AP Calculus AB Name Key

First Semester Review #2 **MULTIPLE CHOICE** **NO CALCULATOR!**

SHOW ALL WORK!! Indicate clearly the methods you use.

1) $\lim_{h \to 0} \dfrac{\cos\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}\right)}{h} =$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f'\left(\frac{\pi}{2}\right) = -1$

a) $-\infty$

b) -1

c) 0

d) 1

e) $+\infty$

2) $\lim_{n \to \infty} \dfrac{3n^3-5n}{n^3-2n^2+1} =$

a) -5

b) -2

c) 1

d) 3

e) nonexistent

3) If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2-4}{x+2}$ when $x \neq -2$, then $f(-2) =$

$f(x) = \dfrac{(x+2)(x-2)}{x+2}$

$= (x-2)$

a) -4

b) -2

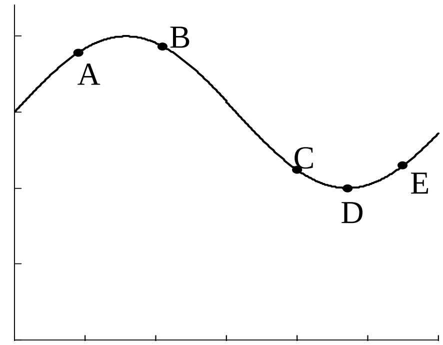
c) -1

d) 0

e) 2

4) At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

↓ cd



a) A b) B c) C d) D e) E

5) $\lim_{\theta \to 0} \frac{1-\cos\theta}{2\sin^2\theta} =$

When finding limits, step 1 is to substitute.

However, we get $\frac{0}{0}$!

So ... use Algebra/Trig

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{2(1-\cos^2\theta)}$$

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{2(1-\cos\theta)(1+\cos\theta)}$$

$$\lim_{\theta \to 0} \frac{1}{2(1+\cos\theta)}$$

$$\frac{1}{2(1+1)}$$

a) 0

b) $\frac{1}{8}$

c) $\frac{1}{4}$

d) 1

e) nonexistent

6) If f is a differentiable function, then $f'(a)$ is given by which of the following?

T I. $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

T II. $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

F III. $\lim_{x \to a} \frac{f(x+h)-f(x)}{h}$

a) I only b) II only c) I and II only d) I and III only e) I, II, and III

AP Calculus AB Name ~~Key~~

First Semester Review **MULTIPLE CHOICE** **CALCULATOR ALLOWED**

SHOW ALL WORK!!

7) If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

a) $-\frac{x^2+y}{x+2y^2}$

b) $-\frac{x^2+y}{x+y^2}$

c) $-\frac{x^2+y}{x+2y}$

d) $-\frac{x^2+y}{2y^2}$

e) $\frac{-x^2}{1+2y^2}$

$3x^2+3x\frac{dy}{dx}+y\cdot 3+6y^2\frac{dy}{dx}=0$

$\frac{dy}{dx}(3x+6y^2)=-3x^2-3y$

$\frac{dy}{dx}=\frac{-3(x^2+y)}{3(x+2y^2)}$

8) The equation of the line tangent to the graph of $y=\frac{2x+3}{3x-2}$ at the point (1, 5) is

a) $13x-y=8$

b) $13x+y=18$

c) $x-13y=64$

d) $x+13y=66$

e) $-2x+3y=13$

$\frac{dy}{dx}=\frac{(3x-2)2-(2x+3)3}{(3x-2)^2}$

$=\frac{6x-4-6x-9}{(3x-2)^2}$

$=\frac{-13}{(3x-2)^2}$

$\left.\frac{dy}{dx}\right|_{(1,5)}=-13$

$y-5=-13(x-1)$

$y=-13x+13+5$

$13x+y=18$

9) If $y=\tan x-\cot x$, then $\frac{dy}{dx}=$

a) $\sec x\cdot\csc x$

b) $\sec x-\csc x$

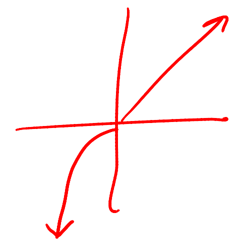
c) $\sec x+\csc x$

d) $\sec^2 x-\csc^2 x$

e) $\sec^2 x+\csc^2 x$

10) Let f be the function defined by $f(x)=\begin{cases} x^3 & \text{for } x \le 0 \\ x & \text{for } x > 0 \end{cases}$.

Which of the following statements about f is true?



a) f is an odd function F

b) f is discontinuous at x = 0 F

c) f has a relative maximum F

d) $f'(0)=0$ F

e) $f'(x)>0$ for $x \ne 0$ T

11) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{x\cos x} =$ $\lim\limits_{x\to 0}\left(\dfrac{\sin 2x}{2x}\cdot\dfrac{2}{\cos x}\right)$

$1 \cdot \dfrac{2}{1}$

a) 0

b) 1

c) $\dfrac{1}{2}$

d) 2

e) Does not exist

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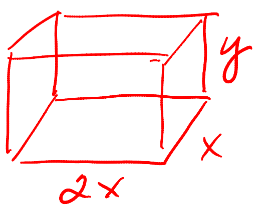
First Semester Review **FREE RESPONSE** **CALCULATOR ALLOWED**

SHOW ALL WORK! Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximation, your answer should be correct to three decimal places.

1) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

$V = 10 m^3$



$2x^2 y = 10$

$y = \frac{5}{x^2}$

Minimize Cost

$Cost = 10 \cdot 2x^2 + 6(2xy + 2 \cdot 2xy)$

$= 20x^2 + 36xy$

$= 20x^2 + 36x\left(\frac{5}{x^2}\right)$

$= 20x^2 + 180x^{-1}$

$\frac{dC}{dx} = 40x - 180x^{-2} = 0$

$40x = \frac{180}{x^2}$

$40x^3 = 180$

$x^3 = 4.5$

$x \approx 1.651$

$\frac{dC}{dx}$ − + (0, 1.651)

There is a minimum cost when $x \approx 1.651$ since $\frac{dC}{dx}$ changes from neg to pos.

Minimum cost = \$163.54

2) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers *c* that satisfy the conclusion of the Mean Value Theorem.

$f(x) = x^3 + x - 1, \ [0, 2]$

$f(x)$ is continuous on $[0,2]$ & differentiable on $(0,2)$.

$f'(x) = 3x^2 + 1$ $\quad f(0) = -1$ $\quad f(2) = 9$

$3c^2 + 1 = \frac{9+1}{2}$

$3c^2 + 1 = 5$

$3c^2 = 4$

$c^2 = \frac{4}{3}$

$c = \pm\frac{2}{\sqrt{3}}$

on $[0,2]$, $c = \frac{2}{\sqrt{3}}$

≈ 1.155

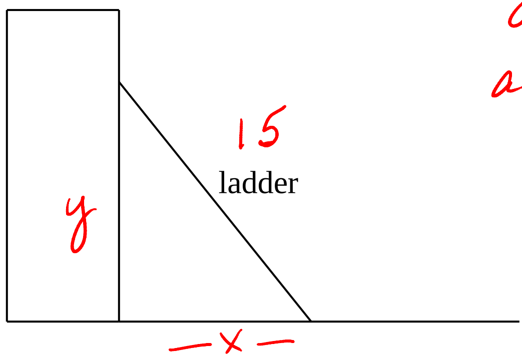
3) A ladder that is 15 feet long is leaning against a building. The bottom of the ladder is moving away from the wall at ½ foot per second.

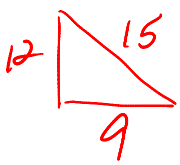
a) Find the rate at which the top of the ladder is moving down the wall when the bottom is 9 feet from the wall.

b) Find the rate of change of the area of the triangle formed by the ladder and the wall when the bottom is 9 feet from the wall.

$\frac{dx}{dt} = \frac{1}{2}$ ft/sec

a) Find $\frac{dy}{dt}$ when $x = 9$ ft





$$x^2 + y^2 = 15^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$9 \cdot \frac{1}{2} = -12\frac{dy}{dt}$$

$$-\frac{3}{4} \cdot \frac{1}{2} \text{ ft/sec} = \frac{dy}{dt}$$

$$-\frac{3}{8} \text{ ft/sec} = \frac{dy}{dt}$$

b) Find $\frac{dA}{dt}$ when $x = 9$ ft

$$A = \frac{1}{2}x \cdot y$$

$$\frac{dA}{dt} = \frac{1}{2}x \cdot \frac{dy}{dt} + \frac{1}{2}y \cdot \frac{dx}{dt}$$

$$= \frac{1}{2} \cdot 9\left(-\frac{3}{8}\right) + \frac{1}{2} \cdot 12 \cdot \frac{1}{2}$$

$$= -\frac{27}{16} + 3$$

$$= -\frac{27}{16} + \frac{48}{16}$$

$$= \frac{21}{16} \text{ ft}^2/\text{sec}$$