Chapter 2
Polynomial, Power, and Rational Functions

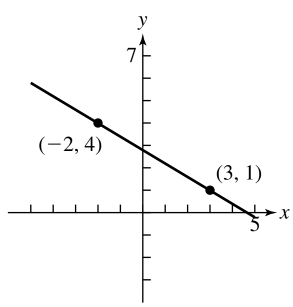
■ Section 2.1 Linear and Quadratic Functions and Modeling

Exploration 1

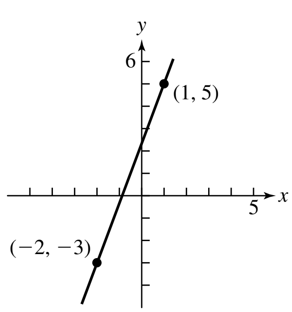
1. $-\$2000$ per year.
2. The equation will have the form $v(t) = mt + b$. The value of the building after 0 year is $v(0) = m(0) + b = b = 50{,}000$.
   The slope m is the rate of change, which is -2000 (dollars per year). So an equation for the value of the building (in dollars) as a function of the time (in years) is $v(t) = -2000t + 50{,}000$.
3. $v(0) = \$50{,}000$ and $v(16) = -2000(16) + 50{,}000 = 18{,}000$ dollars.
4. The equation $v(t) = 39{,}000$ becomes
$$\begin{aligned} -2000t + 50{,}000 &= 39{,}000 \\ -2000t &= -11{,}000 \\ t &= 5.5 \text{ yr} \end{aligned}$$

Quick Review 2.1

1. $y = 8x + 3.6$
2. $y = -1.8x - 2$
3. $y - 4 = -\dfrac{3}{5}(x + 2)$, or $y = -0.6x + 2.8$



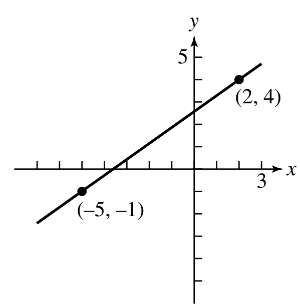
4. $y - 5 = \dfrac{8}{3}(x - 1)$, or $y = \dfrac{8}{3}x + \dfrac{7}{3}$



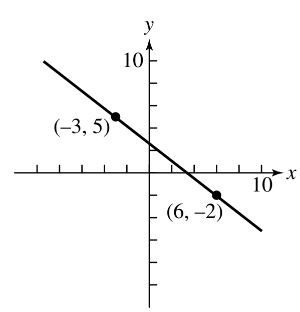
5. $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9$
   $= x^2 + 6x + 9$
6. $(x - 4)^2 = (x - 4)(x - 4) = x^2 - 4x - 4x + 16$
   $= x^2 - 8x + 16$
7. $3(x - 6)^2 = 3(x - 6)(x - 6) = (3x - 18)(x - 6)$
   $= 3x^2 - 18x - 18x + 108 = 3x^2 - 36x + 108$
8. $-3(x + 7)^2 = -3(x + 7)(x + 7)$
   $= (-3x - 21)(x + 7) = -3x^2 - 21x - 21x - 147$
   $= -3x^2 - 42x - 147$
9. $2x^2 - 4x + 2 = 2(x^2 - 2x + 1) = 2(x - 1)(x - 1)$
   $= 2(x - 1)^2$
10. $3x^2 + 12x + 12 = 3(x^2 + 4x + 4) = 3(x + 2)(x + 2)$
    $= 3(x + 2)^2$

Section 2.1 Exercises

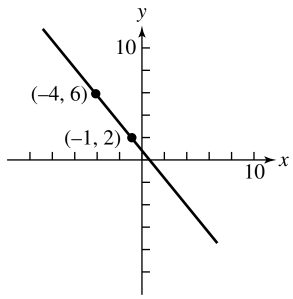
1. Not a polynomial function because of the exponent -5
2. Polynomial of degree 1 with leading coefficient 2
3. Polynomial of degree 5 with leading coefficient 2
4. Polynomial of degree 0 with leading coefficient 13
5. Not a polynomial function because of the radical
6. Polynomial of degree 2 with leading coefficient -5
7. $m = \dfrac{5}{7}$ so $y - 4 = \dfrac{5}{7}(x - 2) \Rightarrow f(x) = \dfrac{5}{7}x + \dfrac{18}{7}$



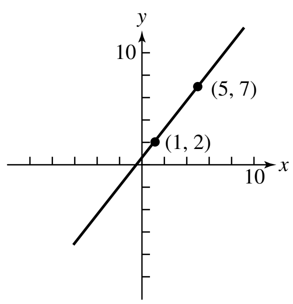
8. $m = -\dfrac{7}{9}$ so $y - 5 = -\dfrac{7}{9}(x + 3) \Rightarrow f(x) = -\dfrac{7}{9}x + \dfrac{8}{3}$



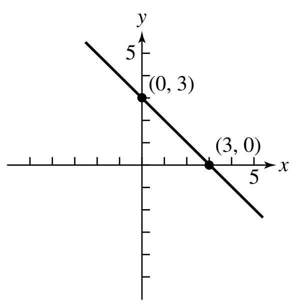
9. $m = -\frac{4}{3}$ so $y - 6 = -\frac{4}{3}(x + 4) \Rightarrow f(x) = -\frac{4}{3}x + \frac{2}{3}$



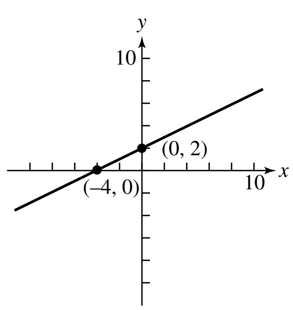
10. $m = \frac{5}{4}$ so $y - 2 = \frac{5}{4}(x - 1) \Rightarrow f(x) = \frac{5}{4}x + \frac{3}{4}$



11. $m = -1$ so $y - 3 = -1(x - 0) \Rightarrow f(x) = -x + 3$



12. $m = \frac{1}{2}$ so $y - 2 = \frac{1}{2}(x - 0) \Rightarrow f(x) = \frac{1}{2}x + 2$



13. (a)—The vertex is at $(-1, -3)$, in Quadrant III, eliminating all but (a) and (d). Since $f(0) = -1$, it must be (a).

14. (d)—The vertex is at $(-2, -7)$, in Quadrant III, eliminating all but (a) and (d). Since $f(0) = 5$, it must be (d).

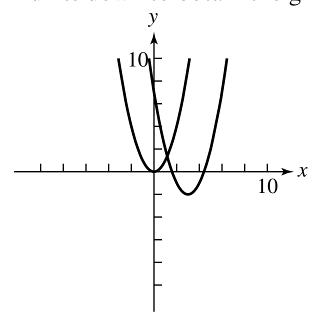
15. (b)—The vertex is in Quadrant I, at $(1, 4)$, meaning it must be either (b) or (f). Since $f(0) = 1$, it cannot be (f): if the vertex in (f) is $(1, 4)$, then the intersection with the y-axis would be about $(0, 3)$. It must be (b).

16. (f)—The vertex is in Quadrant I, at $(1, 12)$, meaning it must be either (b) or (f). Since $f(0) = 10$, it cannot be (b): if the vertex in (b) is $(1, 12)$, then the intersection with the y-axis occurs considerably lower than $(0, 10)$. It must be (f).

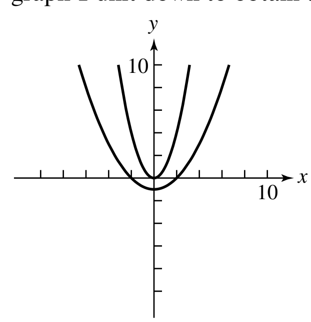
17. (e)—The vertex is at $(1, -3)$ in Quadrant IV, so it must be (e).

18. (c)—The vertex is at $(-1, 12)$ in Quadrant II and the parabola opens down, so it must be (c).

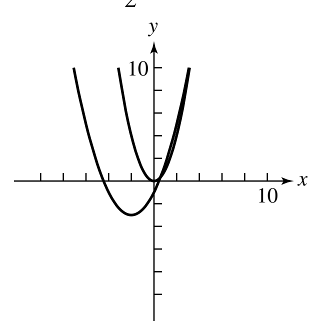
19. Translate the graph of $f(x) = x^2$ 3 units right to obtain the graph of $h(x) = (x - 3)^2$, and translate this graph 2 units down to obtain the graph of $g(x) = (x - 3)^2 - 2$.



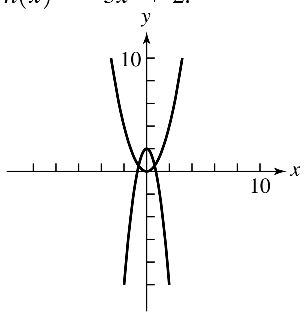
20. Vertically shrink the graph of $f(x) = x^2$ by a factor of $\frac{1}{4}$ to obtain the graph of $g(x) = \frac{1}{4}x^2$, and translate this graph 1 unit down to obtain the graph of $h(x) = \frac{1}{4}x^2 - 1$.



21. Translate the graph of $f(x) = x^2$ 2 units left to obtain the graph of $h(x) = (x + 2)^2$, vertically shrink this graph by a factor of $\frac{1}{2}$ to obtain the graph of $k(x) = \frac{1}{2}(x + 2)^2$, and translate this graph 3 units down to obtain the graph of $g(x) = \frac{1}{2}(x + 2)^2 - 3$.



22. Vertically stretch the graph of $f(x) = x^2$ by a factor of 3 to obtain the graph of $g(x) = 3x^2$, reflect this graph across the x-axis to obtain the graph of $k(x) = -3x^2$, and translate this graph up 2 units to obtain the graph of $h(x) = -3x^2 + 2$.



For #23–32, with an equation of the form $f(x) = a(x - h)^2 + k$, the vertex is (h, k) and the axis is $x = h$.

23. Vertex: $(1, 5)$; axis: $x = 1$

24. Vertex: $(-2, -1)$; axis: $x = -2$

25. Vertex: $(1, -7)$; axis: $x = 1$

26. Vertex: $(\sqrt{3}, 4)$; axis: $x = \sqrt{3}$

27. $f(x) = 3\left(x^2 + \frac{5}{3}x\right) - 4$

$= 3\left(x^2 + 2 \cdot \frac{5}{6}x + \frac{25}{36}\right) - 4 - \frac{25}{12} = 3\left(x + \frac{5}{6}\right)^2 - \frac{73}{12}$

Vertex: $\left(-\frac{5}{6}, -\frac{73}{12}\right)$; axis: $x = -\frac{5}{6}$

28. $f(x) = -2\left(x^2 - \frac{7}{2}x\right) - 3$

$= -2\left(x^2 - 2 \cdot \frac{7}{4}x + \frac{49}{16}\right) - 3 + \frac{49}{8}$

$= -2(x - \frac{7}{4})^2 + \frac{25}{8}$

Vertex: $\left(\frac{7}{4}, \frac{25}{8}\right)$; axis: $x = \frac{7}{4}$

29. $f(x) = -(x^2 - 8x) + 3$
$= -(x^2 - 2 \cdot 4x + 16) + 3 + 16 = -(x - 4)^2 + 19$
Vertex: $(4, 19)$; axis: $x = 4$

30. $f(x) = 4\left(x^2 - \frac{1}{2}x\right) + 6$

$= 4\left(x^2 - 2 \cdot \frac{1}{4}x + \frac{1}{16}\right) + 6 - \frac{1}{4} = 4\left(x - \frac{1}{4}\right)^2 + \frac{23}{4}$

Vertex: $\left(\frac{1}{4}, \frac{23}{4}\right)$; axis: $x = \frac{1}{4}$

31. $g(x) = 5\left(x^2 - \frac{6}{5}x\right) + 4$

$= 5\left(x^2 - 2 \cdot \frac{3}{5}x + \frac{9}{25}\right) + 4 - \frac{9}{5} = 5\left(x - \frac{3}{5}\right)^2 + \frac{11}{5}$

Vertex: $\left(\frac{3}{5}, \frac{11}{5}\right)$; axis: $x = \frac{3}{5}$

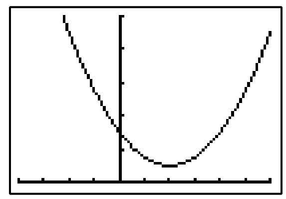
32. $h(x) = -2\left(x^2 + \frac{7}{2}x\right) - 4$

$= -2\left(x^2 + 2 \cdot \frac{7}{4}x + \frac{49}{16}\right) - 4 + \frac{49}{8}$

$= -2\left(x + \frac{7}{4}\right)^2 + \frac{17}{8}$

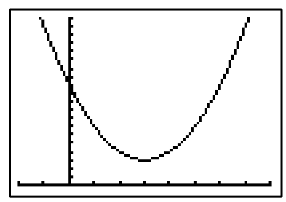
Vertex: $\left(-\frac{7}{4}, \frac{17}{8}\right)$; axis: $x = -\frac{7}{4}$

33. $f(x) = (x^2 - 4x + 4) + 6 - 4 = (x - 2)^2 + 2$.
Vertex: $(2, 2)$; axis: $x = 2$; opens upward; does not intersect x-axis.



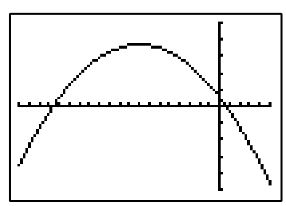
$[-4, 6]$ by $[0, 20]$

34. $g(x) = (x^2 - 6x + 9) + 12 - 9 = (x - 3)^2 + 3$.
Vertex: $(3, 3)$; axis: $x = 3$; opens upward; does not intersect x-axis.



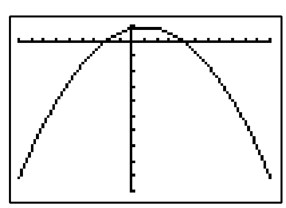
$[-2, 8]$ by $[0, 20]$

35. $f(x) = -(x^2 + 16x) + 10$
$= -(x^2 + 16x + 64) + 10 + 64 = -(x + 8)^2 + 74$.
Vertex: $(-8, 74)$; axis: $x = -8$; opens downward; intersects x-axis at about -16.602 and $0.602 (-8 \pm \sqrt{74})$.



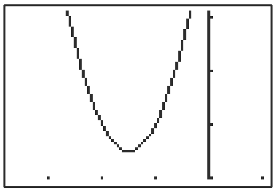
$[-20, 5]$ by $[-100, 100]$

36. $h(x) = -(x^2 - 2x) + 8 = -(x^2 - 2x + 1) + 8 + 1$
$= -(x - 1)^2 + 9$
Vertex: $(1, 9)$; axis: $x = 1$; opens downward; intersects x-axis at -2 and 4.



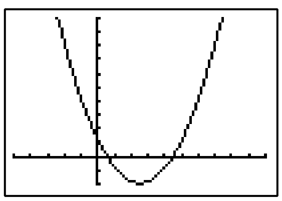
$[-9, 11]$ by $[-100, 10]$

37. $f(x) = 2(x^2 + 3x) + 7$
$= 2\left(x^2 + 3x + \frac{9}{4}\right) + 7 - \frac{9}{2} = 2\left(x + \frac{3}{2}\right)^2 + \frac{5}{2}.$
Vertex: $\left(-\frac{3}{2}, \frac{5}{2}\right)$; axis: $x = -\frac{3}{2}$; opens upward; does not intersect the x-axis; vertically stretched by 2.



$[-3.7, 1]$ by $[2, 5.1]$

38. $g(x) = 5(x^2 - 5x) + 12$
$= 5\left(x^2 - 5x + \frac{25}{4}\right) + 12 - \frac{125}{4}$
$= 5\left(x - \frac{5}{2}\right)^2 - \frac{77}{4}.$
Vertex: $\left(\frac{5}{2}, -\frac{77}{4}\right)$; axis: $x = \frac{5}{2}$; opens upward; intersects x-axis at about 0.538 and 4.462 $\left(\text{or } \frac{5}{2} \pm \frac{1}{10}\sqrt{385}\right)$; vertically stretched by 5.



$[-5, 10]$ by $[-20, 100]$

For #39–44, use the form $y = a(x - h)^2 + k$, taking the vertex (h, k) from the graph or other given information.

39. $h = -1$ and $k = -3$, so $y = a(x + 1)^2 - 3$. Now substitute $x = 1$, $y = 5$ to obtain $5 = 4a - 3$, so $a = 2$: $y = 2(x + 1)^2 - 3$.

40. $h = 2$ and $k = -7$, so $y = a(x - 2)^2 - 7$. Now substitute $x = 0$, $y = 5$ to obtain $5 = 4a - 7$, so $a = 3$: $y = 3(x - 2)^2 - 7$.

41. $h = 1$ and $k = 11$, so $y = a(x - 1)^2 + 11$. Now substitute $x = 4$, $y = -7$ to obtain $-7 = 9a + 11$, so $a = -2$: $y = -2(x - 1)^2 + 11$.

42. $h = -1$ and $k = 5$, so $y = a(x + 1)^2 + 5$. Now substitute $x = 2$, $y = -13$ to obtain $-13 = 9a + 5$, so $a = -2$: $y = -2(x + 1)^2 + 5$.

43. $h = 1$ and $k = 3$, so $y = a(x - 1)^2 + 3$. Now substitute $x = 0$, $y = 5$ to obtain $5 = a + 3$, so $a = 2$: $y = 2(x - 1)^2 + 3$.

44. $h = -2$ and $k = -5$, so $y = a(x + 2)^2 - 5$. Now substitute $x = -4$, $y = -27$ to obtain $-27 = 4a - 5$, so $a = -\frac{11}{2}$: $y = -\frac{11}{2}(x + 2)^2 - 5$.

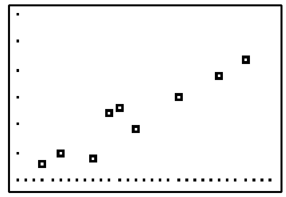
45. Strong positive

46. Strong negative

47. Weak positive

48. No correlation

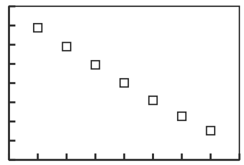
49. **(a)**



$[15, 45]$ by $[20, 50]$

(b) Strong, positive, linear; $r = 0.948$

50. **(a)**



$[0, 80]$ by $[0, 80]$

(b) Strong, negative, linear; $r = -0.999$

51. $m = -\frac{2350}{5} = -470$ and $b = 2350$, so $v(t) = -470t + 2350$.
At $t = 3$, $v(3) = (-470)(3) + 2350 = \$940$.

52. Let x be the number of dolls produced each week and y be the average weekly costs. Then $m = 4.70$, and $b = 350$, so $y = 4.70x + 350$, or $500 = 4.70x + 350$: $x = 32$; 32 dolls are produced each week.

53. **(a)** $y \approx 0.148x + 19.56$. The slope, $m \approx 0.148$, represents the average annual increase in fuel economy for light duty trucks, about 0.15 mpg per year.

(b) Setting $x = 25$ in the regression equation leads to $y \approx 24$ mpg.

54. If the length is x, then the width is $50 - x$, so $A(x) = x(50 - x)$; maximum of 625 ft^2 when $x = 25$ (the dimensions are 25 ft $\times$ 25 ft).

55. **(a)** $[0, 100]$ by $[0, 1000]$ is one possibility.

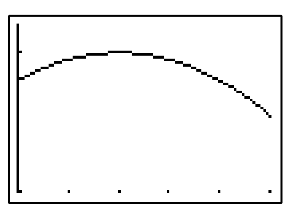
(b) When $x \approx 107.335$ or $x \approx 372.665$ — either 107,335 units or 372,665 units.

56. The area of the picture and the frame, if the width of the picture is x ft, is $A(x) = (x + 2)(x + 5)$ ft^2. This equals 208 when $x = 11$, so the painting is 11 ft $\times$ 14 ft.

57. If the strip is x feet wide, the area of the strip is $A(x) = (25 + 2x)(40 + 2x) - 1000$ ft^2.
This equals 504 ft^2 when $x = 3.5$ ft.

58. **(a)** $R(x) = (800 + 20x)(300 - 5x)$.

(b) $[0, 25]$ by $[200{,}000, 260{,}000]$ is one possibility (shown).

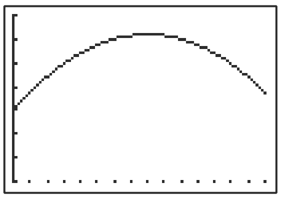


$[0, 25]$ by $[200{,}000, 260{,}000]$

(c) The maximum income — $250,000 — is achieved when $x = 10$, corresponding to rent of $250 per month.

59. (a) $R(x) = (26{,}000 - 1000x)(0.50 + 0.05x)$.

(b) Many choices of Xmax and Ymin are reasonable. Shown is [0, 15] by [10,000, 17,000].



[0, 15] by [10000, 17000]

(c) The maximum revenue — $16,200 — is achieved when $x = 8$; that is, charging 90 cents per can.

60. Total sales would be $S(x) = (30 + x)(50 - x)$ thousand dollars, when x additional salespeople are hired. The maximum occurs when $x = 10$ (halfway between the two zeros, at -30 and 50).

61. (a) $g \approx 32$ ft/sec^2. $s_0 = 83$ ft and $v_0 = 92$ ft/sec. So the models are height $= s(t) = -16t^2 + 92t + 83$ and vertical velocity $= v(t) = -32t + 92$. The maximum height occurs at the vertex of $s(t)$.

$$h = -\frac{b}{2a} = -\frac{92}{2(-16)} = 2.875, \text{ and}$$

$k = s(2.875) = 215.25$. The maximum height of the baseball is about 215 ft above the field.

(b) The amount of time the ball is in the air is a zero of $s(t)$. Using the quadratic formula, we obtain

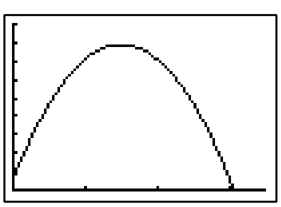
$$t = \frac{-92 \pm \sqrt{92^2 - 4(-16)(83)}}{2(-16)}$$
$$= \frac{-92 \pm \sqrt{13{,}776}}{-32} \approx -0.79 \text{ or } 6.54. \text{ Time is not}$$

negative, so the ball is in the air about 6.54 seconds.

(c) To determine the ball's vertical velocity when it hits the ground, use $v(t) = -32t + 92$, and solve for $t = 6.54$. $v(6.54) = -32(6.54) + 92 \approx -117$ ft/sec when it hits the ground.

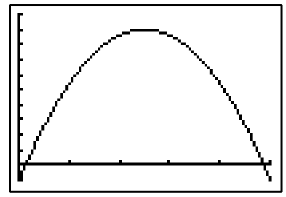
62. (a) $h = -16t^2 + 48t + 3.5$.

(b) The maximum height is 39.5 ft, 1.5 sec after it is thrown.



[0, 3.5] by [0, 45]

63. (a) $h = -16t^2 + 80t - 10$. The graph is shown in the window [0, 5] by [−10, 100].



[0, 5] by [−10, 100]

(b) The maximum height is 90 ft, 2.5 sec after it is shot.

64. The exact answer is $32\sqrt{3}$, or about 55.426 ft/sec. In addition to the guess-and-check strategy suggested, this can be found algebraically by noting that the vertex of the parabola $y = ax^2 + bx + c$ has y coordinate $c - \frac{b^2}{4a} = \frac{b^2}{64}$ (note $a = -16$ and $c = 0$), and setting this equal to 48.

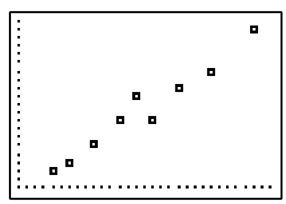
65. The quadratic regression is $y \approx 0.3721x^2 + 2.697x + 111.264$. Plot this curve together with the curve $y = 350$, and then find the intersection to find when the number of patent applications will reach 350,000. Note that we use $y = 350$ because the data were given as a number of thousands. The intersection occurs at $x \approx 22.0$, so the number of applications will reach 350,000 approximately 22 years after 1980—in 2002.

66. (a) $m = \frac{6 \text{ ft}}{100 \text{ ft}} = 0.06$

(b) $r \approx 4167$ ft, or about 0.79 mi.

(c) 2217.6 ft

67. (a)

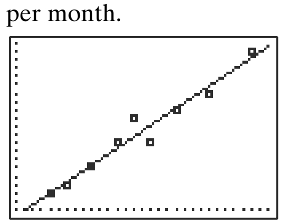


[15, 45] by [20, 40]

(b) $y \approx 0.68x + 9.01$

(c) On average, the children gained 0.68 lb per month.

(d)



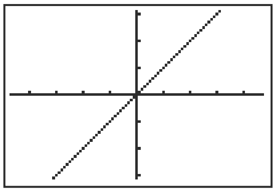
[15, 45] by [20, 40]

(e) ≈ 29.41 lbs

68. (a) The linear regression is $y \approx 255.19x + 4039.43$, where x represents the number of years since 1940.

(b) 2020 is 80 years after 1940, so substitute 80 into the equation to predict the median income of women in 2020.
$y = 255.19(80) + 4039.43 \approx \$24{,}455$.

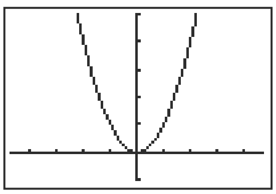
69. The Identity Function $f(x) = x$



[–4.7, 4.7] by [–3.1, 3.1]

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Continuity: The function is continuous on its domain.
Increasing–decreasing behavior: Increasing for all x
Symmetry: Symmetric about the origin
Boundedness: Not bounded
Local extrema: None
Horizontal asymptotes: None
Vertical asymptotes: None
End behavior: $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$

70. The Squaring Function $f(x) = x^2$



[–4.7, 4.7] by [–1, 5]

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Continuity: The function is continuous on its domain.
Increasing–decreasing behavior: Increasing on $[0, \infty)$, decreasing on $(-\infty, 0]$.
Symmetry: Symmetric about the y-axis
Boundedness: Bounded below, but not above
Local extrema: Local minimum of 0 at $x = 0$
Horizontal asymptotes: None
Vertical asymptotes: None
End behavior: $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = \infty$

71. False. For $f(x) = 3x^2 + 2x - 3$, the initial value is $f(0) = -3$.

72. True. By completing the square, we can rewrite $f(x)$ so that $f(x) = \left(x^2 - x + \frac{1}{4}\right) + 1 - \frac{1}{4}$
$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$. Since $f(x) \geq \frac{3}{4}$, $f(x) > 0$ for all x.

73. $m = \frac{1-3}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3}$. The answer is E.

74. $f(x) = mx + b$

$$3 = -\frac{1}{3}(-2) + b$$

$$3 = \frac{2}{3} + b$$

$b = 3 - \frac{2}{3} = \frac{7}{3}$. The answer is C.

For #75–76, $f(x) = 2(x+3)^2 - 5$ corresponds to $f(x) = a(x-h)^2 + k$ with $a = 2$ and $(h, k) = (-3, -5)$.

75. The axis of symmetry runs vertically through the vertex: $x = -3$. The answer is B.

76. The vertex is $(h, k) = (-3, -5)$. The answer is E.

77. (a) Graphs (i), (iii), and (v) are linear functions. They can all be represented by an equation $y = ax + b$, where $a \neq 0$.

(b) In addition to graphs (i), (iii), and (v), graphs (iv) and (vi) are also functions, the difference is that (iv) and (vi) are *constant* functions, represented by $y = b, b \neq 0$.

(c) (ii) is not a function because a single value x (i.e., $x = -2$) results in a multiple number of y-values. In fact, there are infinitely many y-values that are valid for the equation $x = -2$.

78. (a) $\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = 4$

(b) $\frac{f(5) - f(2)}{5 - 2} = \frac{25 - 4}{3} = 7$

(c) $\frac{f(c) - f(a)}{c - a} = \frac{c^2 - a^2}{c - a} = \frac{(c-a)(c+a)}{c - a} = c + a$

(d) $\frac{g(3) - g(1)}{3 - 1} = \frac{11 - 5}{2} = 3$

(e) $\frac{g(4) - g(1)}{4 - 1} = \frac{14 - 5}{3} = 3$

(f) $\frac{g(c) - g(a)}{c - a} = \frac{(3c + 2) - (3a + 2)}{c - a}$
$= \frac{3c - 3a}{c - a} = 3$

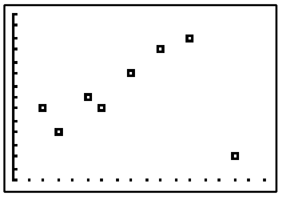
(g) $\frac{h(c) - h(a)}{c - a} = \frac{(7c - 3) - (7a - 3)}{c - a}$
$= \frac{7c - 7a}{c - a} = 7$

(h) $\frac{k(c) - k(a)}{c - a} = \frac{(mc + b) - (ma + b)}{c - a}$
$= \frac{mc - ma}{c - a} = m$

(i) $\frac{l(c) - l(a)}{c - a} = \frac{c^3 - a^3}{c - a} = \frac{-2b}{2a} = \frac{-b}{a} = -\frac{b}{a}$
$= \frac{(c - a)(c^2 + ac + a^2)}{(c - a)} = c^2 + ac + a^2$

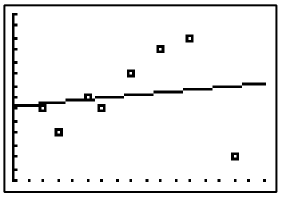
79. The line that minimizes the sum of the squares of vertical distances is nearly always different from the line that minimizes the sum of the squares of horizontal distances to the points in a scatter plot. For the data in Table 2.2, the regression line obtained from reversing the ordered pairs has a slope of $-\frac{1}{15{,}974.90}$; whereas, the inverse of the function in Example 3 has a slope of $-\frac{1}{15{,}358.93}$ — close but not the same slope.

80. (a)



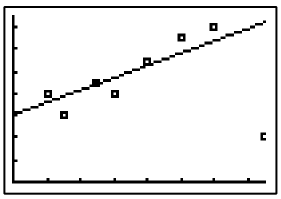
[0, 17] by [2, 16]

(b) $y \approx 0.115x + 8.245$



[0, 17] by [2, 16]

(c) $y \approx 0.556x + 6.093$



[0, 15] by [0, 15]

(d) The median–median line appears to be the better fit, because it approximates more of the data values more closely.

81. (a) If $ax^2 + bx^2 + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ by the quadratic formula. Thus, $x_1 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ and

$$x_1 + x_2 = \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2b}{2a} = \frac{-b}{a} = -\frac{b}{a}.$$

(b) Similarly,

$$x_1 \cdot x_2 = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}.$$

82. $f(x) = (x - a)(x - b) = x^2 - bx - ax + ab$ $= x^2 + (-a - b)x + ab$. If we use the vertex form of a quadratic function, we have $h = -\left(\dfrac{-a - b}{2}\right)$ $= \dfrac{a + b}{2}$. The axis is $x = h = \dfrac{a + b}{2}$.

83. Multiply out $f(x)$ to get $x^2 - (a + b)x + ab$. Complete the square to get $\left(x - \dfrac{a + b}{2}\right)^2 + ab - \dfrac{(a + b)^2}{4}$. The vertex is then (h, k) where $h = \dfrac{a + b}{2}$ and $k = ab - \dfrac{(a + b)^2}{4} = -\dfrac{(a - b)^2}{4}$.

84. x_1 and x_2 are given by the quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; then $x_1 + x_2 = -\dfrac{b}{a}$, and the line of symmetry is $x = -\dfrac{b}{2a}$, which is exactly equal to $\dfrac{x_1 + x_2}{2}$.

85. The Constant Rate of Change Theorem states that a function defined on all real numbers is a linear function if and only if it has a constant nonzero average rate of change between any two points on its graph. To prove this, suppose $f(x) = mx + b$ with m and b constants and $m \neq 0$. Let x_1 and x_2 be real numbers with $x_1 \neq x_2$. Then the average rate of change is

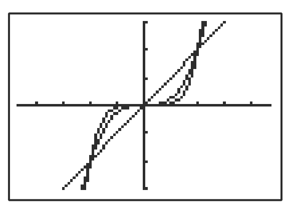
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{(mx_2 + b) - (mx_1 + b)}{x_2 - x_1} =$$
$$\frac{mx_2 - mx_1}{x_2 - x_1} = \frac{m(x_2 - x_1)}{x_2 - x_1} = m, \text{ a nonzero constant.}$$

Now suppose that m and x_1 are constants, with $m \neq 0$. Let x be a real number such that $x \neq x_1$, and let f be a function defined on all real numbers such that $\dfrac{f(x) - f(x_1)}{x - x_1} = m$. Then $f(x) - f(x_1) = m(x - x_1) =$ $mx - mx_1$, and $f(x) = mx + (f(x_1) - mx_1)$. $f(x_1) - mx_1$ is a constant; call it b. Then $f(x_1) - mx_1 = b$; so, $f(x_1) = b + mx_1$ and $f(x) = b + mx$ for all $x \neq x_1$. Thus, f is a linear function.

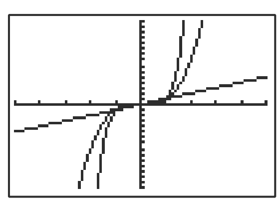
■ Section 2.2 Power Functions with Modeling

Exploration 1

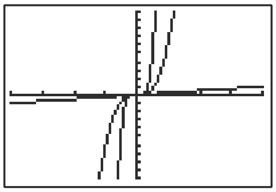
1.



[−2.35, 2.35] by [−1.5, 1.5]



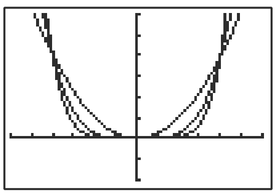
[−5, 5] by [−15, 15]



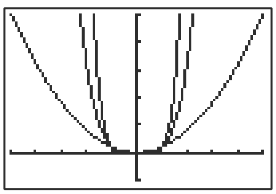
$[-20, 20]$ by $[-200, 200]$

The pairs $(0, 0)$, $(1, 1)$, and $(-1, -1)$ are common to all three graphs. The graphs are similar in that if $x < 0$, $f(x)$, $g(x)$, and $h(x) < 0$ and if $x > 0$, $f(x)$, $g(x)$, and $h(x) > 0$. They are different in that if $|x| < 1$, $f(x)$, $g(x)$, and $h(x) \to 0$ at dramatically different rates, and if $|x| > 1$, $f(x)$, $g(x)$, and $h(x) \to \infty$ at dramatically different rates.

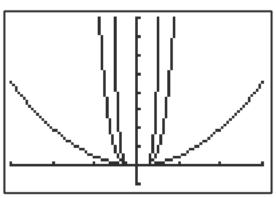
2.



$[-1.5, 1.5]$ by $[-0.5, 1.5]$



$[-5, 5]$ by $[-5, 25]$



$[-15, 15]$ by $[-50, 400]$

The pairs $(0, 0)$, $(1, 1)$, and $(-1, 1)$ are common to all three graphs. The graphs are similar in that for $x \neq 0$, $f(x)$, $g(x)$, and $h(x) > 0$. They are diffferent in that if $|x| < 1$, $f(x)$, $g(x)$, and $h(x) \to 0$ at dramatically different rates, and if $|x| > 1$, $f(x)$, $g(x)$, and $h(x) \to \infty$ at dramatically different rates.

Quick Review 2.2

1. $\sqrt[3]{x^2}$

2. $\sqrt{p^5}$

3. $\dfrac{1}{d^2}$

4. $\dfrac{1}{x^7}$

5. $\dfrac{1}{\sqrt[5]{q^4}}$

6. $\dfrac{1}{\sqrt{m^3}}$

7. $3x^{3/2}$

8. $2x^{5/3}$

9. $\approx 1.71x^{-4/3}$

10. $\approx 0.71x^{-1/2}$

Section 2.2 Exercises

1. power $= 5$, constant $= -\dfrac{1}{2}$

2. power $= \dfrac{5}{3}$, constant $= 9$

3. not a power function

4. power $= 0$, constant $= 13$

5. power $= 1$, constant $= c^2$

6. power $= 5$, constant $= \dfrac{k}{2}$

7. power $= 2$, constant $= \dfrac{g}{2}$

8. power $= 3$, constant $= \dfrac{4\pi}{3}$

9. power $= -2$, constant $= k$

10. power $= 1$, constant $= m$

11. degree $= 0$, coefficient $= -4$

12. not a monomial function; negative exponent

13. degree $= 7$, coefficient $= -6$

14. not a monomial function; variable in exponent

15. degree $= 2$, coefficient $= 4\pi$

16. degree $= 1$, coefficient $= l$

17. $A = ks^2$

18. $V = kr^2$

19. $I = V/R$

20. $V = kT$

21. $E = mc^2$

22. $p = \sqrt{2gd}$

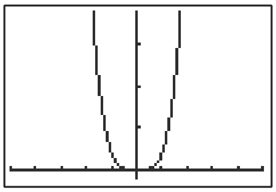
23. The weight w of an object varies directly with its mass m, with the constant of variation g.

24. The circumference C of a circle is proportional to its diameter D, with the constant of variation π.

25. The refractive index n of a medium is inversely proportional to v, the velocity of light in the medium, with constant of variation c, the constant velocity of light in free space.

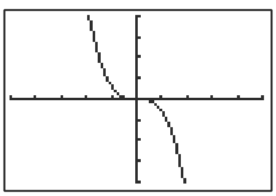
26. The distance d traveled by a free-falling object dropped from rest varies directly with the square of its speed p, with the constant of variation $\dfrac{1}{2g}$.

27. power $= 4$, constant $= 2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Continuous
Decreasing on $(-\infty, 0)$. Increasing on $(0, \infty)$.
Even. Symmetric with respect to y-axis.
Bounded below, but not above
Local minimum at $x = 0$.
Asymptotes: None
End behavior: $\lim_{x\to-\infty} 2x^4 = \infty$, $\lim_{x\to\infty} 2x^4 = \infty$



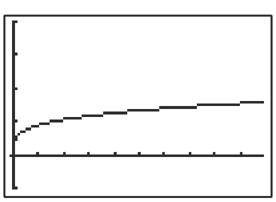
$[-5, 5]$ by $[-1, 49]$

28. power $= 3$, constant $= -3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Continuous
Decreasing for all x.
Odd. Symmetric with respect to origin.
Not bounded above or below
No local extrema
Asymptotes: None
End behavior: $\lim_{x\to-\infty} -3x^3 = \infty$, $\lim_{x\to\infty} -3x^3 = -\infty$



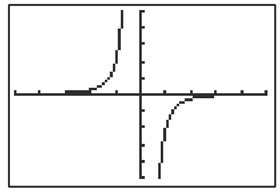
$[-5, 5]$ by $[-20, 20]$

29. power $= \frac{1}{4}$, constant $= \frac{1}{2}$
Domain: $[0, \infty)$
Range: $[0, \infty)$
Continuous
Increasing on $[0, \infty)$.
Bounded below
Neither even nor odd
Local minimum at $(0, 0)$
Asymptotes: None
End behavior: $\lim_{x\to\infty} \frac{1}{2}\sqrt[4]{x} = \infty$



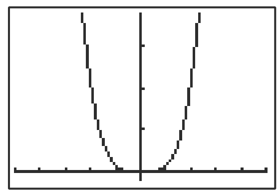
$[-1, 99]$ by $[-1, 4]$

30. power $= -3$, constant $= -2$
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
Discontinuous at $x = 0$
Increasing on $(-\infty, 0)$. Increasing on $(0, \infty)$.
Odd. Symmetric with respect to origin.
Not bounded above or below
No local extrema
Asymptotes at $x = 0$ and $y = 0$
End behavior: $\lim_{x\to-\infty} -2x^{-3} = 0$, $\lim_{x\to\infty} -2x^{-3} = 0$.



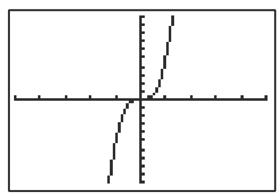
$[-5, 5]$ by $[-5, 5]$

31. Start with $y = x^4$ and shrink vertically by $\frac{2}{3}$. Since $f(-x) = \frac{2}{3}(-x)^4 = \frac{2}{3}x^4$, f is even.



$[-5, 5]$ by $[-1, 19]$

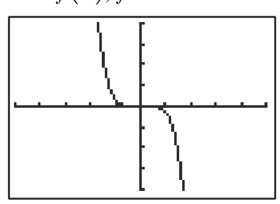
32. Start with $y = x^3$ and stretch vertically by 5. Since $f(-x) = 5(-x)^3 = -5x^3 = -f(x)$, f is odd.



$[-5, 5]$ by $[-20, 20]$

33. Start with $y = x^5$, then stretch vertically by 1.5 and reflect

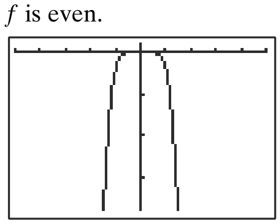
over the x-axis. Since $f(-x) = -1.5(-x)^5 = 1.5x^5$ $= -f(x)$, f is odd.



$[-5, 5]$ by $[-20, 20]$

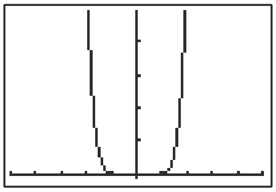
34. Start with $y = x^6$, then stretch vertically by 2 and reflect

over the x-axis. Since $f(-x) = -2(-x)^6 = -2x^6 = f(x)$, f is even.



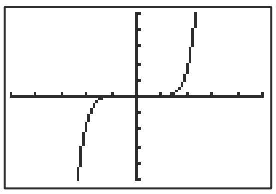
$[-5, 5]$ by $[-19, 1]$

35. Start with $y = x^8$, then shrink vertically by $\frac{1}{4}$. Since $f(-x) = \frac{1}{4}(-x)^8 = \frac{1}{4}x^8 = f(x)$, f is even.



$[-5, 5]$ by $[-1, 49]$

36. Start with $y = x^7$, then shrink vertically by $\frac{1}{8}$. Since $f(-x) = \frac{1}{8}(-x)^7 = -\frac{1}{8}x^7 = -f(x)$, f is odd.



$[-5, 5]$ by $[-50, 50]$

37. (g)

38. (a)

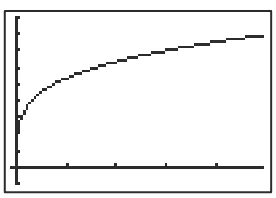
39. (d)

40. (g)

41. (h)

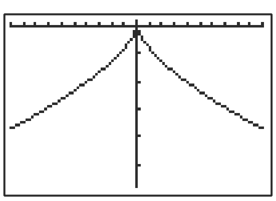
42. (d)

43. $k = 3, a = \frac{1}{4}$. In the first quadrant, the function is increasing. f is undefined for $x < 0$.



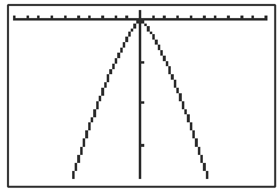
$[-1, 99]$ by $[-1, 10]$

44. $k = -4, a = \frac{2}{3}$. In the fourth quadrant, the function is decreasing. $f(-x) = -4(\sqrt[3]{(-x)^2})$ $= -4\sqrt[3]{x^2} = -4x^{2/3} = f(x)$, so f is even.



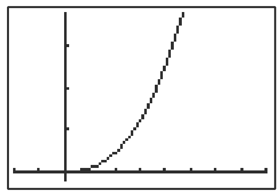
$[-10, 10]$ by $[-29, 1]$

45. $k = -2, a = \frac{4}{3}$. In the fourth quadrant, f is decreasing. $f(-x) = -2(\sqrt[3]{(-x)^4}) = -2(\sqrt[3]{x^4}) = -2x^{4/3} = f(x)$, so f is even.



$[-10, 10]$ by $[-29, 1]$

46. $k = \frac{2}{5}, a = \frac{5}{2}$. In the first quadrant, f is increasing. f is undefined for $x < 0$.

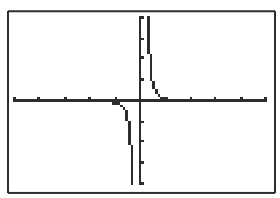


$[-2, 8]$ by $[-1, 19]$

47. $k = \frac{1}{2}, a = -3$. In the first quadrant, f is decreasing.

$$f(-x) = \frac{1}{2}(-x)^{-3} = \frac{1}{2(-x)^3} = -\frac{1}{2}x^{-3} = -f(x),$$

so f is odd.

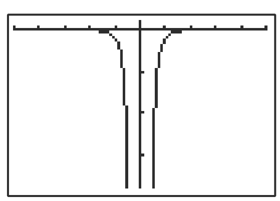


$[-5, 5]$ by $[-20, 20]$

48. $k = -1, a = -4$. In the fourth quadrant, f is increasing.

$$f(-x) = -(-x)^{-4} = -\frac{1}{(-x)^4} = -\frac{1}{x^4} = -x^{-4} = f(x),$$

so f is even.



$[-5, 5]$ by $[-19, 1]$

49. $y = \frac{8}{x^2}$, power $= -2$, constant $= 8$.

50. $y = -2\sqrt{x}$, power $= \frac{1}{2}$, constant $= -2$.

51. $V = \dfrac{kT}{P}$, so $k = \dfrac{PV}{T} = \dfrac{(0.926\text{ atm})(3.46\text{ L})}{302\text{ K}}$

$= 0.0106\,\dfrac{\text{atm} - L}{\text{K}}$

At $P = 1.452$ atm, $V = \dfrac{\left(\dfrac{0.0106\text{ atm} - L}{\text{K}}\right)(302\text{ K})}{1.452\text{ atm}}$

$= 2.21$ L

52. $V = kPT$, so $k = \dfrac{V}{PT} = \dfrac{(3.46\text{ L})}{(0.926\text{ atm})(302\text{ K})}$

$= 0.0124\,\dfrac{\text{L}}{\text{atm} - \text{K}}$

At $T = 338$ K, $V = \left(0.0124\,\dfrac{\text{L}}{\text{atm} - \text{K}}\right)(0.926\text{ atm})$

$(338\text{ K}) = 3.87$ L

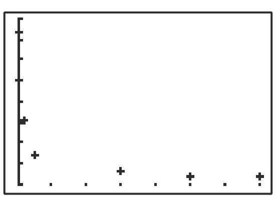
53. $n = \dfrac{c}{v}$, so $v = \dfrac{c}{n} = \dfrac{\left(\dfrac{3.00 \times 10^8\text{ m}}{\text{sec}}\right)}{2.42} = 1.24 \times 10^8\,\dfrac{\text{m}}{\text{sec}}$

54. $P = kv^3$, so $k = \dfrac{P}{v^3} = \dfrac{15\text{ w}}{(10\text{ mph})^3} = 1.5 \times 10^{-2}$

Wind Speed (mph)	Power (W)
10	15
20	120
40	960
80	7680

Since $P = kv^3$ is a cubic, power will increase significantly with only a small increase in wind speed.

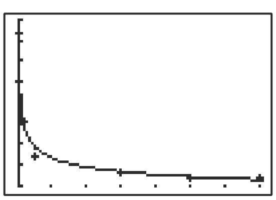
55. (a)



$[-2, 71]$ by $[50, 450]$

(b) $r \approx 231.204 \cdot w^{-0.297}$

(c)



$[-2, 71]$ by $[50, 450]$

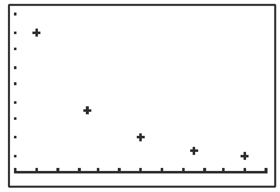
(d) Approximately 37.67 beats/min, which is very close to Clark's observed value.

56. Given that n is an integer, $n \geq 1$:

If n is odd, then $f(-x) = (-x)^n = -(x^n) = -f(x)$ and so $f(x)$ is odd.

If n is even, then $f(-x) = (-x)^n = x^n = f(x)$ and so $f(x)$ is even.

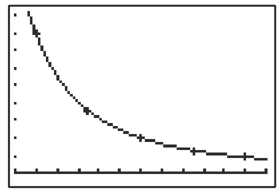
57. (a)



$[0.8, 3.2]$ by $[-0.3, 9.2]$

(b) $y \approx 7.932 \cdot x^{-1.987}$; yes

(c)



$[0.8, 3.2]$ by $[-0.3, 9.2]$

(d) Approximately $2.76\,\dfrac{\text{W}}{\text{m}^2}$ and $0.697\,\dfrac{\text{W}}{\text{m}^2}$, respectively.

58. True, because $f(-x) = (-x)^{-2/3} = [(-x)^2]^{-1/3}$
$= (x^2)^{-1/3} = x^{-2/3} = f(x)$.

59. False. $f(-x) = (-x)^{1/3} = -(x^{1/3}) = -f(x)$ and so the function is odd. It is symmetric about the origin, not the y-axis.

60. $f(4) = 2(4)^{-1/2} = \dfrac{2}{4^{1/2}} = \dfrac{2}{\sqrt{4}} = \dfrac{2}{2} = 1.$

The answer is A.

61. $f(0) = -3(0)^{-1/3} = -3 \cdot \dfrac{1}{0^{1/3}} = -3 \cdot \dfrac{1}{0}$ is undefined.

Also, $f(-1) = -3(-1)^{-1/3} = -3(-1) = 3$,
$f(1) = -3(1)^{-1/3} = -3(1) = -3$, and
$f(3) = -3(3)^{-1/3} \approx -2.08$. The answer is E.

62. $f(-x) = (-x)^{2/3} = [(-x)^2]^{1/3} = (x^2)^{1/3} = x^{2/3} = f(x)$
The function is even. The answer is B.

63. $f(x) = x^{3/2} = (x^{1/2})^3 = (\sqrt{x})^3$ is defined for $x \geq 0$.
The answer is B.

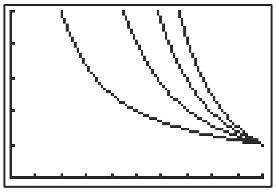
64. Answers will vary. In general, however, students will find

n even: $f(x) = k \cdot x^{m/n} = k \cdot \sqrt[n]{x^m}$, so f is undefined for $x < 0$.

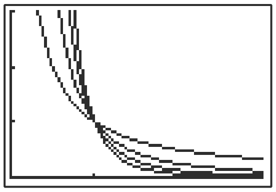
m even, n odd: $f(x) = k \cdot x^{m/n} = k \cdot \sqrt[n]{x^m}$; $f(-x)$
$= k \cdot \sqrt[n]{(-x)^m} = k \cdot \sqrt[n]{x^m} = f(x)$,
so f is even.

m odd, n odd: $f(x) = k \cdot x^{m/n} = k \cdot \sqrt[n]{x^m}$; $f(-x)$
$= k \cdot \sqrt[n]{(-x)^m} = -k \cdot \sqrt[n]{x^m}$
$= -k \cdot x^{m/n} = -f(x)$, so f is odd.

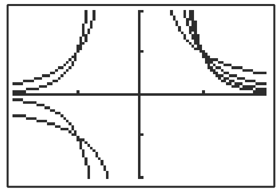
65. (a)



$[0, 1]$ by $[0, 5]$



$[0, 3]$ by $[0, 3]$

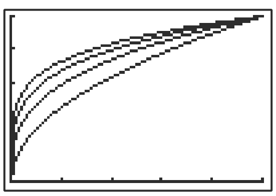


$[-2, 2]$ by $[-2, 2]$

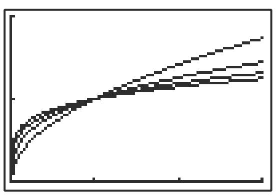
The graphs of $f(x) = x^{-1}$ and $h(x) = x^{-3}$ are similar and appear in the 1st and 3rd quadrants only. The graphs of $g(x) = x^{-2}$ and $k(x) = x^{-4}$ are similar and appear in the 1st and 2nd quadrants only. The pair $(1, 1)$ is common to all four functions.

	f	g	h	k
Domain	$x \neq 0$	$x \neq 0$	$x \neq 0$	$x \neq 0$
Range	$y \neq 0$	$y > 0$	$y \neq 0$	$y > 0$
Continuous	yes	yes	yes	yes
Increasing		$(-\infty, 0)$		$(-\infty, 0)$
Decreasing	$(-\infty, 0), (0, \infty)$	$(0, \infty)$	$(-\infty, 0), (0, \infty)$	$(0, \infty)$
Symmetry	w.r.t. origin	w.r.t. y-axis	w.r.t. origin	w.r.t. y-axis
Bounded	not	below	not	below
Extrema	none	none	none	none
Asymptotes	x-axis, y-axis	x-axis, y-axis	x-axis, y-axis	x-axis, y-axis
End Behavior	$\lim_{x \to \pm\infty} f(x) = 0$	$\lim_{x \to \pm\infty} g(x) = 0$	$\lim_{x \to \pm\infty} h(x) = 0$	$\lim_{x \to \pm\infty} k(x) = 0$

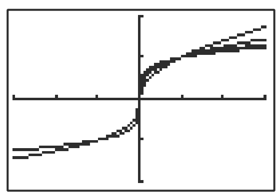
(b)



$[0, 1]$ by $[0, 1]$



$[0, 3]$ by $[0, 2]$

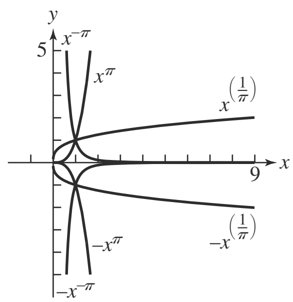


$[-3, 3]$ by $[-2, 2]$

The graphs of $f(x) = x^{1/2}$ and $h(x) = x^{1/4}$ are similar and appear in the 1st quadrant only. The graphs of $g(x) = x^{1/3}$ and $k(x) = x^{1/5}$ are similar and appear in the 1st and 3rd quadrants only. The pairs $(0, 0)$, $(1, 1)$ are common to all four functions.

	f	g	h	k
Domain	$[0, \infty)$	$(-\infty, \infty)$	$[0, \infty)$	$(-\infty, \infty)$
Range	$y \geq 0$	$(-\infty, \infty)$	$y \geq 0$	$(-\infty, \infty)$
Continuous	yes	yes	yes	yes
Increasing	$[0, \infty)$	$(-\infty, \infty)$	$[0, \infty)$	$(-\infty, \infty)$
Decreasing				
Symmetry	none	w.r.t. origin	none	w.r.t. origin
Bounded	below	not	below	not
Extrema	min at $(0, 0)$	none	min at $(0, 0)$	none
Asymptotes	none	none	none	none
End Behavior	$\lim_{x \to \infty} f(x) = \infty$	$\lim_{x \to \infty} g(x) = \infty$ $\lim_{x \to -\infty} g(x) = -\infty$	$\lim_{x \to \infty} h(x) = \infty$	$\lim_{x \to \infty} k(x) = \infty$ $\lim_{x \to -\infty} k(x) = -\infty$

66.



The graphs look like those shown in Figure 2.14 on page 177.

$f(x) = x^{\pi}$ looks like the red graph in Figure 2.14(a) because $k = 1 > 0$ and $a = \pi > 1$.

$f(x) = x^{1/\pi}$ looks like the blue graph in Figure 2.14(a) because $k = 1 > 0$ and $0 < a = 1/\pi < 1$.

$f(x) = x^{-\pi}$ looks like the green graph in Figure 2.14(a) because $k = 1 < 0$ and $a = -\pi < 0$.

$f(x) = -x^{\pi}$ looks like the red graph in Figure 2.14(b) because $k = -1 < 0$ and $a = \pi > 1$.

$f(x) = -x^{1/\pi}$ looks like the blue graph in Figure 2.14(b) because $k = -1 < 0$ and $a = -\pi < 0$.

$f(x) = -x^{-\pi}$ looks like the green graph in Figure 2.14(b) because $k = -1 < 0$ and $a = -\pi < 0$.

67. Our new table looks like:

Table 2.10 (revised) Average Distances and Orbit Periods for the Six Innermost Planets

Planet	Average Distance from Sun (Au)	Period of Orbit (yrs)
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1	1
Mars	1.52	1.88
Jupiter	5.20	11.86
Saturn	9.54	29.46

Source: Shupe, Dorr, Payne, Hunsiker, et al., National Geographic Atlas of the World (rev. 6th ed.). Washington, DC: National Geographic Society, 1992, plate 116.

Using these new data, we find a power function model of: $y \approx 0.99995 \cdot x^{1.50115} \approx x^{1.5}$. Since y represents years, we set $y = T$ and since x represents distance, we set $x = a$, then $y = x^{1.5} \Rightarrow T = a^{3/2} \Rightarrow (T)^2 = (a^{3/2})^2 \Rightarrow T^2 = a^3$.

68. Using the free-fall equations in Section 2.1, we know that $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$ and that $v(t) = -gt + v_0$. If $t = 0$ is the time at which the object is dropped, then $v_0 = 0$. So $d = s_0 - s = s_0 - \left(-\frac{1}{2}gt^2 + s_0\right) = \frac{1}{2}gt^2$ and $p = |v| = |-gt|$. Solving $d = \frac{1}{2}gt^2$ for t, we have $t = \sqrt{\frac{2d}{g}}$. Then $p = \left|-g\sqrt{\frac{2d}{g}}\right| = \sqrt{\frac{2dg^2}{g}} = \sqrt{2dg}$.
The results of Example 6 approximate this formula.

69. If f is even,

$f(x) = f(-x)$, so $\frac{1}{f(x)} = \frac{1}{f(-x)}$, $(f(x) \neq 0)$.

Since $g(x) = \frac{1}{f(x)} = \frac{1}{f(-x)} = g(-x)$, g is also even.

If g is even,

$g(x) = g(-x)$, so $g(-x) = \frac{1}{f(-x)} = g(x) = \frac{1}{f(x)}$.

Since $\frac{1}{f(-x)} = \frac{1}{f(x)}$, $f(-x) = f(x)$, and f is even.

If f is odd,

$f(x) = -f(x)$, so $\frac{1}{f(x)} = -\frac{1}{f(x)}$, $f(x) \neq 0$.

Since $g(x) = \frac{1}{f(x)} = -\frac{1}{f(x)} = -g(x)$, g is also odd.

If g is odd,

$g(x) = g(-x)$, so $g(-x) = \frac{1}{f(-x)} = -g(x) = -\frac{1}{f(x)}$.

Since $\frac{1}{f(-x)} = -\frac{1}{f(x)}$, $f(-x) = -f(x)$, and f is odd.

70. Let $g(x) = x^{-a}$ and $f(x) = x^a$. Then $g(x) = 1/x^a = 1/f(x)$. Exercise 69 shows that $g(x) = 1/f(x)$ is even if and only if $f(x)$ is even, and $g(x) = 1/f(x)$ is odd if and only if $f(x)$ is odd. Therefore, $g(x) = x^{-a}$ is even if and only if $f(x) = x^a$ is even, and that $g(x) = x^{-a}$ is odd if and only if $f(x) = x^a$ is odd.

71. (a) The force F acting on an object varies jointly as the mass m of the object and the acceleration a of the object.

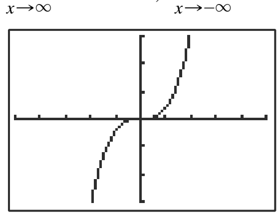
(b) The kinetic energy KE of an object varies jointly as the mass m of the object and the square of the velocity v of the object.

(c) The force of gravity F acting on two objects varies jointly as their masses m_1 and m_2 and inversely as the square of the distance r between their centers, with the constant of variation G, the universal gravitational constant.

■ Section 2.3 Polynomial Functions of Higher Degree with Modeling

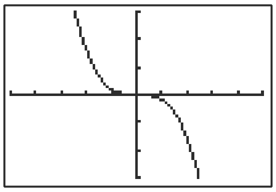
Exploration 1

1. (a) $\lim_{x\to\infty} 2x^3 = \infty$, $\lim_{x\to-\infty} 2x^3 = -\infty$



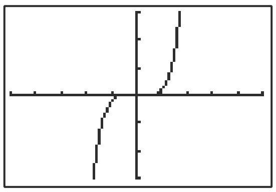
[−5, 5] by [−15, 15]

(b) $\lim_{x\to\infty}(-x^3) = -\infty,\ \lim_{x\to-\infty}(-x^3) = \infty$



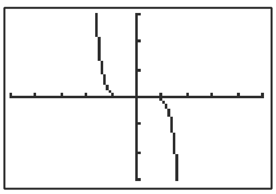
[−5, 5] by [−15, 15]

(c) $\lim_{x\to\infty} x^5 = \infty,\ \lim_{x\to-\infty} x^5 = -\infty$



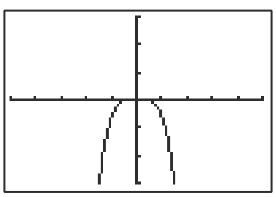
[−5, 5] by [−15, 15]

(d) $\lim_{x\to\infty}(-0.5x^7) = -\infty,\ \lim_{x\to-\infty}(-0.5x^7) = \infty$



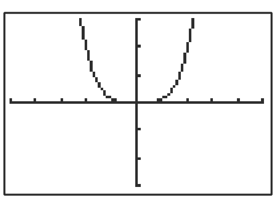
[−5, 5] by [−15, 15]

2. (a) $\lim_{x\to\infty}(-3x^4) = -\infty,\ \lim_{x\to-\infty}(-3x^4) = -\infty$



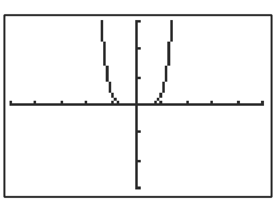
[−5, 5] by [−15, 15]

(b) $\lim_{x\to\infty} 0.6x^4 = \infty,\ \lim_{x\to-\infty} 0.6x^4 = \infty$



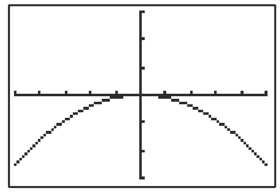
[−5, 5] by [−15, 15]

(c) $\lim_{x\to\infty} 2x^6 = \infty,\ \lim_{x\to-\infty} 2x^6 = \infty$



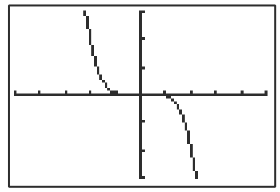
[−5, 5] by [−15, 15]

(d) $\lim_{x\to\infty}(-0.5x^2) = -\infty,\ \lim_{x\to-\infty}(-0.5x^2) = -\infty$



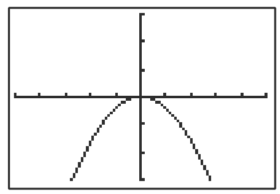
[−5, 5] by [−15, 15]

3. (a) $\lim_{x\to\infty}(-0.3x^5) = -\infty,\ \lim_{x\to-\infty}(-0.3x^5) = \infty$



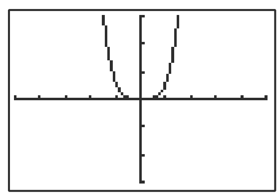
[−5, 5] by [−15, 15]

(b) $\lim_{x\to\infty}(-2x^2) = -\infty,\ \lim_{x\to-\infty}(-2x^2) = -\infty$



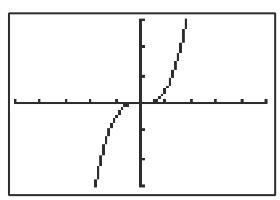
[−5, 5] by [−15, 15]

(c) $\lim_{x\to\infty} 3x^4 = \infty,\ \lim_{x\to-\infty} 3x^4 = \infty$



[−5, 5] by [−15, 15]

(d) $\lim_{x\to\infty} 2.5x^3 = \infty,\ \lim_{x\to-\infty} 2.5x^3 = -\infty$

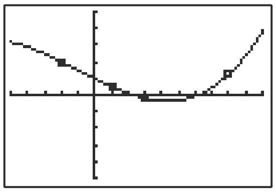


[−5, 5] by [−15, 15]

If $a_n > 0$ and $n > 0$, $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$. If $a_n < 0$ and $n > 0$, $\lim_{x\to\infty} f(x) = -\infty$ and $\lim_{x\to-\infty} f(x) = \infty$.

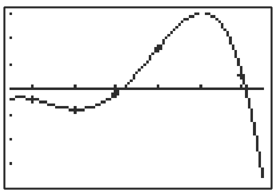
Exploration 2

1. $y = 0.0061x^3 + 0.0177x^2 - 0.5007x + 0.9769$. It is an exact fit, which we expect with only 4 data points!



$[-5, 10]$ by $[-5, 5]$

2. $y = -0.375x^4 + 6.917x^3 - 44.125x^2 + 116.583x - 111$. It is an exact fit, exactly what we expect with only 5 data points!



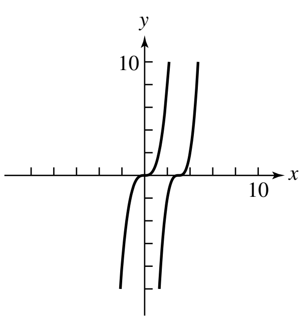
$[2.5, 8.5]$ by $[-18, 15]$

Quick Review 2.3

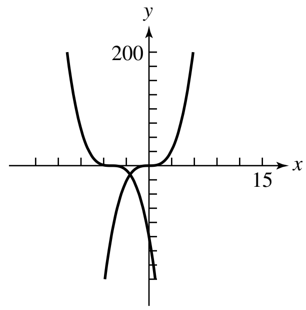
1. $(x - 4)(x + 3)$
2. $(x - 7)(x - 4)$
3. $(3x - 2)(x - 3)$
4. $(2x - 1)(3x - 1)$
5. $x(3x - 2)(x - 1)$
6. $2x(3x - 2)(x - 3)$
7. $x = 0, x = 1$
8. $x = 0, x = -2, x = 5$
9. $x = -6, x = -3, x = 1.5$
10. $x = -6, x = -4, x = 5$

Section 2.3 Exercises

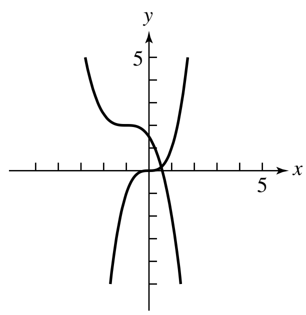
1. Start with $y = x^3$, shift to the right by 3 units, and then stretch vertically by 2. y-intercept: $(0, -54)$.



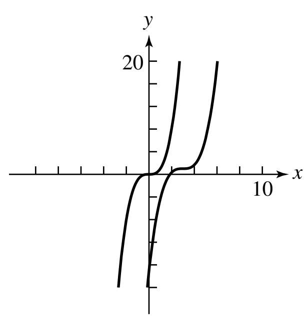
2. Start with $y = x^3$, shift to the left by 5 units, and then reflect over the x-axis. y-intercept: $(0, -125)$.



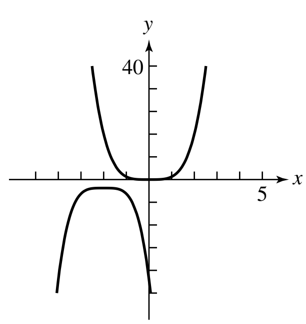
3. Start with $y = x^3$, shift to the left by 1 unit, vertically shrink by $\frac{1}{2}$, reflect over the x-axis, and then vertically shift up 2 units. y-intercept: $\left(0, \frac{3}{2}\right)$.



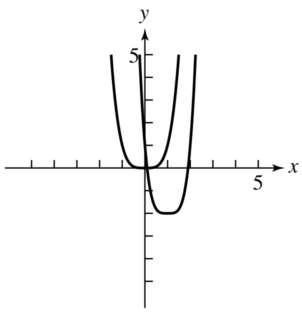
4. Start with $y = x^3$, shift to the right by 3 units, vertically shrink by $\frac{2}{3}$, and vertically shift up 1 unit. y-intercept: $(0, -17)$.



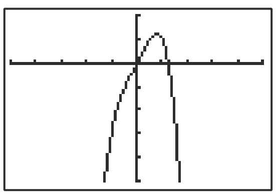
5. Start with $y = x^4$, shift to the left 2 units, vertically stretch by 2, reflect over the x-axis, and vertically shift down 3 units. y-intercept: $(0, -35)$.



6. Start with $y = x^4$, shift to the right 1 unit, vertically stretch by 3, and vertically shift down 2 units. y-intercept: $(0, 1)$.

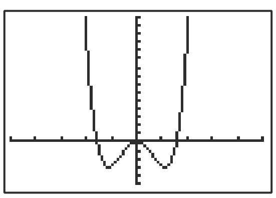


7. Local maximum: $\approx (0.79, 1.19)$, zeros: $x = 0$ and $x \approx 1.26$. The general shape of f is like $y = -x^4$, but near the origin, f behaves a lot like its other term, $2x$. f is neither even nor odd.



$[-5, 5]$ by $[-5, 2]$

8. Local maximum at $(0, 0)$ and local minima at $(1.12, -3.13)$ and $(-1.12, -3.13)$, zeros: $x = 0$, $x \approx 1.58$, $x \approx -1.58$.
f behaves a lot like $y = 2x^4$ except in the interval $[-1.58, 1.58]$, where it behaves more like its second building block term, $-5x^2$.



$[-5, 5]$ by $[-5, 15]$

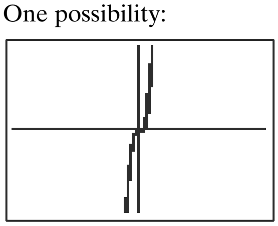
9. Cubic function, positive leading coefficient. The answer is (c).

10. Cubic function, negative leading coefficient. The answer is (b).

11. Higher than cubic, positive leading coefficient. The answer is (a).

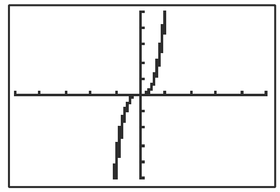
12. Higher than cubic, negative leading coefficient. The answer is (d).

13. One possibility:



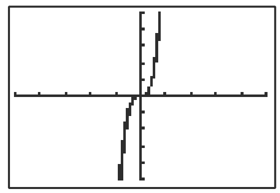
$[-100, 100]$ by $[-1000, 1000]$

14. One possibility:



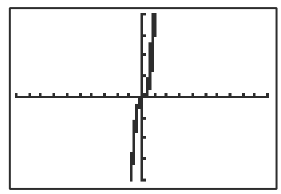
$[-50, 50]$ by $[-1000, 1000]$

15. One possibility:



$[-50, 50]$ by $[-1000, 1000]$

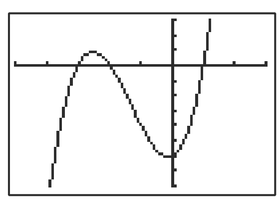
16. One possibility:



$[-100, 100]$ by $[-2000, 2000]$

For #17–24, when one end of a polynomial function's graph curves up into Quadrant I or II, this indicates a limit at ∞. And when an end curves down into Quadrant III or IV, this indicates a limit at $-\infty$.

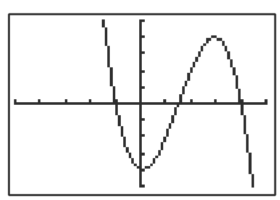
17.



$[-5, 3]$ by $[-8, 3]$

$$\lim_{x\to\infty} f(x) = \infty$$
$$\lim_{x\to-\infty} f(x) = -\infty$$

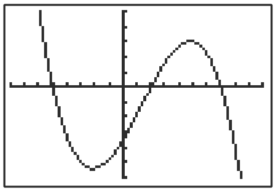
18.



$[-5, 5]$ by $[-15, 15]$

$$\lim_{x\to\infty} f(x) = -\infty$$
$$\lim_{x\to-\infty} f(x) = \infty$$

19.

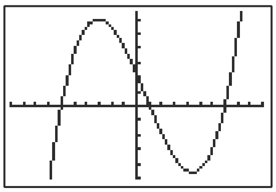


$[-8, 10]$ by $[-120, 100]$

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = \infty$

20.

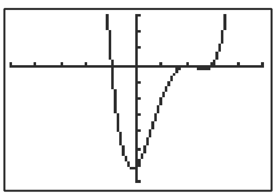


$[-10, 10]$ by $[-100, 130]$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

21.

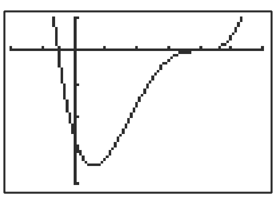


$[-5, 5]$ by $[-14, 6]$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

22.

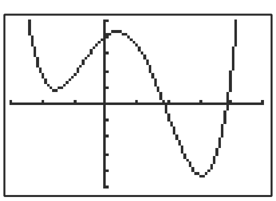


$[-2, 6]$ by $[-100, 25]$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

23.

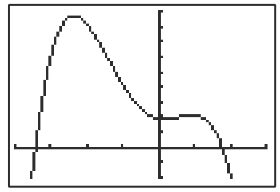


$[-3, 5]$ by $[-50, 50]$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

24.



$[-4, 3]$ by $[-20, 90]$

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = -\infty$

For #25–28, the end behavior of a polynomial is governed by the highest-degree term.

25. $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = \infty$

26. $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = \infty$

27. $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = \infty$

28. $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = -\infty$

29. (a); There are 3 zeros: they are −2.5, 1, and 1.1.

30. (b); There are 3 zeros: they are 0.4, approximately 0.429 (actually 3/7), and 3.

31. (c); There are 3 zeros: approximately −0.273 (actually −3/11), −0.25, and 1.

32. (d); There are 3 zeros: −2, 0.5, and 3.

For #33–35, factor or apply the quadratic formula.

33. −4 and 2

34. −2 and 2/3

35. 2/3 and −1/3

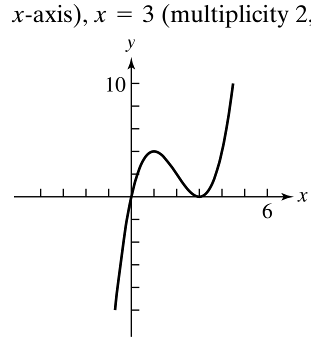
For #36–38, factor out x, then factor or apply the quadratic formula.

36. 0, −5, and 5

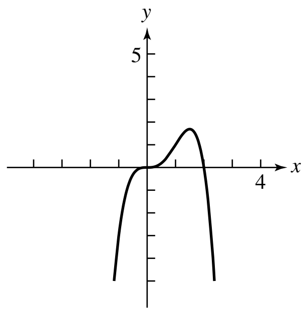
37. 0, −2/3, and 1

38. 0, −1, and 2

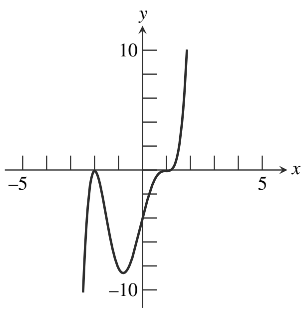
39. Degree 3; zeros: $x = 0$ (multiplicity 1, graph crosses x-axis), $x = 3$ (multiplicity 2, graph is tangent).



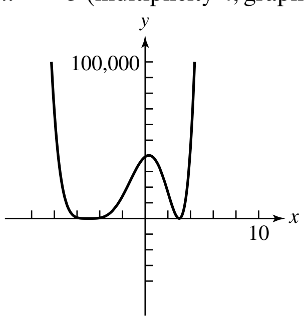
40. Degree 4; zeros: $x = 0$ (multiplicity 3, graph crosses x-axis), $x = 2$ (multiplicity 1, graph crosses x-axis).



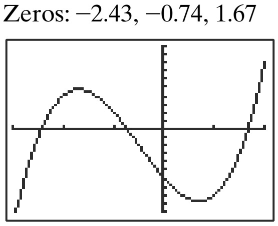
41. Degree 5; zeros: $x = 1$ (multiplicity 3, graph crosses x-axis), $x = -2$ (multiplicity 2, graph is tangent).



42. Degree 6; zeros: $x = 3$ (multiplicity 2, graph is tangent), $x = -5$ (multiplicity 4, graph is tangent).

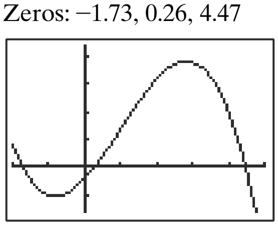


43. Zeros: −2.43, −0.74, 1.67



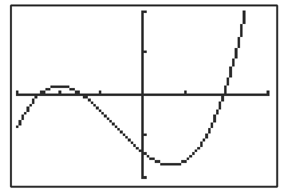
[−3, 2] by [−10, 10]

44. Zeros: −1.73, 0.26, 4.47



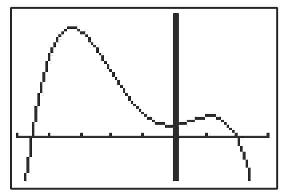
[−2, 5] by [−8, 22]

45. Zeros: −2.47, −1.46, 1.94



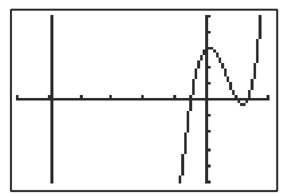
[−3, 3] by [−10, 10]

46. Zeros: −4.53, 2



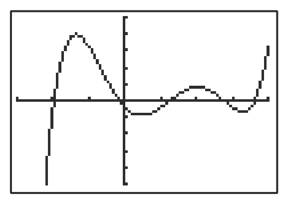
[−5, 3] by [−20, 90]

47. Zeros: −4.90, −0.45, 1, 1.35



[−6, 2] by [−5, 5]

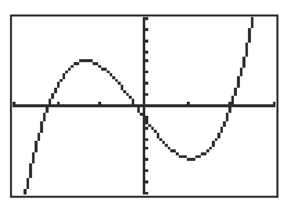
48. Zeros: −1.98, −0.16, 1.25, 2.77, 3.62



[−3, 4] by [−100, 100]

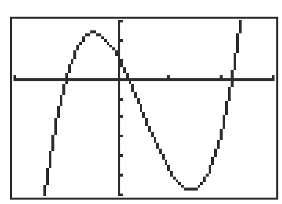
49. 0, −6, and 6. Algebraically — factor out x first.

50. −11, −1, and 10. Graphically. Cubic equations *can* be solved algebraically, but methods of doing so are more complicated than the quadratic formula.



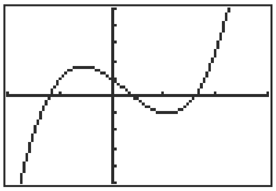
[–15, 15] by [–800, 800]

51. −5, 1, and 11. Graphically.



[–10, 15] by [–300, 150]

52. −6, 2, and 8. Graphically.



[–10, 15] by [–500, 500]

For #53–56, the "minimal" polynomials are given; any constant (or any other polynomial) can be multiplied by the answer given to give another answer.

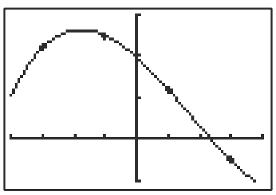
53. $f(x) = (x - 3)(x + 4)(x - 6)$
$= x^3 - 5x^2 - 18x + 72$

54. $f(x) = (x + 2)(x - 3)(x + 5) = x^3 + 4x^2 - 11x - 30$

55. $f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - 4)$
$= (x^2 - 3)(x - 4) = x^3 - 4x^2 - 3x + 12$

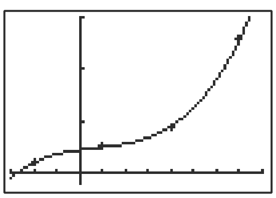
56. $f(x) = (x - 1)(x - 1 - \sqrt{2})(x - 1 + \sqrt{2})$
$= (x - 1)[(x - 1)^2 - 2] = x^3 - 3x^2 + x + 1$

57. $y = 0.25x^3 - 1.25x^2 - 6.75x + 19.75$



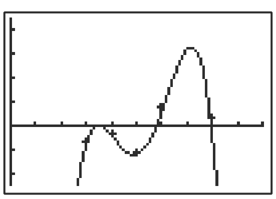
$[-4, 4]$ by $[-10, 30]$

58. $y = 0.074x^3 - 0.167x^2 + 0.611x + 4.48$



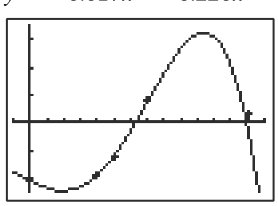
$[-3, 8]$ by $[-2, 30]$

59. $y = -2.21x^4 + 45.75x^3 - 339.79x^2 + 1075.25x - 1231$



$[0, 10]$ by $[-25, 45]$

60. $y = -0.017x^4 + 0.226x^3 + 0.289x^2 - 3.202x - 21$

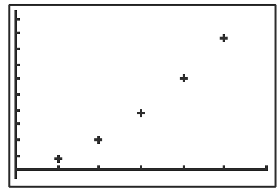


$[-1, 14]$ by $[-25, 35]$

61. $f(x) = x^7 + x + 100$ has an odd-degree leading term, which means that in its end behavior it will go toward $-\infty$ at one end and toward ∞ at the other. Thus the graph must cross the x-axis at least once. That is to say, $f(x)$ takes on both positive and negative values, and thus by the Intermediate Value Theorem, $f(x) = 0$ for some x.

62. $f(x) = x^9 - x + 50$ has an odd-degree leading term, which means that in its end behavior it will go toward $-\infty$ at one end and toward ∞ at the other. Thus the graph must cross the x-axis at least once. That is to say, $f(x)$ takes on both positive and negative values, and thus by the Intermediate Value Theorem, $f(x) = 0$ for some x.

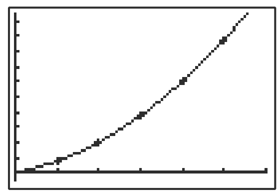
63. (a)



$[0, 60]$ by $[-10, 210]$

(b) $y = 0.051x^2 + 0.97x + 0.26$

(c)



$[0, 60]$ by $[-10, 210]$

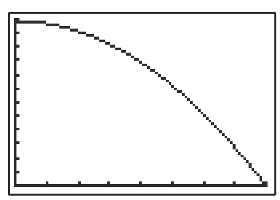
(d) $y(25) \approx 56.39$ ft

(e) Using the quadratic equation to solve $0 = 0.051x^2 + 0.97x + (0.26 - 300)$, we find two answers: $x = 67.74$ mph and $x = -86.76$ mph. Clearly the negative value is extraneous.

64. (a) $P(x) = R(x) - C(x)$ is positive if $29.73 < x < 541.74$ (approx.), so they need between 30 and 541 customers.

(b) $P(x) = 60{,}000$ when $x = 200.49$ or $x = 429.73$. Either 201 or 429 customers gives a profit slightly over \$60,000; 200 or 430 customers both yield slightly less than \$60,000.

65. (a)

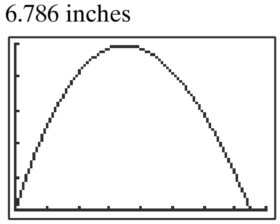


$[0, 0.8]$ by $[0, 1.20]$

(b) 0.3391 cm from the center of the artery

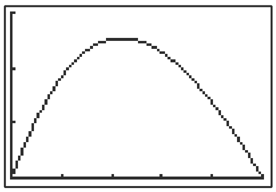
66. (a) The height of the box will be x, the width will be $15 - 2x$, and the length $60 - 2x$.

(b) Any value of x between approximately 0.550 and 6.786 inches



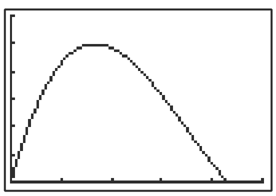
$[0, 8]$ by $[0, 1500]$

67. The volume is $V(x) = x(10 - 2x)(25 - 2x)$; use any x with $0 < x \leq 0.929$ or $3.644 \leq x < 5$.



$[0, 5]$ by $[0, 300]$

68. Determine where the function is positive: $0 < x < 21.5$. (The side lengths of the rectangle are 43 and 62 units.)



$[0, 25]$ by $[0, 12000]$

69. True. Because f is continuous and $f(1) = (1)^3 - (1)^2 - 2 = -2 < 0$ while $f(2) = (2)^3 - (2)^2 - 2 = 2 > 0$, the Intermediate Value Theorem assures us that the graph of f crosses the x-axis ($f(x) = 0$) somewhere between $x = 1$ and $x = 2$.

70. False. If $a > 0$, the graph of $f(x) = (x + a)^2$ is obtained by translating the graph of $f(x) = x^2$ to the left by a units. Translation to the right corresponds to $a < 0$.

71. When $x = 0$, $f(x) = 2(x - 1)^3 + 5 = 2(-1)^3 + 5 = 3$. The answer is C.

72. In $f(x) = (x - 2)^2(x + 2)^3(x + 3)^7$, the factor $x - 2$ occurs twice. So $x = 2$ is a zero of multiplicity 2, and the answer is B.

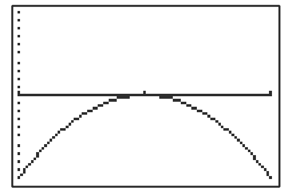
73. The graph indicates three zeros, each of multiplicity 1: $x = -2$, $x = 0$, and $x = 2$. The end behavior indicates a negative leading coefficient. So $f(x) = -x(x + 2)(x - 2)$, and the answer is B.

74. The graph indicates four zeros: $x = -2$ (multiplicity 2), $x = 0$ (multiplicity 1), and $x = 2$ (multiplicity 2). The end behavior indicates a positive leading coefficient. So $f(x) = x(x + 2)^2(x - 2)$, and the answer is A.

75. The first view shows the end behavior of the function but obscures the fact that there are two local maxima and a local minimum (and 4 x-axis intersections) between -3 and 4. These are visible in the second view, but missing is the minimum near $x = 7$ and the x-axis intersection near $x = 9$. The second view suggests a degree 4 polynomial rather than degree 5.

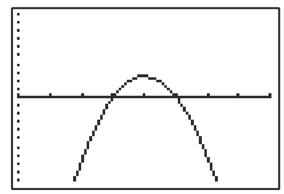
76. The figure at left shows the end behavior and a zero of $x \approx -5$, but hides the other three zeros. The figure at right shows the zero at 1 and the zeros near 0.83 and 1.22, but hides the fourth zero and the end behavior.

77. The exact behavior near $x = 1$ is hard to see. A zoomed–in view around the point $(1, 0)$ suggests that the graph just touches the x-axis at 0 without actually crossing it — that is, $(1, 0)$ is a local maximum. One possible window is $[0.9999, 1.0001]$ by $[-1 \times 10^{-7}, 1 \times 10^{-7}]$.



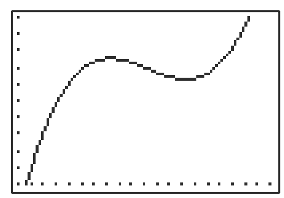
$[0.9999, 1.0001]$ by $[-1 \times 10^{-7}, 1 \times 10^{-7}]$

78. This also has a maximum near $x = 1$ — but this time a window such as $[0.6, 1.4]$ by $[-0.1, 0.1]$ reveals that the graph actually rises above the x-axis and has a maximum at $(0.999, 0.025)$.



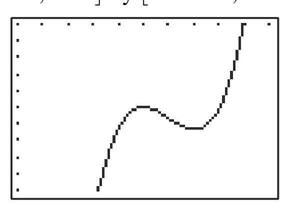
$[0.6, 1.4]$ by $[-0.1, 0.1]$

79. A maximum and minimum are not visible in the standard window, but can be seen on the window $[0.2, 0.4]$ by $[5.29, 5.3]$.



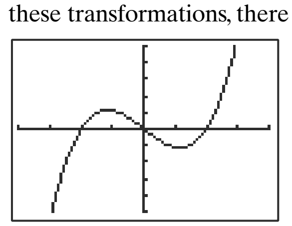
$[0.2, 0.4]$ by $[5.29, 5.30]$

80. A maximum and minimum are not visible in the standard window, but can be seen on the window $[0.95, 1.05]$ by $[-6.0005, -5.9995]$.



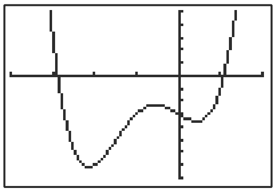
$[0.95, 1.05]$ by $[-6.0005, -5.9995]$

81. The graph of $y = 3(x^3 - x)$ (shown on the window $[-2, 2]$ by $[-5, 5]$) increases, then decreases, then increases; the graph of $y = x^3$ only increases. Therefore, this graph cannot be obtained from the graph of $y = x^3$ by the transformations studied in Chapter 1 (translations, reflections, and stretching/shrinking). Since the right side includes only these transformations, there can be no solution.



$[-2, 2]$ by $[-5, 5]$

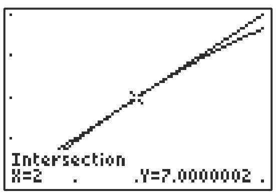
82. The graph of $y = x^4$ has a "flat" bottom while the graph of $y = x^4 + 3x^3 - 2x - 3$ (shown on $[-4, 2]$ by $[-8, 5]$) is "bumpy." Therefore this graph cannot be obtained from the graph of $y = x^4$ through the transformations of Chapter 1 (translations, reflections, and stretching/shrinking). Since the right side includes only these transformations, there can be no solution.



$[-4, 2]$ by $[-8, 5]$

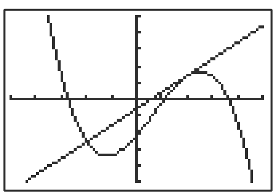
83. **(a)** Substituting $x = 2$, $y = 7$, we find that $7 = 5(2 - 2) + 7$, so Q is on line L, and also $f(2) = -8 + 8 + 18 - 11 = 7$, so Q is on the graph of $f(x)$.

(b) Window $[1.8, 2.2]$ by $[6, 8]$. Calculator output will not show the detail seen here.



$[1.8, 2.2]$ by $[6, 8]$

(c) The line L also crosses the graph of $f(x)$ at $(-2, -13)$.

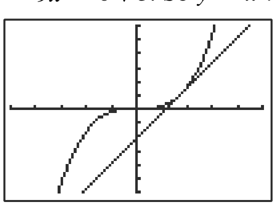


$[-5, 5]$ by $[-25, 25]$

84. **(a)** Note that $f(a) = a^n$ and $f(-a) = -a^n$; $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-a^n - a^n}{-a - a} = \dfrac{-2a^n}{-2a} = a^{n-1}$.

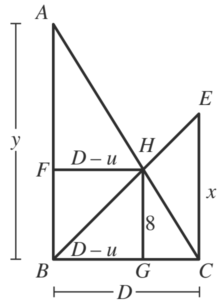
(b) First observe that $f(x_0) = (a^{1/(n-1)})^n = a^{n/(n-1)}$. Using point–slope form: $y - a^{n/(n-1)} = a^{n-1}(x - a^{1/(n-1)})$.

(c) With $n = 3$ and $a = 3$, this equation becomes $y - 3^{3/2} = 3^2(x - 3^{1/2})$, or $y = 9(x - \sqrt{3}) + 3\sqrt{3} = 9x - 6\sqrt{3}$. So $y = x^3$.



$[-5, 5]$ by $[-30, 30]$

85. **(a)** Label the points of the diagram as shown, adding the horizontal segment $\overline{FH}$. Therefore, ΔECB is similar (in the geometric sense) to ΔHGB, and also ΔABC is similar to ΔAFH. Therefore: $\dfrac{HG}{EC} = \dfrac{BG}{BC}$, or $\dfrac{8}{x} = \dfrac{D - u}{D}$, and also $\dfrac{AF}{AB} = \dfrac{FH}{BC}$, or $\dfrac{y - 8}{y} = \dfrac{D - u}{D}$. Then $\dfrac{8}{x} = \dfrac{y - 8}{y}$.



(b) Equation (a) says $\dfrac{8}{x} = 1 - \dfrac{8}{y}$. Multiply both sides by xy: $8y = xy - 8x$. Subtract xy from both sides and factor: $y(8 - x) = -8$. Divide both sides by $8 - x$: $y = \dfrac{-8}{8 - x}$. Factor out -1 from numerator and denominator: $y = \dfrac{8}{x - 8}$.

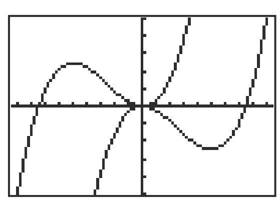
(c) Applying the Pythagorean Theorem to ΔEBC and ΔABC, we have $x^2 + D^2 = 20^2$ and $y^2 + D^2 = 30^2$, which combine to give $D^2 = 400 - x^2 = 900 - y^2$, or $y^2 - x^2 = 500$. Substituting $y = 8x/(x - 8)$, we get $\left(\dfrac{8x}{x - 8}\right)^2 - x^2 = 500$, so that $\dfrac{64x^2}{(x - 8)^2} - x^2 = 500$, or $64x^2 - x^2(x - 8)^2 = 500(x - 8)^2$. Expanding this gives $500x^2 - 8000x + 32{,}000 = 64x^2 - x^4 + 16x^3 - 64x^2$. This is equivalent to $x^4 - 16x^3 + 500x^2 - 8000x + 32{,}000 = 0$.

(d) The two solutions are $x \approx 5.9446$ and $x \approx 11.7118$. Based on the figure, x must be between 8 and 20 for this problem, so $x \approx 11.7118$. Then $D = \sqrt{20^2 - x^2} \approx 16.2121$ ft.

86. **(a)** Regardless of the value of b, $f(-b) = 1 - b$, $\lim_{x\to\infty} f(x) = +\infty$, $\lim_{x\to-\infty} f(x) = -\infty$, and the graph of f has a y-intercept of 1. If $|b| \le \sqrt{3}$, the graph of f is strictly increasing. If $|b| > \sqrt{3}$, f has one local maximum and one local minimum. If $|b|$ is large, the graph of f *appears* to have a double root at 0 and a single root at $-b$, because $f(x) = x^3 + bx^2 = x^2(x + b)$ for large x.

(b) Answers will vary.

(c)



$[-18.8, 18.8]$ by $[-1000, 1000]$

■ Section 2.4 Real Zeros of Polynomial Functions

Quick Review 2.4

1. $x^2 - 4x + 7$
2. $x^2 - \frac{5}{2}x - 3$
3. $7x^3 + x^2 - 3$
4. $2x^2 - \frac{2}{3}x + \frac{7}{3}$
5. $x(x^2 - 4) = x(x^2 - 2^2) = x(x + 2)(x - 2)$
6. $6(x^2 - 9) = 6(x^2 - 3^2) = 6(x + 3)(x - 3)$
7. $4(x^2 + 2x - 15) = 4(x + 5)(x - 3)$
8. $x(15x^2 - 22x + 8) = x(3x - 2)(5x - 4)$
9. $(x^3 + 2x^2) - (x + 2) = x^2(x + 2) - 1(x + 2)$
   $= (x + 2)(x^2 - 1) = (x + 2)(x + 1)(x - 1)$
10. $x(x^3 + x^2 - 9x - 9) = x[(x^3 + x^2) - (9x + 9)]$
    $= x([x^2(x + 1) - 9(x + 1)]$
    $= x(x + 1)(x^2 - 9) = x(x + 1)(x^2 - 3^2)$
    $= x(x + 1)(x + 3)(x - 3)$

Section 2.4 Exercises

1.
$$\begin{array}{r} x - 1 \\ x - 1\overline{)x^2 - 2x + 3} \\ \underline{x^2 - x} \\ -x + 3 \\ \underline{-x + 1} \\ 2 \end{array}$$
$f(x) = (x - 1)^2 + 2;\ \frac{f(x)}{x - 1} = x - 1 + \frac{2}{x - 1}$

2.
$$\begin{array}{r} x^2 - x + 1 \\ x + 1\overline{)x^3 + 0x^2 + 0x - 1} \\ \underline{x^3 + x^2} \\ -x^2 + 0x \\ \underline{-x^2 - x} \\ x - 1 \\ \underline{x + 1} \\ -2 \end{array}$$
$f(x) = (x^2 - x + 1)(x + 1) - 2;$
$\frac{f(x)}{x + 1} = x^2 - x + 1 - \frac{2}{x + 1}$

3.
$$\begin{array}{r} x^2 + x + 4 \\ x + 3\overline{)x^3 + 4x^2 + 7x - 9} \\ \underline{x^3 + 3x^2} \\ x^2 + 7x \\ \underline{x^2 + 3x} \\ 4x - 9 \\ \underline{4x + 12} \\ -21 \end{array}$$
$f(x) = (x^2 + x + 4)(x + 3) - 21;$
$\frac{f(x)}{x + 3} = x^2 + x + 4 - \frac{21}{x + 3}$

4.
$$\begin{array}{r} 2x^2 - 5x + \frac{7}{2} \\ 2x + 1\overline{)4x^3 - 8x^2 + 2x - 1} \\ \underline{4x^3 + 2x^2} \\ -10x^2 + 2x \\ \underline{-10x^2 - 5x} \\ 7x - 1 \\ \underline{7x + \frac{7}{2}} \\ -\frac{9}{2} \end{array}$$
$f(x) = \left(2x^2 - 5x + \frac{7}{2}\right)(2x + 1) - \frac{9}{2};$
$\frac{f(x)}{2x + 1} = 2x^2 - 5x + \frac{7}{2} - \frac{9/2}{2x + 1}$

5.
$$\begin{array}{r} x^2 - 4x + 12 \\ x^2 + 2x - 1\overline{)x^4 - 2x^3 + 3x^2 - 4x + 6} \\ \underline{x^4 + 2x^3 - x^2} \\ -4x^3 + 4x^2 - 4x \\ \underline{-4x^3 - 8x^2 + 4x} \\ 12x^2 - 8x + 6 \\ \underline{12x^2 + 24x - 12} \\ -32x + 18 \end{array}$$
$f(x) = (x^2 - 4x + 12)(x^2 + 2x - 1) - 32x + 18;$
$\frac{f(x)}{x^2 + 2x - 1} = x^2 - 4x + 12 + \frac{-32x + 18}{x^2 + 2x - 1}$

6.
$$\begin{array}{r} x^2 - 3x + 5 \\ x^2 + 1\overline{)x^4 - 3x^3 + 6x^2 - 3x + 5} \\ \underline{x^4 + x^2} \\ -3x^3 + 5x^2 - 3x \\ \underline{-3x^3 - 3x} \\ 5x^2 + 5 \\ \underline{5x^2 + 5} \\ 0 \end{array}$$
$f(x) = (x^2 - 3x + 5)(x^2 + 1);\ \frac{f(x)}{x^2 + 1} = x^2 - 3x + 5$

7. $\frac{x^3 - 5x^2 + 3x - 2}{x + 1} = x^2 - 6x + 9 + \frac{-11}{x + 1}$
$$\begin{array}{r|rrrr} -1 & 1 & -5 & 3 & -2 \\ & & -1 & 6 & -9 \\ \hline & 1 & -6 & 9 & -11 \end{array}$$

8. $\frac{2x^4 - 5x^3 + 7x^2 - 3x + 1}{x - 3}$
$= 2x^3 + x^2 + 10x + 27 + \frac{82}{x - 3}$
$$\begin{array}{r|rrrrr} 3 & 2 & -5 & 7 & -3 & 1 \\ & & 6 & 3 & 30 & 81 \\ \hline & 2 & 1 & 10 & 27 & 82 \end{array}$$

9. $\frac{9x^3 + 7x^2 - 3x}{x - 10} = 9x^2 + 97x + 967 + \frac{9670}{x - 10}$
$$\begin{array}{r|rrrr} 10 & 9 & 7 & -3 & 0 \\ & & 90 & 970 & 9670 \\ \hline & 9 & 97 & 967 & 9670 \end{array}$$

10. $\dfrac{3x^4 + x^3 - 4x^2 + 9x - 3}{x + 5}$

$= 3x^3 - 14x^2 + 66x - 321 + \dfrac{1602}{x + 5}$

$$\begin{array}{r|rrrrr} -5 & 3 & 1 & -4 & 9 & -3 \\ & & -15 & 70 & -330 & 1605 \\ \hline & 3 & -14 & 66 & -321 & 1602 \end{array}$$

11. $\dfrac{5x^4 - 3x + 1}{4 - x}$

$= -5x^3 - 20x^2 - 80x - 317 + \dfrac{-1269}{4 - x}$

$$\begin{array}{r|rrrrr} 4 & -5 & 0 & 0 & 3 & -1 \\ & & -20 & -80 & -320 & -1268 \\ \hline & -5 & -20 & -80 & -317 & -1269 \end{array}$$

12. $\dfrac{x^8 - 1}{x + 2}$

$= x^7 - 2x^6 + 4x^5 - 8x^4 + 16x^3 - 32x^2 + 64x - 128$
$+ \dfrac{255}{x + 2}$

$$\begin{array}{r|rrrrrrrrr} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ & & -2 & 4 & -8 & 16 & -32 & 64 & -128 & 256 \\ \hline & 1 & -2 & 4 & -8 & 16 & -32 & 64 & -128 & 255 \end{array}$$

13. The remainder is $f(2) = 3$.

14. The remainder is $f(1) = -4$.

15. The remainder is $f(-3) = -43$.

16. The remainder is $f(-2) = 2$.

17. The remainder is $f(2) = 5$.

18. The remainder is $f(-1) = 23$.

19. Yes: 1 is a zero of the second polynomial.

20. Yes: 3 is a zero of the second polynomial.

21. No: When $x = 2$, the second polynomial evaluates to 10.

22. Yes: 2 is a zero of the second polynomial.

23. Yes: -2 is a zero of the second polynomial.

24. No: When $x = -1$, the second polynomial evaluates to 2.

25. From the graph it appears that $(x + 3)$ and $(x - 1)$ are factors.

$$\begin{array}{r|rrrr} -3 & 5 & -7 & -49 & 51 \\ & & -15 & 66 & -51 \\ \hline 1 & 5 & -22 & -17 & 0 \\ & & 5 & -17 & \\ \hline & 5 & -17 & 0 & \end{array}$$

$f(x) = (x + 3)(x - 1)(5x - 17)$

26. From the graph it appears that $(x + 2)$ and $(x - 3)$ are factors.

$$\begin{array}{r|rrrr} -2 & 5 & -12 & -23 & 42 \\ & & -10 & 44 & -42 \\ \hline 3 & 5 & -22 & 21 & 0 \\ & & 15 & -21 & \\ \hline & 5 & -7 & 0 & \end{array}$$

$f(x) = (x + 2)(x - 3)(5x - 7)$

27. $2(x + 2)(x - 1)(x - 4) = 2x^3 - 6x^2 - 12x + 16$

28. $2(x + 1)(x - 3)(x + 5) = 2x^3 + 6x^2 - 26x - 30$

29. $2(x - 2)\left(x - \dfrac{1}{2}\right)\left(x - \dfrac{3}{2}\right)$

$= \dfrac{1}{2}(x - 2)(2x - 1)(2x - 3)$

$= 2x^3 - 8x^2 + \dfrac{19}{2}x - 3$

30. $2(x + 3)(x + 1)(x)\left(x - \dfrac{5}{2}\right)$

$= x(x + 3)(x + 1)(2x - 5)$

$= 2x^4 + 3x^3 - 14x^2 - 15x$

31. Since $f(-4) = f(3) = f(5) = 0$, it must be that $(x + 4)$, $(x - 3)$, and $(x - 5)$ are factors of f. So $f(x) = k(x + 4)(x - 3)(x - 5)$ for some constant k.
Since $f(0) = 180$, we must have $k = 3$. So $f(x) = 3(x + 4)(x - 3)(x - 5)$.

32. Since $f(-2) = f(1) = f(5) = 0$, it must be that $(x + 2)$, $(x - 1)$, and $(x - 5)$ are factors of f. So $f(x) = k(x + 2)(x - 1)(x - 5)$ for some constant k.
Since $f(-1) = 24$, we must have $k = 2$, so $f(x) = 2(x + 2)(x - 1)(x - 5)$.

33. Possible rational zeros: $\dfrac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm\dfrac{1}{2}, \pm\dfrac{1}{3}, \pm\dfrac{1}{6}$; 1 is a zero.

34. Possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 7, \pm 14}{\pm 1, \pm 3}$, or $\pm 1, \pm 2, \pm 7, \pm 14, \pm\dfrac{1}{3}, \pm\dfrac{2}{3}, \pm\dfrac{7}{3}, \pm\dfrac{14}{3}$; $\dfrac{7}{3}$ is a zero.

35. Possible rational zeros: $\dfrac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}$, or $\pm 1, \pm 3, \pm 9, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{9}{2}$; $\dfrac{3}{2}$ is a zero.

36. Possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{1}{3}, \pm\dfrac{2}{3}, \pm\dfrac{4}{3}, \pm\dfrac{1}{6}$; $-\dfrac{4}{3}$ and $\dfrac{3}{2}$ are zeros.

37.
$$\begin{array}{r|rrrr} 3 & 2 & -4 & 1 & -2 \\ & & 6 & 6 & 21 \\ \hline & 2 & 2 & 7 & 19 \end{array}$$

Since all numbers in the last line are ≥ 0, 3 is an upper bound for the zeros of f.

38.
$$\begin{array}{r|rrrr} 5 & 2 & -5 & -5 & -1 \\ & & 10 & 25 & 100 \\ \hline & 2 & 5 & 20 & 99 \end{array}$$

Since all values in the last line are ≥ 0, 5 is an upper bound for the zeros of $f(x)$.

39.
$$\begin{array}{r|rrrrr} 2 & 1 & -1 & 1 & 1 & -12 \\ & & 2 & 2 & 6 & 14 \\ \hline & 1 & 1 & 3 & 7 & 2 \end{array}$$

Since all values in the last line are ≥ 0, 2 is an upper bound for the zeros of $f(x)$.

40. $$\begin{array}{r|rrrrr} 3 & 4 & -6 & -7 & 9 & 2 \\ & & 12 & 18 & 33 & 126 \\ \hline & 4 & 6 & 11 & 42 & 128 \end{array}$$

Since all values in the last line are ≥ 0, 3 is an upper bound for the zeros of $f(x)$.

41. $$\begin{array}{r|rrrr} -1 & 3 & -4 & 1 & 3 \\ & & -3 & 7 & -8 \\ \hline & 3 & -7 & 8 & -5 \end{array}$$

Since the values in the last line alternate signs, -1 is a lower bound for the zeros of $f(x)$.

42. $$\begin{array}{r|rrrr} -3 & 1 & 2 & 2 & 5 \\ & & -3 & 3 & -15 \\ \hline & 1 & -1 & 5 & -10 \end{array}$$

Since the values in the last line alternate signs, -3 is a lower bound for the zeros of $f(x)$.

43. $$\begin{array}{r|rrrr} 0 & 1 & -4 & 7 & -2 \\ & & 0 & 0 & 0 \\ \hline & 1 & -4 & 7 & -2 \end{array}$$

Since the values in the last line alternate signs, 0 is a lower bound for the zeros of $f(x)$.

44. $$\begin{array}{r|rrrr} -4 & 3 & -1 & -5 & -3 \\ & & -12 & 52 & -188 \\ \hline & 3 & -13 & 47 & -191 \end{array}$$

Since the values in the last line alternate signs, -4 is a lower bound for the zeros of $f(x)$.

45. By the Upper and Lower Bound Tests, -5 is a lower bound and 5 is an upper bound. No zeros outside window.

$$\begin{array}{r|rrrrr} -5 & 6 & -11 & -7 & 8 & -34 \\ & & -30 & 205 & -990 & 4910 \\ \hline & 6 & -41 & 198 & -982 & 4876 \end{array}$$

$$\begin{array}{r|rrrrr} 5 & 6 & -11 & -7 & 8 & -34 \\ & & 30 & 95 & 440 & 2240 \\ \hline & 6 & 19 & 88 & 448 & 2206 \end{array}$$

46. By the Upper and Lower Bound Tests, -5 is a lower bound and 5 is an upper bound. No zeros outside window.

$$\begin{array}{r|rrrrrr} -5 & 1 & -1 & 0 & 21 & 19 & -3 \\ & & -5 & 30 & -150 & 645 & -3320 \\ \hline & 1 & -6 & 30 & -129 & 664 & -3323 \end{array}$$

$$\begin{array}{r|rrrrrr} 5 & 1 & -1 & 0 & 21 & 19 & -3 \\ & & 5 & 20 & 100 & 605 & 3120 \\ \hline & 1 & 4 & 20 & 121 & 624 & 3117 \end{array}$$

47. Synthetic division shows that the Upper and Lower Bound Tests were not met. There *are* zeros not shown (approx. -11.002 and 12.003), because -5 and 5 are not bounds for zeros of $f(x)$.

$$\begin{array}{r|rrrrrr} -5 & 1 & -4 & -129 & 396 & -8 & 3 \\ & & -5 & 45 & 420 & -4080 & 20{,}440 \\ \hline & 1 & -9 & -84 & 816 & -4088 & 20{,}443 \end{array}$$

$$\begin{array}{r|rrrrrr} 5 & 1 & -4 & -129 & 396 & -8 & 3 \\ & & 5 & 5 & -620 & -1120 & -5640 \\ \hline & 1 & 1 & -124 & -224 & -1128 & -5637 \end{array}$$

48. Synthetic division shows that the lower/upper bounds tests were not met. There *are* zeros not shown (approx. -8.036 and 9.038), because -5 and 5 are not bounds for zeros of $f(x)$.

$$\begin{array}{r|rrrrrr} -5 & 2 & -5 & -141 & 216 & -91 & 25 \\ & & -10 & 75 & 330 & -2730 & 14{,}105 \\ \hline & 2 & -15 & -66 & 546 & -2821 & 14{,}130 \end{array}$$

$$\begin{array}{r|rrrrrr} 5 & 2 & -5 & -141 & 216 & -91 & 25 \\ & & 10 & 25 & -580 & -1820 & -9555 \\ \hline & 2 & 5 & -116 & -364 & -1911 & -9530 \end{array}$$

For #49–56, determine the rational zeros using a grapher (and the Rational Zeros Test as necessary). Use synthetic division to reduce the function to a quadratic polynomial, which can be solved with the quadratic formula (or otherwise). The first two are done in detail; for the rest, we show only the synthetic division step(s).

49. Possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$, or

$\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$. The only rational zero is $\frac{3}{2}$. Synthetic division (below) leaves $2x^2 - 4$, so the irrational zeros are $\pm\sqrt{2}$.

$$\begin{array}{r|rrrr} 3/2 & 2 & -3 & -4 & 6 \\ & & 3 & 0 & -6 \\ \hline & 2 & 0 & -4 & 0 \end{array}$$

50. Possible rational zeros: $\pm 1, \pm 3, \pm 9$. The only rational zero is -3. Synthetic division (below) leaves $x^2 - 3$, so the irrational zeros are $\pm\sqrt{3}$.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -3 & -9 \\ & & -3 & 0 & 9 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

51. Rational: -3; irrational: $1 \pm \sqrt{3}$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -8 & -6 \\ & & -3 & 6 & 6 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

52. Rational: 4; irrational: $1 \pm \sqrt{2}$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 7 & 4 \\ & & 4 & -8 & -4 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

53. Rational: -1 and 4; irrational: $\pm\sqrt{2}$

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & -6 & 6 & 8 \\ & & -1 & 4 & 2 & -8 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

54. Rational: -1 and 2; irrational: $\pm\sqrt{5}$

$-1\rfloor$	1	-1	-7	5	10
		-1	2	5	-10
	1	-2	-5	10	0

$2\rfloor$	1	-2	-5	10
		2	0	-10
	1	0	-5	0

55. Rational: $-\frac{1}{2}$ and 4; irrational: none

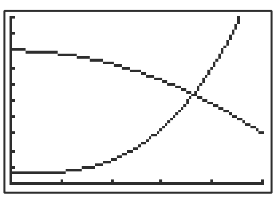
$4\rfloor$	2	-7	-2	-7	-4
		8	4	8	4
	2	1	2	1	0

$-1/2\rfloor$	2	1	2	1
		-1	0	-1
	2	0	2	0

56. Rational: $\frac{2}{3}$; irrational: about -0.6823

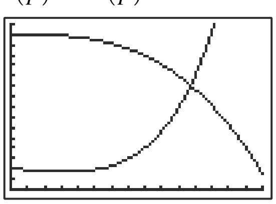
$2/3\rfloor$	3	-2	3	1	-2
		2	0	2	2
	3	0	3	3	0

57. The supply and demand graphs are shown on the window $[0, 50]$ by $[0, 100]$. They intersect when $p = \$36.27$, at which point the supply and demand equal 54.



$[0, 50]$ by $[0, 100]$

58. The supply and demand graphs, shown on the window $[0, 150]$ by $[0, 1600]$, intersect when $p = \$106.99$. There $S(p) = D(p) = 1010$.



$[0, 150]$ by $[0, 1600]$

59. Using the Remainder Theorem, the remainder is $(-1)^{40} - 3 = -2$.

60. Using the Remainder Theorem, the remainder is $1^{63} - 17 = -16$.

61. (a) Lower bound:

$-5\rfloor$	1	2	-11	-13	38
		-5	15	-20	165
	1	-3	4	-33	203

Upper bound:

$4\rfloor$	1	2	-11	-13	38
		4	24	52	156
	1	6	13	39	194

The Upper and Lower Bound Tests are met, so all real zeros of f lie on the interval $[-5, 4]$.

(b) Potential rational zeros:
$\frac{\textit{Factors of } 38}{\textit{Factors of } 1} : \frac{\pm 1, \pm 2, \pm 19, \pm 38}{\pm 1}$

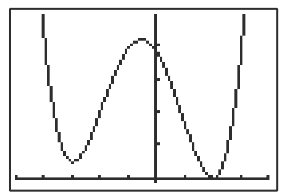
A graph shows that 2 is most promising, so we verify with synthetic division:

$2\rfloor$	1	2	-11	-13	38
		2	8	-6	-38
	1	4	-3	-19	0

Use the Remainder Theorem:

$f(-2) = 20 \neq 0$ $\quad f(-38) = 1{,}960{,}040$
$f(-1) = 39 \neq 0$ $\quad f(38) = 2{,}178{,}540$
$f(1) = 17 \neq 0$ $\quad f(-19) = 112{,}917$
$f(19) = 139{,}859$

Since all possible rational roots besides 2 yield non-zero function values, there are no other rational roots.



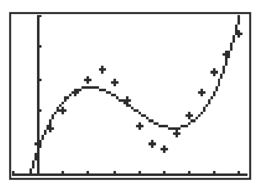
$[-5, 4]$ by $[-1, 49]$

(c) $f(x) = (x - 2)(x^3 + 4x^2 - 3x - 19)$

(d) From our graph, we find that one irrational zero of x is $x \approx 2.04$.

(e) $f(x) \approx (x - 2)(x - 2.04)(x^2 + 6.04x + 9.3216)$

62. (a) $D \approx 0.0669t^3 - 0.7420t^2 + 2.1759t + 0.8250$



[1, 8.25] by [0, 5]

(b) When $t = 0$, $D \approx 0.8250$ m.

(c) The graph changes direction at $t \approx 2.02$ and at $t \approx 5.38$. Lewis is approximately 2.74 m from the motion detector at $t = 2.02$ and 1.47 m from the motion detector at $t = 5.38$.

63. False. $x - a$ is a factor if and only if $f(a) = 0$. So $(x + 2)$ is a factor if and only if $f(-2) = 0$.

64. True. By the Remainder Theorem, the remainder when $f(x)$ is divided by $x - 1$ is $f(1)$, which equals 3.

65. The statement $f(3) = 0$ means that $x = 3$ is a zero of $f(x)$ and that 3 is an x-intercept of the graph of $f(x)$. And it follows that $x - 3$ is a factor of $f(x)$ and thus that the remainder when $f(x)$ is divided by $x - 3$ is zero. So the answer is A.

66. By the Rational Zeros Theorem, every rational root of $f(x)$ must be among the numbers $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$.
The answer is E.

67. $f(x) = (x + 2)(x^2 + x - 1) - 3$ yields a remainder of -3 when divided by either $x + 2$ or $x^2 + x - 1$, from which it follows that $x + 2$ is not a factor of $f(x)$ and that $f(x)$ is not evenly divisible by $x + 2$. The answer is B.

68. Answers A through D can be verified to be true. And because $f(x)$ is a polynomial function of odd degree, its graph must cross the x-axis somewhere. The answer is E.

69. (a) The volume of a sphere is $V = \frac{4}{3}\pi r^3$. In this case, the radius of the buoy is 1, so the buoy's volume is $\frac{4}{3}\pi$.

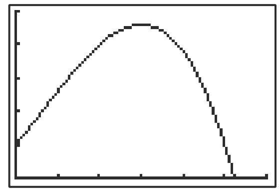
(b) Total weight $=$ volume $\cdot \dfrac{\text{weight}}{\text{unit volume}}$ $=$ volume $\cdot$ density. In this case, the density of the buoy is $\frac{1}{4}d$, so, the weight W_b of the buoy is
$W_b = \frac{4\pi}{3} \cdot \frac{1}{4} d = \frac{d\pi}{3}$.

(c) The weight of the displaced water is $W_{H_2O} =$ volume $\cdot$ density. We know from geometry that the volume of a spherical cap is
$V = \frac{\pi}{6}(3r^2 + h^2)\,h$, so,
$W_{H_2O} = \frac{\pi}{6}(3r^2 + x^2)\,x \cdot d = \frac{\pi d}{6}x(3r^2 + x^2)$.

(d) Setting the two weights equal, we have:
$W_b = W_{H_2O}$
$\frac{\pi d}{3} = \frac{\pi d}{6}(3r^2 + x^2)x$
$2 = (3r^2 + x^2)x$
$0 = (6x - 3x^2 + x^2)x - 2$
$0 = -2x^3 + 6x^2 - 2$
$0 = x^3 - 3x^2 + 1.$
Solving graphically, we find that $x \approx 0.6527$ m, the depth that the buoy will sink.

70. The weight of the buoy, W_b, with density $\frac{1}{5}d$, is
$W_b = \frac{4\pi}{3} \cdot \frac{1}{5} d = \frac{4\pi d}{15}$. So,
$W_b = W_{H_2O}$
$\frac{4\pi d}{15} = \frac{\pi d}{6}(3r^2 + x^2)x$
$\frac{24}{15} = (3r^2 + x^2)x$
$0 = (6x - 3x^2 + x^2)x - \frac{8}{5}$
$0 = -2x^3 + 6x^2 - \frac{8}{5}$
$0 = x^3 - 3x^2 + \frac{4}{5}$
Solving graphically, we find that $x \approx 0.57$ m, the depth the buoy would sink.

71. (a) Shown is one possible view, on the window $[0, 600]$ by $[0, 500]$.



$[0, 600]$ by $[0, 500]$

(b) The maximum population, after 300 days, is 460 turkeys.

(c) $P = 0$ when $t \approx 523.22$ — about 523 days after release.

(d) Answers will vary. One possibility: After the population increases to a certain point, they begin to compete for food and eventually die of starvation.

72. (a) d is the independent variable.

(b) A good choice is $[0, 172]$ by $[0, 5]$.

(c) $s = 1.25$ when $d \approx 95.777$ ft (found graphically).

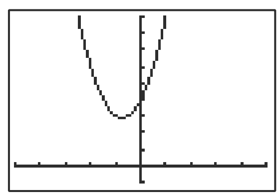
73. (a) 2 sign changes in $f(x)$, 1 sign change in $f(-x) = -x^3 + x^2 + x + 1$; 0 or 2 positive zeros, 1 negative zero.

(b) No positive zeros, 1 or 3 negative zeros.

(c) 1 positive zero, no negative zeros.

(d) 1 positive zero, 1 negative zero.

74.



$[-5, 5]$ by $[-1, 9]$

The functions are not exactly the same, when $x \neq 3$, we have $f(x) = \dfrac{(x - 3)(2x^2 + 3x + 4)}{(x - 3)}$
$= 2x^2 + 3x + 4 = g(x)$

The domain of f is $(-\infty, 3) \cup (3, \infty)$ while the domain of g is $(-\infty, \infty)$. f is discontinuous at $x = 3$. g is continuous.

75. $\dfrac{4x^3 - 5x^2 + 3x + 1}{2x - 1}$

$= \dfrac{2x^3 - \frac{5}{2}x^2 + \frac{3}{2}x + \frac{1}{2}}{x - \frac{1}{2}}$ Divide numerator and denominator by 2.

zero of divisor $\frac{1}{2}$	2	$-\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	Write coefficients of dividend.
line for products		1	$-\frac{3}{4}$	$\frac{3}{8}$	
line for sums	2	$-\frac{3}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	Quotient, remainder

Copy 2 into the first quotient position. Multiply $2 \cdot \frac{1}{2} = 1$ and add this to $-\frac{5}{2}$. Multiply $-\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}$ and add this to $\frac{3}{2}$. Multiply $\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$ and add this to $\frac{1}{2}$. The last line tells us $\left(x - \frac{1}{2}\right)\left(2x^2 - \frac{3}{2}x + \frac{3}{4}\right) + \frac{7}{8}$

$= 2x^3 - \frac{5}{4}x^2 + \frac{3}{2}x + \frac{1}{2}.$

76. Use the zero or root finder feature to locate the zero near $x = -3$. Then regraph the function in a smaller window, such as $[0, 2]$ by $[-0.5, 0.5]$, and locate the other three zeros of the function.

77. (a) $g(x) = 3f(x)$, so the zeros of f and the zeros of g are identical. If the coefficients of a polynomial are rational, we may multiply that polynomial by the least common multiple (LCM) of the denominators of the coefficients to obtain a polynomial, with integer coefficients, that has the same zeros as the original.

(b) The zeros of $f(x)$ are the same as the zeros of $6f(x) = 6x^3 - 7x^2 - 40x + 21$. Possible rational zeros: $\frac{\pm 1, \pm 3, \pm 7, \pm 21}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm 3, \pm 7, \pm 21$, $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}, \pm\frac{1}{3}, \pm\frac{7}{3}, \pm\frac{1}{6}, \pm\frac{7}{6}$.
The actual zeros are $-7/3$, $1/2$, and 3.

(c) The zeros of $f(x)$ are the same as the zeros of $12f(x) = 12x^3 - 30x^2 - 37x + 30$.
Possible rational zeros:
$\frac{\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30}{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}$, or
$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$,
$\pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{6}$,
$\pm\frac{5}{6}, \pm\frac{1}{12}, \pm\frac{5}{12}$.
There are no rational zeros.

78. Let $f(x) = x^2 - 2 = (x + \sqrt{2})(x - \sqrt{2})$. Notice that $\sqrt{2}$ is a zero of f. By the Rational Zeros Theorem, the only possible rational zeros of f are ± 1 and ± 2. Because $\sqrt{2}$ is none of these, it must be irrational.

79. (a) Approximate zeros: $-3.126, -1.075, 0.910, 2.291$

(b) $f(x) \approx g(x)$
$= (x + 3.126)(x + 1.075)(x - 0.910)(x - 2.291)$

(c) Graphically: Graph the original function and the approximate factorization on a variety of windows and observe their similarity. Numerically: Compute $f(c)$ and $g(c)$ for several values of c.

■ Section 2.5 Complex Zeros and the Fundamental Theorem of Algebra

Exploration 1

1. $f(2i) = (2i)^2 - i(2i) + 2 = -4 + 2 + 2 = 0$;
$f(-i) = (-i)^2 - i(-i) + 2 = -1 - 1 + 2 = 0$; no.

2. $g(i) = i^2 - i + (1 + i) = -1 - i + 1 + i = 0$;
$g(1 - i) = (1 - i)^2 - (1 - i) + (1 + i) =$
$-2i + 2i = 0$; no.

3. The Complex Conjugate Zeros Theorem does not necessarily hold true for a polynomial function with *complex* coefficients.

Quick Review 2.5

1. $(3 - 2i) + (-2 + 5i) = (3 - 2) + (-2 + 5)i$
$= 1 + 3i$

2. $(5 - 7i) - (3 - 2i) = (5 - 3) + (-7 - (-2))i$
$= 2 - 5i$

3. $(1 + 2i)(3 - 2i) = 1(3 - 2i) + 2i(3 - 2i)$
$= 3 - 2i + 6i - 4i^2$
$= 7 + 4i$

4. $\frac{2 + 3i}{1 - 5i} = \frac{2 + 3i}{1 - 5i} \cdot \frac{1 + 5i}{1 + 5i}$
$= \frac{2 + 10i + 3i + 15i^2}{1^2 + 5^2}$
$= \frac{-13 + 13i}{26}$
$= -\frac{1}{2} + \frac{1}{2}i$

5. $(2x - 3)(x + 1)$

6. $(3x + 1)(2x - 5)$

7. $x = \frac{5 \pm \sqrt{25 - 4(1)(11)}}{2} = \frac{5 \pm \sqrt{-19}}{2}$
$= \frac{5}{2} \pm \frac{\sqrt{19}}{2}i$

8. $x = \frac{-3 \pm \sqrt{9 - 4(2)(7)}}{4} = \frac{-3 \pm \sqrt{-47}}{4}$
$= -\frac{3}{4} \pm \frac{\sqrt{47}}{4}i$

9. $\frac{\pm 1, \pm 2}{\pm 1, \pm 3}$, or $\pm 1, \pm 2, \pm\frac{1}{3}, \pm\frac{2}{3}$

10. $\frac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 4}$, or $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

Section 2.5 Exercises

1. $(x - 3i)(x + 3i) = x^2 - (3i)^2 = x^2 + 9$. The factored form shows the zeros to be $x = \pm 3i$. The absence of real zeros means that the graph has no x-intercepts.

2. $(x + 2)(x - \sqrt{3}\,i)(x + \sqrt{3}\,i) = (x + 2)(x^2 + 3)$
$= x^3 + 2x^2 + 3x + 6$. The factored form shows the zeros to be $x = -2$ and $x = \pm\sqrt{3}i$. The real zero $x = -2$ is the x-intercept of the graph.

3. $(x - 1)(x - 1)(x + 2i)(x - 2i)$
$= (x^2 - 2x + 1)(x^2 + 4)$
$= x^4 - 2x^3 + 5x^2 - 8x + 4$. The factored form shows the zeros to be $x = 1$ (multiplicity 2) and $x = \pm 2i$. The real zero $x = 1$ is the x-intercept of the graph.

4. $x(x - 1)(x - 1 - i)(x - 1 + i)$
$= (x^2 - x)[x - (1 + i)][x - (1 - i)]$
$= (x^2 - x)[x^2 - (1 - i + 1 + i)\,x + (1 + 1)]$
$= (x^2 - x)(x^2 - 2x + 2) = x^4 - 3x^3 + 4x^2 - 2x$.
The factored form shows the zeros to be $x = 0$, $x = 1$, and $x = 1 \pm i$. The real zeros $x = 0$ and $x = 1$ are the x-intercepts of the graph.