum graphically.



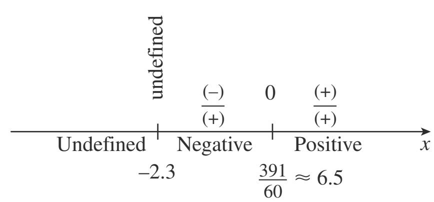

$[0, 15]$ by $[0, 1000]$

The minimum surface area is about 348.73 $\text{cm}^2$.

**62. (a)** $\dfrac{1}{R} = \dfrac{1}{2.3} + \dfrac{1}{x}$
$2.3x = Rx + 2.3R = R(x + 2.3)$
$R = \dfrac{2.3x}{x + 2.3}$

**(b)**
$$\begin{aligned} R \ge 1.7 \Rightarrow \frac{2.3x}{x + 2.3} &\ge 1.7 \\ \frac{2.3x}{x + 2.3} - 1.7 &\ge 0 \\ \frac{2.3x - 1.7(x + 2.3)}{x + 2.3} &\ge 0 \\ \frac{0.6x - 3.91}{x + 2.3} &\ge 0 \end{aligned}$$

The function $f(x) = \dfrac{0.6x - 3.91}{x + 2.3}$ has a point of potential sign change at $x = \dfrac{391}{60} \approx 6.5$. Note that the domain of $f$ is $(-2.3, \infty)$.




By the sign chart, the solution of $f(x) \ge 0$ is about $[6.5, \infty)$. The resistance in the second resistor is at least 6.5 ohms.

**63. (a)** $y \approx 2.723x + 282.043$

**(b)** From the graph of $y = 2.723x + 282.043$, we find that $y = 320$ when $x \approx 13.94$, which is approximately 14 years. The population will exceed 320 million shortly before July 1, 2014.

**64. (a)** $y \approx 4.02x^2 - 77.84x^2 + 595.72$

**(b)** From the graph of $y \approx 4.02x^2 - 77.84x + 595.72$, we find that $y = 247{,}900$ when $x \approx 12.4$. The median cost of a new home will return to $247,900 during November 2012.

**65.** False. Because the factor $x^4$ has an even power, it does not change sign at $x = 0$.

**66.** True. Because the denominator factor $(x + 2)$ has an odd power (namely 1), it changes sign at $x = -2$.

**67.** $x$ must be positive but less than 1. The answer is C.

**68.** The statement is true so long as the numerator does not equal zero. The answer is B.

**69.** The statement is true so long as the denominator is negative and the numerator is nonzero. Thus $x$ must be less than 3 but nonzero. The answer is D.

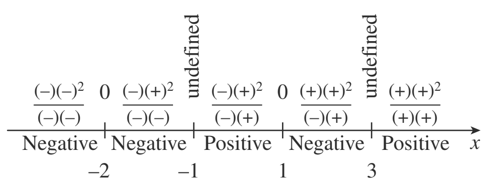
**70.** The expression $(x^2 - 1)^2$ cannot be negative for any real $x$, and it can equal zero only for $x = \pm 1$. The answer is A.

**71.** $f(x) = \dfrac{(x-1)(x+2)^2}{(x-3)(x+1)}$

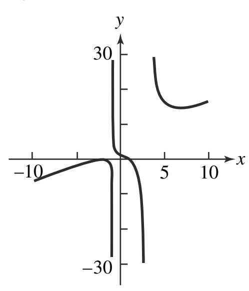
Vertical asymptotes: $x = -1$, $x = 3$

$x$-intercepts: $(-2, 0)$, $(1, 0)$

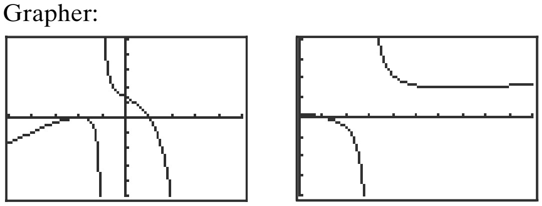
$y$-intercept: $\left(0, \dfrac{4}{3}\right)$




By hand:




Grapher:



[−5, 5] by [−5, 5]

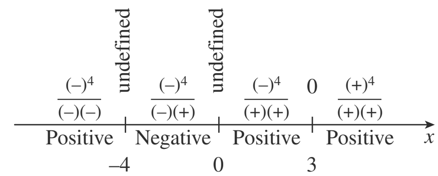
[0, 10] by [−40, 40]

**72.** $g(x) = \dfrac{(x-3)^4}{x^2 + 4x} = \dfrac{(x-3)^4}{x(x+4)}$

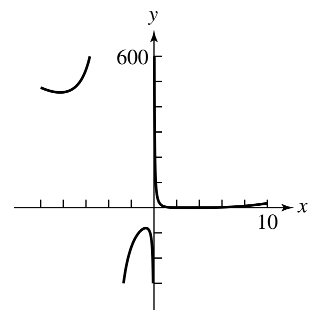
Vertical asymptotes: $x = -4$, $x = 0$

$x$-intercept: $(3, 0)$

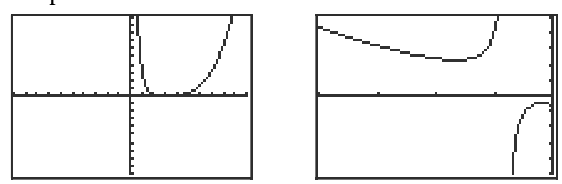
$y$-intercept: None




Sketch:




Grapher:



[−10, 10] by [−10, 10]

[−20, 0] by [−1000, 1000]

**73. (a)** $|x - 3| < 1/3 \Rightarrow |3x - 9| < 1 \Rightarrow$
$|3x - 5 - 4| < 1 \Rightarrow |f(x) - 4| < 1$.

For example:

$|f(x) - 4| = |(3x - 5) - 4| = |3x - 9|$

$= 3|x - 3| < 3\left(\dfrac{1}{3}\right) = 1.$

**(b)** If $x$ stays within the dashed vertical lines, $f(x)$ will stay within the dashed horizontal lines. For the example in part (a), the graph shows that for $\dfrac{8}{3} < x < \dfrac{10}{3}$ $\left(\text{that is, } |x - 3| < \dfrac{1}{3}\right)$, we have $3 < f(x) < 5$ (that is, $|f(x) - 4| < 1$).

**(c)** $|x - 3| < 0.01 \Rightarrow |3x - 9| < 0.03 \Rightarrow$
$|3x - 5 - 4| < 0.03 \Rightarrow |f(x) - 4| < 0.03$. The dashed lines would be closer to $x = 3$ and $y = 4$.

**74.** When $x^2 - 4 \geq 0$, $y = 1$, and when $x^2 - 4 \ngeq 0$, $y = 0$.

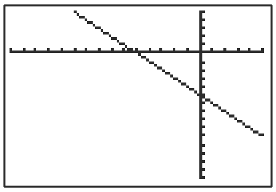
**75.** One possible answer: Given $0 < a < b$, multiplying both sides of $a < b$ by $a$ gives $a^2 < ab$; multiplying by $b$ gives $ab < b^2$. Then, by the transitive property of inequality, we have $a^2 < b^2$.

**76.** One possible answer: Given $0 < a < b$, multiplying both sides of $a < b$ by $\dfrac{1}{ab}$ gives $\dfrac{1}{b} < \dfrac{1}{a}$, which is equivalent to $\dfrac{1}{a} > \dfrac{1}{b}$.

## ■ Chapter 2 Review

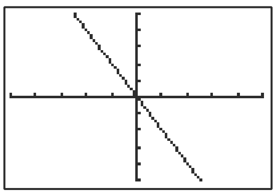
For #1 and 2, first find the slope of the line. Then use algebra to put into $y = mx + b$ format.

**1.** $m = \dfrac{-9 - (-2)}{4 - (-3)} = \dfrac{-7}{7} = -1$, $(y + 9) = -1(x - 4)$, $y = -x - 5$



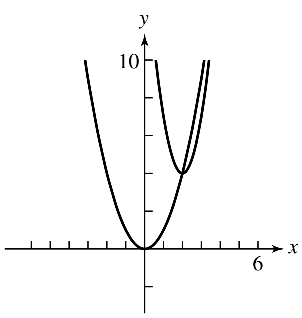
[−15, 5] by [−15, 5]

**2.** $m = \dfrac{-2 - 6}{1 - (-3)} = \dfrac{-8}{4} = -2$, $(y + 2) = -2(x - 1)$, $y = -2x$

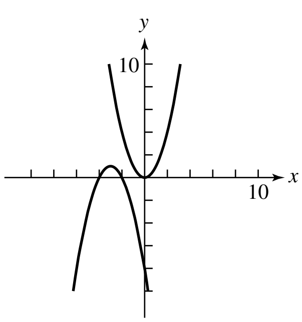


[−5, 5] by [−5, 5]

**3.** Starting from $y = x^2$, translate right 2 units and vertically stretch by 3 (either order), then translate up 4 units.




**4.** Starting from $y = x^2$, translate left 3 units and reflect across $x$-axis (either order), then translate up 1 unit.




**5.** Vertex: $(-3, 5)$; axis: $x = -3$

**6.** Vertex: $(5, -7)$; axis: $x = 5$

**7.** $f(x) = -2(x^2 + 8x) - 31$
$= -2(x^2 + 8x + 16) + 32 - 31 = -2(x + 4)^2 + 1$;
Vertex: $(-4, 1)$; axis: $x = -4$

**8.** $g(x) = 3(x^2 - 2x) + 2 = 3(x^2 - 2x + 1) - 3 + 2 = 3(x - 1)^2 - 1$; Vertex: $(1, -1)$; axis: $x = 1$

For #9–12, use the form $y = a(x - h)^2 + k$, where $(h, k)$, the vertex, is given.

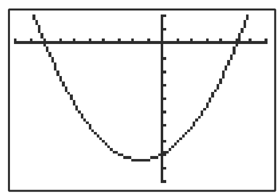
**9.** $h = -2$ and $k = -3$ are given, so $y = a(x + 2)^2 - 3$.
Using the point $(1, 2)$, we have $2 = 9a - 3$, so $a = \dfrac{5}{9}$;
$y = \dfrac{5}{9}(x + 2)^2 - 3$.

**10.** $h = -1$ and $k = 1$ are given, so $y = a(x + 1)^2 + 1$.
Using the point $(3, -2)$, we have $-2 = 16a + 1$, so
$a = -\dfrac{3}{16}$; $y = -\dfrac{3}{16}(x + 1)^2 + 1$.

**11.** $h = 3$ and $k = -2$ are given, so $y = a(x - 3)^2 - 2$.
Using the point $(5, 0)$, we have $0 = 4a - 2$, so $a = \dfrac{1}{2}$;
$y = \dfrac{1}{2}(x - 3)^2 - 2$.

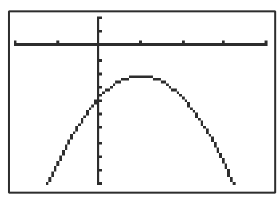
**12.** $h = -4$ and $k = 5$ are given, so $y = a(x + 4)^2 + 5$.
Using the point $(0, -3)$, we have $-3 = 16a + 5$,
so $a = -\dfrac{1}{2}$; $y = -\dfrac{1}{2}(x + 4)^2 + 5$.

**13.**



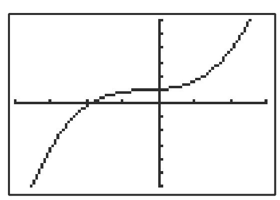
[−10, 7] by [−50, 10]

**14.**



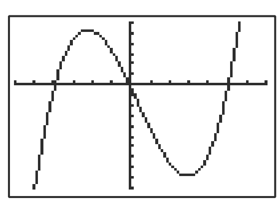
[−2, 4] by [−50, 10]

**15.**



[−4, 3] by [−30, 30]

**16.**



[−6, 7] by [−50, 30]

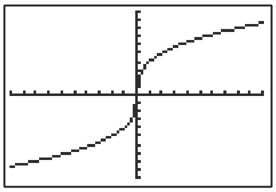
**17.** $S = kr^2 \quad (k = 4\pi)$

**18.** $F = \dfrac{k}{d^2} \quad (k = \text{gravitational constant})$

**19.** The force $F$ needed varies directly with the distance $x$ from its resting position, with constant of variation $k$.

**20.** The area of a circle $A$ varies directly with the square of its radius.

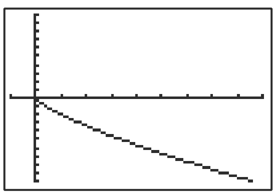
**21.** $k = 4$, $a = \frac{1}{3}$. In Quadrant I, $f(x)$ is increasing and concave down since $0 < a < 1$.



$[-10, 10]$ by $[-10, 10]$

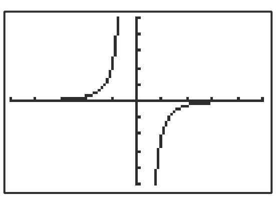
$f(-x) = 4(-x)^{1/3} = -4x^{1/3} = -f(x)$, so $f$ is odd.

**22.** $k = -2$, $a = \frac{3}{4}$. In Quadrant IV, $f(x)$ is decreasing and concave up since $0 < a < 1$. $f$ is not defined for $x < 0$.



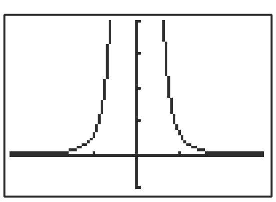
$[-1, 9]$ by $[-10, 10]$

**23.** $k = -2$, $a = -3$. In Quadrant IV, $f$ is increasing and concave down. $f(-x) = -2(-x)^{-3} = \frac{-2}{(-x)^3} = \frac{-2}{-x^3}$ $= \frac{2}{x^3} = 2x^{-3} = -f(x)$, so $f$ is odd.



$[-5, 5]$ by $[-5, 5]$

**24.** $k = \frac{2}{3}$, $a = -4$. In Quadrant I, $f(x)$ is decreasing and concave up. $f(-x) = \frac{2}{3}(-x)^{-4} = \frac{2}{3} \cdot \frac{1}{(-x)^4}$ $= \frac{2}{3x^4} = \frac{2}{3}x^{-4} = f(x)$, so $f$ is even.



$[-3, 3]$ by $[-1, 4]$

**25.** $\frac{2x^3 - 7x^2 + 4x - 5}{x - 3} = 2x^2 - x + 1 - \frac{2}{x - 3}$

$$\begin{array}{r|l} & 2x^2 - x + 1 \\ x - 3 & 2x^3 - 7x^2 + 4x - 5 \\ & \underline{2x^3 - 6x^2} \\ & -x^2 + 4x \\ & \underline{-x^2 + 3x} \\ & x - 5 \\ & \underline{x - 3} \\ & -2 \end{array}$$

**26.** $\frac{x^4 + 3x^3 + x^2 - 3x + 3}{x + 2} = x^3 + x^2 - x - 1 + \frac{5}{x + 2}$

$$\begin{array}{r|l} & x^3 + x^2 - x - 1 \\ x + 2 & x^4 + 3x^3 + x^2 - 3x + 3 \\ & \underline{x^4 + 2x^3} \\ & x^3 + x^2 \\ & \underline{x^3 + 2x^2} \\ & -x^2 - 3x \\ & \underline{-x^2 - 2x} \\ & -x + 3 \\ & \underline{-x - 2} \\ & 5 \end{array}$$

**27.** $\frac{2x^4 - 3x^3 + 9x^2 - 14x + 7}{x^2 + 4}$

$= 2x^2 - 3x + 1 + \frac{-2x + 3}{x^2 + 4}$

$$\begin{array}{r|l} & 2x^2 - 3x + 1 \\ x^2 + 4 & 2x^4 - 3x^3 + 9x^2 - 14x + 7 \\ & \underline{2x^4 \quad\quad + 8x^2} \\ & -3x^3 + x^2 - 14x + 7 \\ & \underline{-3x^3 \quad\quad - 12x} \\ & x^2 - 2x + 7 \\ & \underline{x^2 \quad\quad + 4} \\ & -2x + 3 \end{array}$$

**28.** $\frac{3x^4 - 5x^3 - 2x^2 + 3x - 6}{3x + 1} = x^3 - 2x^2 + 1 + \frac{-7}{3x + 1}$

$$\begin{array}{r|l} & x^3 - 2x^2 \quad\quad + 1 \\ 3x + 1 & 3x^4 - 5x^3 - 2x^2 + 3x - 6 \\ & \underline{3x^4 + x^3} \\ & -6x^3 - 2x^2 + 3x - 6 \\ & \underline{-6x^3 - 2x^2} \\ & 3x - 6 \\ & \underline{3x + 1} \\ & -7 \end{array}$$

**29.** Remainder: $f(-2) = -39$

**30.** Remainder: $f(3) = -2$

**31.** Yes: 2 is a zero of the second polynomial.

**32.** No: $x = -3$ yields 1 from the second polynomial.

**33.**

| 5 | 1 | −5 | 3 | 4 |
|---|---|---|---|---|
| | | 5 | 0 | 15 |
| | 1 | 0 | 3 | 19 |

Yes, $x = 5$ is an upper bound for the zeros of $f(x)$ because all entries on the bottom row are nonnegative.

**34.**

| 4 | 4 | −16 | 8 | 16 | −12 |
|---|---|---|---|---|---|
| | | 16 | 0 | 32 | 192 |
| | 4 | 0 | 8 | 48 | 180 |

Yes, $x = 4$ is an upper bound for the zeros of $f(x)$ because all entries on the bottom row are nonnegative.

**35.**

| −3 | 4 | 4 | −15 | −17 | −2 |
|---|---|---|---|---|---|
| | | −12 | 24 | −27 | 132 |
| | 4 | −8 | 9 | −44 | 130 |

Yes, $x = -3$ is a lower bound for the zeros of $f(x)$ because all entries on the bottom row alternate signs.

**36.**

| −3 | 2 | 6 | 1 | −6 |
|---|---|---|---|---|
| | | −6 | 0 | −3 |
| | 2 | 0 | 1 | −9 |

Yes, $x = -3$ is a lower bound for the zeros of $f(x)$ because all entries on the bottom row alternate signs (remember that $0 = -0$).

**37.** Possible rational zeros: $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$,

or $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$; $-\frac{3}{2}$ and 2 are zeros.

**38.** Possible rational zeros: $\frac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$,

or $\pm 1, \pm 7, \pm\frac{1}{2}, \pm\frac{7}{2}, \pm\frac{1}{3}, \pm\frac{7}{3}, \pm\frac{1}{6}, \pm\frac{7}{6}$; $\frac{7}{3}$ is a zero.

**39.** $(1 + i)^3 = (1 + 2i + i^2)(1 + i) = (2i)(1 + i) = -2 + 2i$

**40.** $(1 + 2i)^2(1 - 2i)^2 = [(1 + 2i)(1 - 2i)]^2 = (1 + 2^2)^2 = 25$

**41.** $i^{29} = i$

**42.** $\sqrt{-16} = 4i$

For #43 and 44, use the quadratic formula.

**43.** $x = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$

**44.** $x = \frac{2 \pm \sqrt{4 - 16}}{2} = \frac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i$

**45.** (c) $f(x) = (x - 2)^2$ is a quadratic polynomial that has vertex (2, 0) and $y$-intercept (0, 4), so its graph must be graph (c).

**46.** (d) $f(x) = (x - 2)^3$ is a cubic polynomial that passes through (2, 0) and (0, −8), so its graph must be graph (d).

**47.** (b) $f(x) = (x - 2)^4$ is a quartic polynomial that passes through (2, 0) and (0, 16), so its graph must be graph (b).

**48.** (a) $f(x) = (x - 2)^5$ is a quintic polynomial that passes through (2, 0) and (0, −32), so its graph must be graph (a).

In #49–52, use a graph and the Rational Zeros Test to determine zeros.

**49.** Rational: 0 (multiplicity 2) — easily seen by inspection. Irrational: $5 \pm \sqrt{2}$ (using the quadratic formula, after taking out a factor of $x^2$). No nonreal zeros.

**50.** Rational: ±2. Irrational: $\pm\sqrt{3}$. No nonreal zeros. These zeros may be estimated from a graph, or by dividing $k(t)$ by $t - 2$ and $t + 2$ then applying the quadratic formula, or by using the quadratic formula on $k(t)$ to determine that $t^2 = \frac{7 \pm \sqrt{49 - 48}}{2}$, i.e., $t^2$ is 3 or 4.

**51.** Rational: none. Irrational: approximately −2.34, 0.57, 3.77. No nonreal zeros.

**52.** Rational: none. Irrational: approximately −3.97, −0.19. Two nonreal zeros.

**53.** The only rational zero is $-\frac{3}{2}$. Dividing by $x + \frac{3}{2}$ (below) leaves $2x^2 - 12x + 20$, which has zeros $\frac{12 \pm \sqrt{144 - 160}}{4} = 3 \pm i$. Therefore

$$f(x) = (2x + 3)[x - (3 - i)][x - (3 + i)] = (2x + 3)(x - 3 + i)(x - 3 - i).$$

| −3/2 | 2 | −9 | 2 | 30 |
|---|---|---|---|---|
| | | −3 | 18 | −30 |
| | 2 | −12 | 20 | 0 |

**54.** The only rational zero is $\frac{4}{5}$. Dividing by $x - \frac{4}{5}$ (below) leaves $5x^2 - 20x - 15$, which has zeros $\frac{20 \pm \sqrt{400 + 300}}{10} = 2 \pm \sqrt{7}$. Therefore

$$f(x) = (5x - 4)(x - (2 + \sqrt{7}))(x - (2 - \sqrt{7})) = (5x - 4)(x - 2 - \sqrt{7})(x - 2 + \sqrt{7}).$$

| 4/5 | 5 | −24 | 1 | 12 |
|---|---|---|---|---|
| | | 4 | −16 | −12 |
| | 5 | −20 | −15 | 0 |

**55.** All zeros are rational: 1, −1, $\frac{2}{3}$, and $-\frac{5}{2}$. Therefore $f(x) = (3x - 2)(2x + 5)(x - 1)(x + 1)$; this can be confirmed by multiplying out the terms or graphing the original function and the factored form of the function.

**56.** Since all coefficients are real, $1 - 2i$ is also a zero. Dividing synthetically twice leaves the quadratic $x^2 - 6x + 10$, which has zeros $3 \pm i$.
$f(x) = [x - (1 + 2i)][x - (1 - 2i)][x - (3 + i)]$
$[x - (3 - i)] = (x - 1 - 2i)(x - 1 + 2i)$
$(x - 3 - i)(x - 3 + i)$

| | | | | | |
|---|---|---|---|---|---|
| $1 + 2i$ | 1 | −8 | 27 | −50 | 50 |
| | | $1 + 2i$ | $-11 - 12i$ | $40 + 20i$ | −50 |
| | 1 | $-7 + 2i$ | $16 - 12i$ | $-10 + 20i$ | 0 |

| | | | | |
|---|---|---|---|---|
| $1 - 2i$ | 1 | $-7 + 2i$ | $16 - 12i$ | $-10 + 20i$ |
| | | $1 - 2i$ | $-6 + 12i$ | $10 - 20i$ |
| | 1 | −6 | 10 | 0 |

In #57–60, determine rational zeros (graphically or otherwise) and divide synthetically until a quadratic remains. If more real zeros remain, use the quadratic formula.

**57.** The only real zero is 2; dividing by $x - 2$ leaves the quadratic factor $x^2 + x + 1$, so $f(x) = (x - 2)(x^2 + x + 1)$.

| | | | | |
|---|---|---|---|---|
| 2 | 1 | −1 | −1 | −2 |
| | | 2 | 2 | 2 |
| | 1 | 1 | 1 | 0 |

**58.** The only rational zero is −1; dividing by $x + 1$ leaves the quadratic factor $9x^2 - 12x - 1$, which has zeros $\frac{12 \pm \sqrt{144 + 36}}{18} = \frac{2}{3} \pm \frac{1}{3}\sqrt{5}$. Then $f(x) = (x + 1)(9x^2 - 12x - 1)$.

| | | | | |
|---|---|---|---|---|
| −1 | 9 | −3 | −13 | −1 |
| | | −9 | 12 | 1 |
| | 9 | −12 | −1 | 0 |

**59.** The two real zeros are 1 and $\frac{3}{2}$; dividing by $x - 1$ and $x - \frac{3}{2}$ leaves the quadratic factor $2x^2 - 4x + 10$, so $f(x) = (2x - 3)(x - 1)(x^2 - 2x + 5)$.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | −9 | 23 | −31 | 15 |
| | | 2 | −7 | 16 | −15 |
| | 2 | −7 | 16 | −15 | 0 |

| | | | | |
|---|---|---|---|---|
| 3/2 | 2 | −7 | 16 | −15 |
| | | 3 | −6 | 15 |
| | 2 | −4 | 10 | 0 |

**60.** The two real zeros are −1 and $-\frac{2}{3}$; dividing by $x + 1$ and $x + \frac{2}{3}$ leaves the quadratic factor $3x^2 - 12x + 15$, so $f(x) = (3x + 2)(x + 1)(x^2 - 4x + 5)$.

| | | | | | |
|---|---|---|---|---|---|
| −1 | 3 | −7 | −3 | 17 | 10 |
| | | −3 | 10 | −7 | −10 |
| | 3 | −10 | 7 | 10 | 0 |

| | | | | |
|---|---|---|---|---|
| −2/3 | 3 | −10 | 7 | 10 |
| | | −2 | 8 | −10 |
| | 3 | −12 | 15 | 0 |

**61.** $(x - \sqrt{5})(x + \sqrt{5})(x - 3) = x^3 - 3x^2 - 5x + 15$. Other answers may be found by multiplying this polynomial by any real number.

**62.** $(x + 3)^2 = x^2 + 6x + 9$. (This may be multiplied by any real number.)

**63.** $(x - 3)(x + 2)(3x - 1)(2x + 1)$
$= 6x^4 - 5x^3 - 38x^2 - 5x + 6$. (This may be multiplied by any real number.)

**64.** The third zero must be $1 - i$:
$(x - 2)(x - 1 - i)(x - 1 + i) = x^3 - 4x^2 + 6x - 4$.
(This may be multiplied by any real number.)

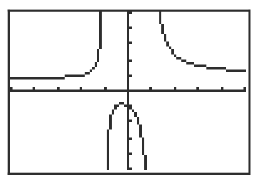
**65.** $(x + 2)^2(x - 4)^2 = x^4 - 4x^3 - 12x^2 + 32x + 64$.
(This may be multiplied by any real number.)

**66.** The third zero must be $2 + i$, so
$f(x) = a(x + 1)(x - 2 - i)(x - 2 + i)$.
Since $f(2) = 6$, $a = 2$:
$f(x) = 2(x + 1)(x - 2 - i)(x - 2 + i)$
$= 2x^3 - 6x^2 + 2x + 10$.

**67.** $f(x) = -1 + \frac{2}{x - 5}$; translate right 5 units and vertically stretch by 2 (either order), then translate down 1 unit.
Horizontal asymptote: $y = -1$; vertical asymptote: $x = 5$.

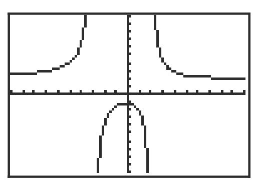
**68.** $f(x) = 3 - \frac{1}{x + 2}$; translate left 2 units and reflect across $x$-axis (either order), then translate up 3 units.
Horizontal asymptote: $y = 3$; vertical asymptote: $x = -2$.

**69.** Asymptotes: $y = 1$, $x = -1$, and $x = 1$.
Intercept: $(0, -1)$.



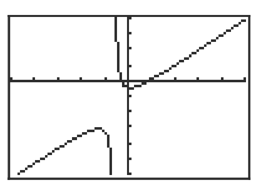
[–5, 5] by [–5, 5]

**70.** Asymptotes: $y = 2$, $x = -3$, and $x = 2$.
Intercept: $\left(0, -\frac{7}{6}\right)$.



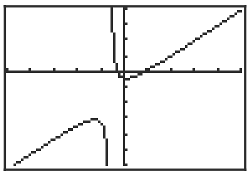
[–10, 10] by [–10, 10]

**71.** End-behavior asymptote: $y = x - 7$.
Vertical asymptote: $x = -3$. Intercept: $\left(0, \frac{5}{3}\right)$.



[–25, 25] by [–30, 20]

**72.** End-behavior asymptote: $y = x - 6$.
Vertical asymptote: $x = -3$. Intercepts: approx. $(-1.54, 0)$, $(4.54, 0)$, and $\left(0, -\frac{7}{3}\right)$.



[–25, 25] by [–30, 20]

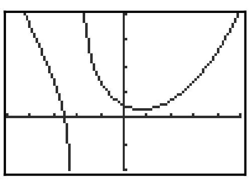
**73.** $f(x) = \frac{x^3 + x^2 - 2x + 5}{x + 2}$ has only one $x$-intercept, and we can use the graph to show that it is about $-2.552$. The $y$-intercept is $f(0) = 5/2$. The denominator is zero when $x = -2$, so the vertical asymptote is $x = -2$. Because we can rewrite $f(x)$ as

$f(x) = \frac{x^3 + x^2 - 2x + 5}{x + 2} = x^2 - x + \frac{5}{x + 2}$,

we know that the end-behavior asymptote is $y = x^2 - x$. The graph supports this information and allows us to conclude that

$\lim_{x \to -2^-} = -\infty$, $\lim_{x \to -2^+} = \infty$.

The graph also shows a local minimum of about 1.63 at about $x = 0.82$.



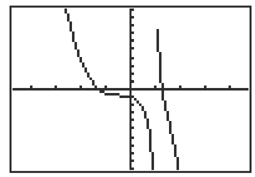
[–5, 5] by [–10, 20]

$y$-intercept: $\left(0, \frac{5}{2}\right)$
$x$-intercept: $(-2.55, 0)$
Domain: All real $x \neq -2$
Range: $(-\infty, \infty)$
Continuity: All real $x \neq -2$
Increasing on $[0.82, \infty)$
Decreasing on $(-\infty, -2)$, $(-2, 0.82]$
Not symmetric
Unbounded
Local minimum: $(0.82, 1.63)$
No horizontal asymptote. End-behavior asymptote: $y = x^2 - x$
Vertical asymptote: $x = -2$.
End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = \infty$

**74.** $f(x) = \frac{-x^4 + x^2 + 1}{x - 1}$ has two $x$-intercepts, and we can use the graph to show that they are about $-1.27$ and $1.27$. The $y$-intercept is $f(0) = -1$. The denominator is zero when $x = 1$, so the vertical asymptote is $x = 1$. Because we can rewrite $f(x)$ as

$f(x) = \frac{-x^4 + x^2 + 1}{x - 1} = -x^3 - x^2 + \frac{1}{x - 1}$,

we know that the end-behavior asymptote is $y = -x^3 - x^2$. The graph supports this information and allows us to conclude that $\lim_{x \to 1^-} = -\infty$ and $\lim_{x \to 1^+} = \infty$. The graph shows no local extrema.



[–4.7, 4.7] by [–10, 10]

$y$-intercept: $(0, 1)$
$x$-intercepts: $(-1.27, 0)$, $(1.27, 0)$
Domain: All real $x \neq 1$
Range: $(-\infty, \infty)$
Continuity: All real $x \neq 1$
Never increasing
Decreasing on $(-\infty, 1)$, $(1, \infty)$
No symmetry
Unbounded
No local extrema
No horizontal asymptote. End-behavior asymptote: $y = -x^3 - x^2$
Vertical asymptote: $x = 1$
End behavior: $\lim_{x \to -\infty} f(x) = \infty$; $\lim_{x \to \infty} f(x) = -\infty$

**75.** Multiply by $x$: $2x^2 - 11x + 12 = 0$, so $x = \frac{3}{2}$ or $x = 4$.

**76.** Multiply by $(x + 2)(x - 3) = x^2 - x - 6$: $x(x - 3) + 5(x + 2) = 25$, or $x^2 + 2x - 15 = 0$, so $x = -5$ or $x = 3$. The latter is extraneous; the only solution is $x = -5$.

For #77–78, find the zeros of $f(x)$ and then determine where the function is positive or negative by creating a sign chart.

**77.** $f(x) = (x - 3)(2x + 5)(x + 2)$, so the zeros of $f(x)$ are $x = \left\{-\frac{5}{2}, -2, 3\right\}$.

| (–)(–)(–) | (–)(+)(–) | (–)(+)(+) | (+)(+)(+) |
|---|---|---|---|
| Negative | Positive | Negative | Positive |

Boundaries on the $x$-axis: $-\frac{5}{2}$, $-2$, $3$

As our sign chart indicates, $f(x) < 0$ on the interval $\left(-\infty, -\frac{5}{2}\right) \cup (-2, 3)$.

**78.** $f(x) = (x - 2)^2(x + 4)(3x + 1)$, so the zeros of $f(x)$ are $x = \left\{-4, -\frac{1}{3}, 2\right\}$.

| (+)(−)(−) | (+)(+)(−) | (+)(+)(+) | (+)(+)(+) |
|---|---|---|---|
| Positive | Negative | Positive | Positive |

Boundaries on the x-axis: $-4$, $-\frac{1}{3}$, $2$

As our sign chart indicates, $f(x) \geq 0$ on the interval $(-\infty, -4] \cup \left[-\frac{1}{3}, \infty\right)$.

**79.** Zeros of numerator and denominator: −3, −2, and 2. Choose −4, −2.5, 0, and 3; $\frac{x+3}{x^2-4}$ is positive at −2.5 and 3, and equals 0 at −3, so the solution is $[-3, -2) \cup (2, \infty)$.

**80.** $\frac{x^2 - 7}{x^2 - x - 6} - 1 = \frac{x - 1}{x^2 - x - 6}$. Zeros of numerator and denominator: −2, 1, and 3. Choose −3, 0, 2, and 4; $\frac{x - 1}{x^2 - x - 6}$ is negative at −3 and 2, so the solution is $(-\infty, -2) \cup (1, 3)$.

**81.** Since the function is always positive, we need only worry about the equality $(2x - 1)^2|x + 3| = 0$. By inspection, we see this holds true only when $x = \left\{-3, \frac{1}{2}\right\}$.

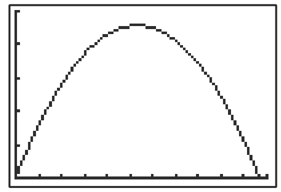
**82.** $\sqrt{x + 3}$ exists only when $x \geq -3$, so we are concerned only with the interval $(-3, \infty)$. Further $|x - 4|$ is always 0 or positive, so the only possible value for a sign change is $x = 1$. For $-3 < x < 1$, $\frac{(x - 1)|x - 4|}{\sqrt{x + 3}}$ is negative, and for $1 < x < 4$ or $4 < x < \infty$, $\frac{(x - 1)|x + 4|}{\sqrt{x + 3}}$ is positive. So the solution is $(1, 4) \cup (4, \infty)$.

**83.** Synthetic division reveals that we *cannot* conclude that 5 is an upper bound (since there are both positive and negative numbers on the bottom row), while −5 *is* a lower bound (because all numbers on the bottom row alternate signs). Yes, there is another zero (at $x \approx 10.0002$).

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 1 | −10 | −3 | 28 | 20 | −2 |
| | | 5 | −25 | −140 | −560 | −2700 |
| | 1 | −5 | −28 | −112 | −540 | −2702 |

| | | | | | | |
|---|---|---|---|---|---|---|
| −5 | 1 | −10 | −3 | 28 | 20 | −2 |
| | | −5 | 75 | −360 | 1660 | −8400 |
| | 1 | −15 | 72 | −332 | 1680 | −8402 |

**84.** **(a)** $h = -16t^2 + 170t + 6$



[0, 11] by [0, 500]

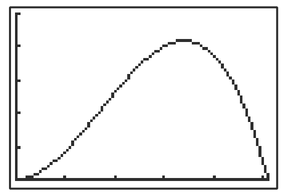
**(b)** When $t \approx 5.3125$, $h \approx 457.5625$.

**(c)** The rock will hit the ground after about 10.66 sec.

**85.** **(a)** $V = $ (height)(width)(length) $= x(30 - 2x)(70 - 2x)$ in$^3$

**(b)** Either $x \approx 4.57$ in. or $x \approx 8.63$ in.

**86.** **(a)** and **(b)**



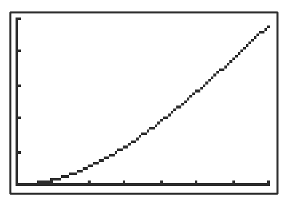
[0, 255] by [0, 2.5]

**(c)** When $d \approx 170$ ft, $s \approx 2.088$ ft.

**(d)** One possibility: The beam may taper off (become thinner) from west to east — e.g., perhaps it measures 8 in by 8 in at the west end, but only 7 in by 7 in on the east end. Then we would expect the beam to bend more easily closer to the east end (though not at the extreme east end, since it is anchored to the piling there). Another possibility: The two pilings are made of different materials.

**87.** **(a)** The tank is made up of a cylinder, with volume $\pi x^2(140 - 2x)$, and a sphere, with volume $\frac{4}{3}\pi x^3$. Thus, $V = \frac{4}{3}\pi x^3 + \pi x^2(140 - 2x)$.

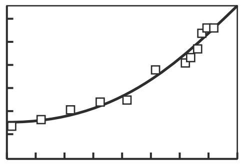
**(b)**



[0, 70] by [0, 1,500,000]

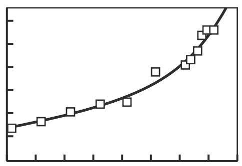
**(c)** The largest volume occurs when $x = 70$ (so it is actually a sphere). This volume is $\frac{4}{3}\pi(70)^3 \approx 1{,}436{,}755$ ft$^3$.

**88. (a)** $y = 3.404x^2 - 13.495x + 1578.73$



[0, 40] by [0, 6000]

**(b)** $y = 0.0023x^4 - 0.0305x^3 - 0.4917x^2 + 70.456x + 1319.51$

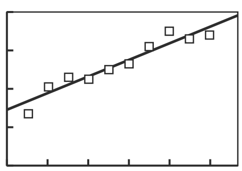


[0, 40] by [0, 6000]

**(c)** For $x = 40$, the quadratic model yields $y \approx \$6485$ and the quartic model yields $y \approx \$7374$.

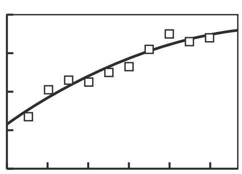
**(d)** The quadratic and quartic models, which both have a positive leading coefficient, predict that the amount of the Pell Grant will always increase.

**89. (a)** $y = 0.435x + 26.016$



[2, 13] by [24, 32]

**(b)** $y = -0.246x^2 + 0.804x + 24.835$



[2, 13] by [24, 32]

**(c)** Linear: Spending will continue to increase.

Quadratic: Spending is nearing its maximum and will then begin to decrease.

**90. (a)** Each shinguard costs \$4.32 plus a fraction of the overhead: $C = 4.32 + 4000/x$.

**(b)** Solve $x(5.25 - 4.32 - 4000/x) = 8000$: $0.93x = 12{,}000$, so $x \approx 12{,}903.23$ — round up to 12,904.

**91. (a)** $P(15) = 325$, $P(70) = 600$, $P(100) = 648$

**(b)** $y = \dfrac{640}{0.8} = 800$

**(c)** The deer population approaches (but never equals) 800.

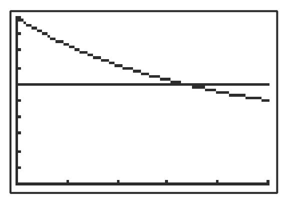
**92. (a)** $\dfrac{1}{1.2} = \dfrac{1}{x} + \dfrac{1}{R_2}$, so $\dfrac{1}{R_2} = \dfrac{1}{1.2} - \dfrac{1}{x} = \dfrac{x - 1.2}{1.2x}$.

Then, $R_2 = \dfrac{1.2x}{x - 1.2}$.

**(b)** When $x = 3$, $R_2 = \dfrac{3.6}{3 - 1.2} = \dfrac{3.6}{1.8} = 2$ ohms.

**93. (a)** $C(x) = \dfrac{50}{50 + x}$

**(b)** Shown is the window $[0, 50]$ by $[0, 1]$, with the graphs of $y = C(x)$ and $y = 0.6$. The two graphs cross when $x \approx 33.33$ ounces of distilled water.



$[0, 50]$ by $[0, 1]$

**(c)** Algebraic solution of $\dfrac{50}{50 + x} = 0.6$ leads to $50 = 0.6(50 + x)$, so that $0.6x = 20$, or $x = \dfrac{100}{3} \approx 33.33$.

**94. (a)** Let $h$ be the height (in cm) of the can; we know the volume is 1 L $= 1000\text{ cm}^3 = \pi x^2 h$, so $h = \dfrac{1000}{\pi x^2}$. Then $S = 2\pi x^2 + 2\pi x h = 2\pi x^2 + 2000/x$.

**(b)** Solve $2\pi x^2 + 2000/x = 900$, or equivalently, $2\pi x^3 - 900x + 2000 = 0$. Graphically we find that either $x \approx 2.31$ cm and $h \approx 59.75$ cm, or $x \approx 10.65$ and $h \approx 2.81$ cm.

**(c)** Approximately $2.31 < x < 10.65$ (graphically) and $2.81 < h < 59.75$.

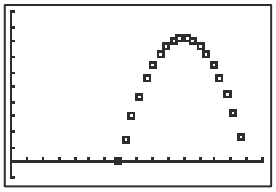
**95. (a)** Let $y$ be the height of the tank; $1000 = x^2 y$, so $y = 1000/x^2$. The surface area equals the area of the base plus 4 times the area of one side. Each side is a rectangle with dimensions $x \times y$, so $S = x^2 + 4xy = x^2 + 4000/x$.

**(b)** Solve $x^2 + 4000/x = 600$, or $x^3 - 600x + 4000 = 0$ (a graphical solution is easiest): Either $x = 20$, giving dimensions 20 ft by 20 ft by 2.5 ft or $x \approx 7.32$, giving approximate dimensions 7.32 by 7.32 by 18.66.

**(c)** $7.32 < x < 20$ (lower bound approximate), so $y$ must be between 2.5 ft and about 18.66 ft.

## Chapter 2 Project

Answers are based on the sample data shown in the table.

1. 

[0, 1.6] by [−0.1, 1]

2. We estimate the vertex to lie halfway between the two data points with the greatest height, so that $h$ is the average of 1.075 and 1.118, or about 1.097. We estimate $k$ to be 0.830, which is slightly greater than the greatest height in the data, 0.828.
Noting that $y = 0$ when $x = 0.688$, we solve $0 = a(0.688 - 1.097)^2 + 0.830$ to find $a \approx -4.962$. So the estimated quadratic model is $y = -4.962(x - 1.097)^2 + 0.830$.

3. The sign of $a$ affects the direction the parabola opens. The magnitude of $a$ affects the vertical stretch of the graph. Changes to $h$ cause horizontal shifts to the graph, while changes to $k$ cause vertical shifts.

4. $y \approx -4.962x^2 + 10.887x - 5.141$

5. $y \approx -4.968x^2 + 10.913x - 5.160$

6. $$\begin{aligned} y &\approx -4.968x^2 + 10.913x - 5.160 \\ &\approx -4.968\,(x^2 - 2.1967x + 1.0386) \\ &= -4.968\left[x^2 - 2.1967x + \left(\frac{2.1967}{2}\right)^2 - \left(\frac{2.1967}{2}\right)^2 + 1.0386\right] \\ &= -4.968\left[\left(x - \frac{2.1967}{2}\right)^2 - 0.1678\right] \\ &\approx -4.968\,(x - 1.098)^2 + 0.833 \end{aligned}$$