AP Calculus AB
First Semester Review

Name Key

NO CALCULATOR ALLOWED!!

Section I, Part A

Directions: Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given and circle your answer choice.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

$y' = x^2 + 10x$
$y'' = 2x + 10$
$x = -5$

2. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

$4 + 2y = 10$
$y = 3$

$2x + x\frac{dy}{dx} + y = 0$
$4 + 2\frac{dy}{dx} + 3 = 0$
$\frac{dy}{dx} = \frac{-7}{2}$

3. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

$h'(x) = f(x)g'(x) + g(x)f'(x) = f(x)g'(x)$
$\therefore g(x)f'(x) = 0$
$\therefore f'(x) = 0$ so $f(x)$ is constant

4. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

$$f'(x) = \frac{(x-1)2x - (x^2-2)}{(x-1)^2} = \frac{2x^2 - 2x - x^2 + 2}{(x-1)^2} =$$

$$= \frac{x^2 - 2x + 2}{(x-1)^2} \Rightarrow f'(2) = \frac{4-4+2}{1} = 2$$

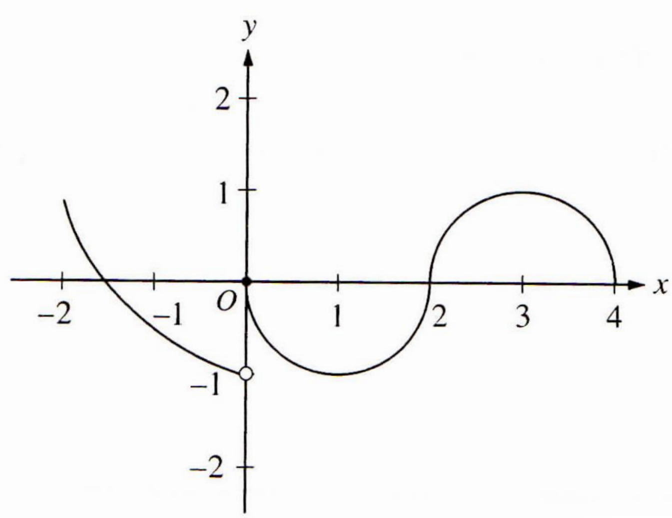
5. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

$$\lim_{x \to 2^-} f(x) = \ln 2$$

$$\lim_{x \to 2^+} f(x) = 4 \ln 2$$

6. The graph of the function f shown in the figure below has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, -1) and (3, 1). For what values of x, $-2 < x < 4$, is f not differentiable?



(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

7. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$v(t) = 2t - 6 = 0$

$t = 3$

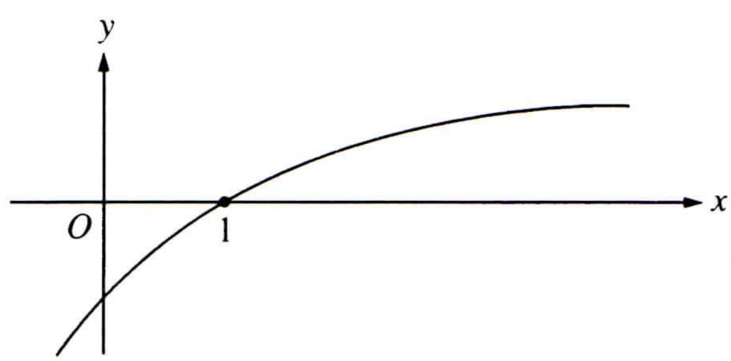
8. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

(A) $-\cos\left(e^{-x}\right)$ (B) $\cos\left(e^{-x}\right) + e^{-x}$ (C) $\cos\left(e^{-x}\right) - e^{-x}$

(D) $e^{-x}\cos\left(e^{-x}\right)$ (E) $-e^{-x}\cos\left(e^{-x}\right)$

$f'(x) = \cos(e^{-x})\, e^{-x}\, (-1)$

9. The graph of a twice-differentiable function f is shown in the figure below. Which of the following is true?



(A) $f(1) < f'(1) < f''(1)$ (B) $f(1) < f''(1) < f'(1)$ (C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$ (E) $f''(1) < f'(1) < f(1)$

f(x) is increasing, so $f'(x) > 0$

f(x) is concave down, so $f''(x) < 0$

10. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

$y' = 1 - \sin x$

$y'\big|_{x=0} = 1$

$y = x + 1$

11. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

$f''(x)$

--+ -++ +++ +++

cu -1 cd 0 cu 2 cu

12. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$ (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (C) $(0, \infty)$ (D) $(-\infty, 0)$ (E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

$f'(x) = 4x^3 + 2x$

$2x(2x^2 + 1) = 0$

$x = 0$

f' — +

f is → dec 0 inc

13. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

To max acceleration, evaluate $a'(t)$

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

$a(t) = 3t^2 - 6t + 12$

$a'(t) = 6t - 6 = 0$

$t = 1$

$a'(t)$

- +

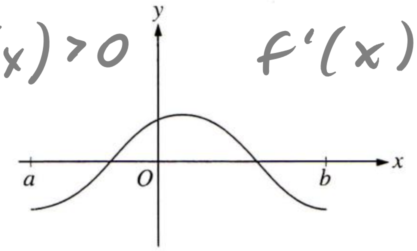
0 max 1 min 3 max

$a(0) = 12$

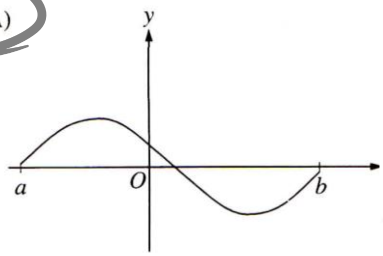
$a(3) = 21$ *

14. The graph of f is shown in the figure below. Which of the following could be the graph of the derivative of f?

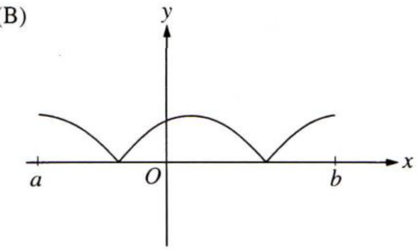
$f'(x) > 0$ $f'(x) < 0$



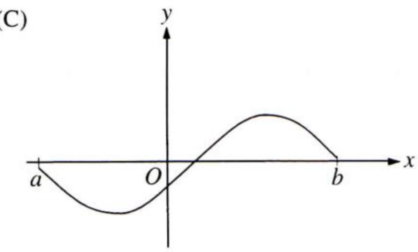
(A)



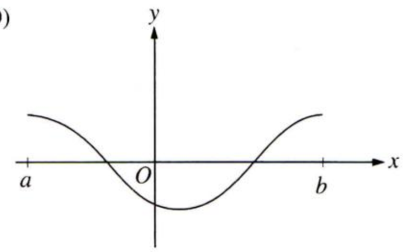
(B)



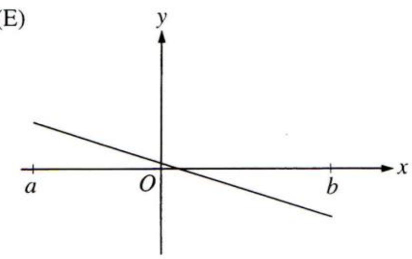
(C)



(D)



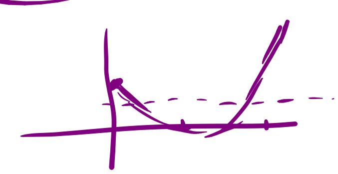
(E)



15. The function f is continuous on the closed interval [0, 2] and has values that are given in the table below. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if k =

x	0	1	2
$f(x)$	1	k	2

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3



k must be less than $\frac{1}{2}$

16. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

$f'(x) = \sec^2(2x) \cdot 2$

$f'\left(\frac{\pi}{6}\right) = 2\sec^2\left(\frac{\pi}{3}\right)$

$= 2 \cdot 2^2$

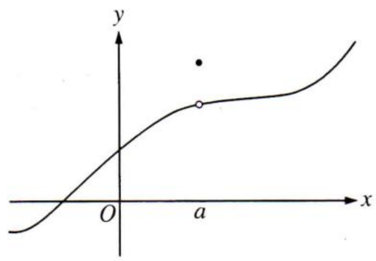
AP Calculus AB
First Semester Review

Name Key
Section I, Part B

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

In this test: The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

1. The graph of a function f is shown below. Which of the following statements about f is false?



(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x\to a^+} f(x)$ is equal to $\lim_{x\to a^-} f(x)$.

(E) $\lim_{x\to a} f(x)$ exists.

2. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701 (B) -0.567 (C) -0.391 (D) -0.302 (E) -0.258

$f'(x) = 6e^{2x}$
$g'(x) = 18x^2$

$f'(x) = g'(x)$

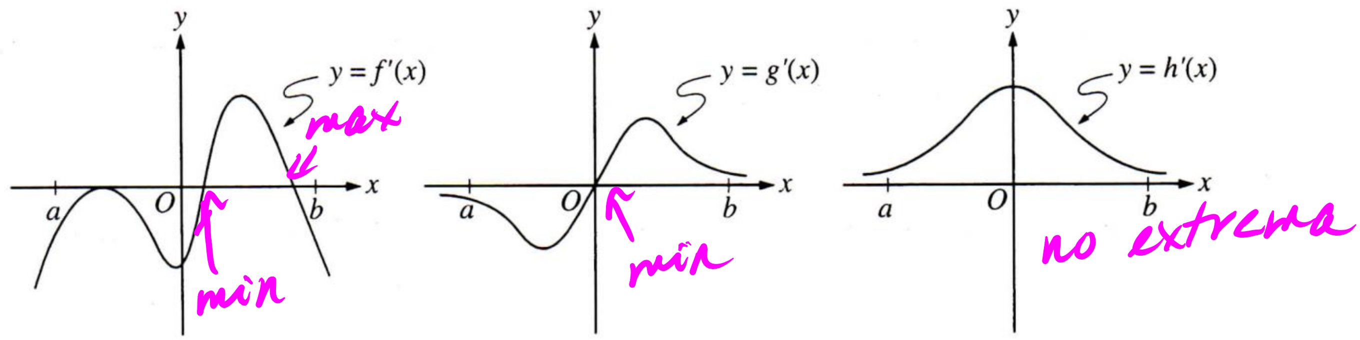
3. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$ (B) $-(0.1)C$ (C) $-\frac{(0.1)C}{2\pi}$ (D) $(0.1)^2 C$ (E) $(0.1)^2 \pi C$

$\frac{dr}{dt} = -1$ cm/sec

$A = \pi r^2$
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
$= C(-0.1)$

4. The graphs of the derivatives of the functions f, g, and h are shown below. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?



(A) f only (B) g only (C) h only (D) f and g only (E) f, g, and h

5. The first derivative of the function f is given by $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$. How many critical values does f have on the open interval (0, 10)?

(A) One (B) Three (C) Four (D) Five (E) Seven

$f'(x) = 0$

6. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

T I. f is continuous at $x = 0$.
F II. f is differentiable at $x = 0$.
T III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

7. If $a \neq 0$, then $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

$$\lim_{x \to a} \frac{x^2 - a^2}{(x^2 - a^2)(x^2 + a^2)} = \lim_{x \to a} \frac{1}{x^2 + a^2} = \frac{1}{2a^2}$$

8. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

$y - \quad = 1(x - .237)$

~~(A)~~ $y = 8x - 5$ (B) $y = x + 7$ (C) $y = x + 0.763$

(D) $y = x - 0.122$ (E) $y = x - 2.146$

$f'(x) = 4x^3 + 4x = 1$

$x \approx 0.237$ $y \approx 0.11522$

9. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

(A) f has a relative maximum at x = -2 and a relative minimum at x = 2.
(B) f has a relative minimum at x = -2 and a relative maximum at x = 2.
(C) f has a relative minima at x = -2 and at x = 2.
(D) f has a relative maxima at x = -2 and at x = 2.
(E) It cannot be determined if f has any relative extrema.

$f'(x) = (x-2)(x+2)g(x)$

− − − min + − max + −

↓ −2 ↑ 2 ↓

10. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing. (B) A is always decreasing.
(C) A is decreasing only when $b < h$. (D) A is decreasing only when $b > h$.
(E) A remains constant.

$\frac{dA}{dt} = \frac{1}{2}\left(b\frac{dh}{dt} + h\frac{db}{dt}\right)$

$= \frac{1}{2}(-3b + 3h)$

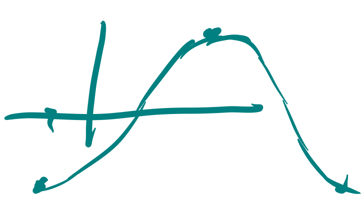
$\frac{dA}{dt} > 0$ when $h > b$

$\frac{dA}{dt} < 0$ when $b > h$

11. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

T I. f has at least 2 zeros.
T II. The graph of f has at least one horizontal tangent.
T III. For some c, $2 < c < 5$, $f(c) = 3$.

(A) None (B) I only (C) I and II only (D) I and III only (E) I, II, and III



AP Calculus AB
First Semester Review

Name Key
Section II

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

SHOW ALL YOUR WORK. YOU WILL BE GRADED ON THE CORRECTNESS AND COMPLETENESS OF YOUR METHODS AS WELL AS THE ACCURACY OF YOUR FINAL ANSWERS. CORRECT ANSWERS WITHOUT SUPPORTING WORK MAY NOT RECEIVE CREDIT. IF YOUR ANSWER IS GIVEN AS A DECIMAL APPROXIMATION, IT SHOULD BE CORRECT TO THREE PLACES AFTER THE DECIMAL POINT.

1. Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

$$\lim_{x \to -\infty} f(x) = 0 \qquad \lim_{x \to \infty} f(x) = \infty$$

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

$$f'(x) = 2xe^{2x} \cdot 2 + e^{2x} \cdot 2$$
$$= 2e^{2x}(2x+1)$$

$f'(x) = 0$ when $x = \frac{-1}{2}$

$f'(x)$

$+\,-$ | $+\,+$

$-\frac{1}{2}$

$$f\left(-\tfrac{1}{2}\right) = -1e^{-1} = \frac{-1}{e} \approx -0.3678$$

There is an absolute min at $x = -\frac{1}{2}$ since $f'(x)$ changes from negative to positive at $x = -\frac{1}{2}$, the only critical value.

(c) What is the range of f?

$$\left[-\frac{1}{e}, \infty\right)$$

(d) Consider the family of functions defined by $f(x) = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

$$f'(x) = bxe^{bx} \cdot b + e^{bx} \cdot b$$
$$= be^{bx}(bx+1)$$

$f'(x) = 0$ when $x = \frac{-1}{b}$

$f'(x)$: $-$ | $+$ at $-\frac{1}{b}$

There is an abs. min value at $x = \frac{-1}{b}$ since $f'(x)$ changes from neg to pos at $x = \frac{-1}{b}$, the only critical value.

$f\left(-\frac{1}{b}\right) = -e^{-1}$

$\therefore$ Absolute min value is $\frac{-1}{e}$.

2. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where $x = 1$.

slope of f at $x = 1$:

$$f'(1) = \frac{3 \cdot 1 + 1}{2 \cdot 4} = \frac{1}{2}$$

(b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

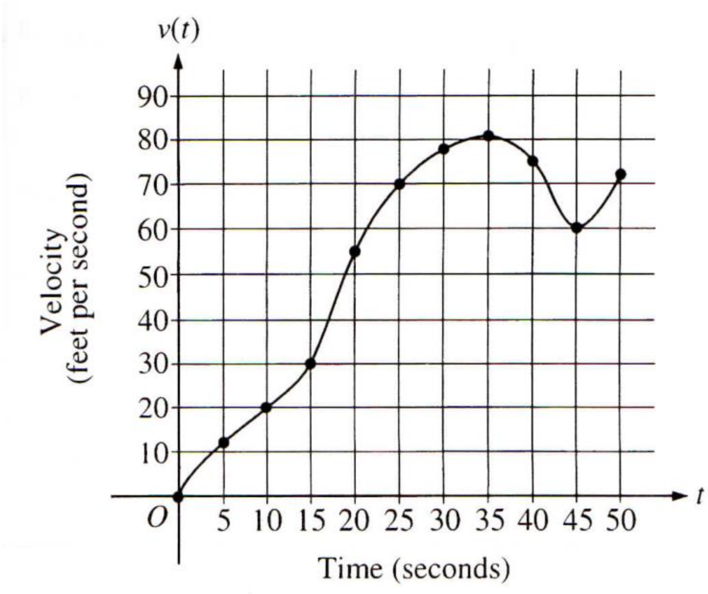
$y - 4 = \frac{1}{2}(x - 1)$

OR

$y = \frac{1}{2}x + 3\frac{1}{2}$

$$f(1.2) \approx \frac{1}{2}(1.2) + 3.5$$
$$= 0.6 + 3.5$$
$$= 4.1$$

3. The graph of the velocity $v(t)$, in ft/sec of a car traveling on a straight road, for $0 \le t \le 50$, is shown below. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

The acceleration is positive on (0, 35) and (45, 50). This is because the acceleration is positive when the velocity is increasing.

(b) Find the average acceleration of the car, in ft/sec², over the interval $0 \le t \le 50$.

$$\text{av acc} = \frac{72-0}{50-0}$$

$$= 1.44 \text{ ft/sec}^2$$

(c) Find one approximation for the acceleration of the car, in ft/sec², at $t = 40$. Show the computations you used to arrive at your answer.

$$a(40) \approx \frac{v(45)-v(35)}{45-35} = \frac{60-81}{10} = -2.1 \text{ ft/sec}^2$$

OR $$a(40) \approx \frac{v(45)-v(40)}{45-40} = \frac{60-75}{5} = -3 \text{ ft/sec}^2$$

OR $$a(40) \approx \frac{v(40)-v(35)}{40-35} = \frac{75-81}{5} = -1.2 \text{ ft/sec}^2$$

4. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.

$$6y^2\frac{dy}{dx} + 6x^2\frac{dy}{dx} + y\cdot 12x - 24x + 6\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$$

$$\frac{dy}{dx} = \frac{6(4x - 2xy)}{6(x^2 + y^2 + 1)}$$

$$\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$$

(b) Write an equation of each horizontal tangent line to the curve.

$\frac{dy}{dx} = 0$ when $4x - 2xy = 0$

$2x(2 - y) = 0$

$x = 0,\ y = 2$

$x = 0$

$2y^3 + 6y = 1$

$y \approx 0.165$

$\boxed{y = 0.165}$

$y = 2$

$16 + 12x^2 - 12x^2 + 12 = 1$

$28 = 1$

$y \neq 2$

(c) The line through the origin with slope -1 is tangent to the curve at point *P*. Find the *x*- and *y*-coordinates of point *P*.

$y = -x$ is tangent to curve at P.

From (a): $\dfrac{4x - 2xy}{x^2 + y^2 + 1} = -1$

$4x - 2xy = -x^2 - y^2 - 1$

$4x - 2x(-x) = -x^2 - (-x)^2 - 1$

$4x + 2x^2 = x^2 - x^2 - 1$

$4x^2 + 4x + 1 = 0$

$(2x + 1)^2 = 0$

$x = -\frac{1}{2} \Rightarrow y = +\frac{1}{2}$ $\quad\left(-\frac{1}{2}, \frac{1}{2}\right)$

AP Calculus AB **AP Calculus AB** Key

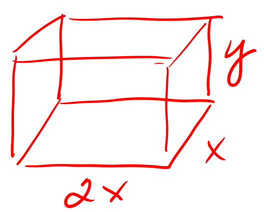
Name________________________________

First Semester Review **FREE RESPONSE** **CALCULATOR ALLOWED**

SHOW ALL WORK! Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximation, your answer should be correct to three decimal places.

1) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

$V = 10m^3$



$2x^2 y = 10$

$y = \frac{5}{x^2}$

Minimize Cost

$$\text{Cost} = 10 \cdot 2x^2 + 6(2xy + 2 \cdot 2xy)$$
$$= 20x^2 + 36xy$$
$$= 20x^2 + 36x\left(\frac{5}{x^2}\right)$$
$$= 20x^2 + 180x^{-1}$$

$$\frac{dC}{dx} = 40x - 180x^{-2} = 0$$
$$40x = \frac{180}{x^2}$$
$$40x^3 = 180$$
$$x^3 = 4.5$$
$$x \approx 1.651$$

$\frac{dC}{dx}$: $-$ on $(0, 1.651)$, $+$ after 1.651

There is a minimum cost when $x \approx 1.651$ since $\frac{dC}{dx}$ changes from neg to pos.

Minimum cost = \$163.54

2) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers *c* that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = x^3 + x - 1, \ [0, 2]$$

$f(x)$ is continuous on $[0,2]$ & differentiable on $(0,2)$.

$f'(x) = 3x^2 + 1$ $\quad f(0) = -1$ $\quad f(2) = 9$

$$3c^2 + 1 = \frac{9+1}{2}$$
$$3c^2 + 1 = 5$$
$$3c^2 = 4$$
$$c^2 = \frac{4}{3}$$

$c = \pm\frac{2}{\sqrt{3}}$

on $[0,2]$, $c = \frac{2}{\sqrt{3}}$

≈ 1.155

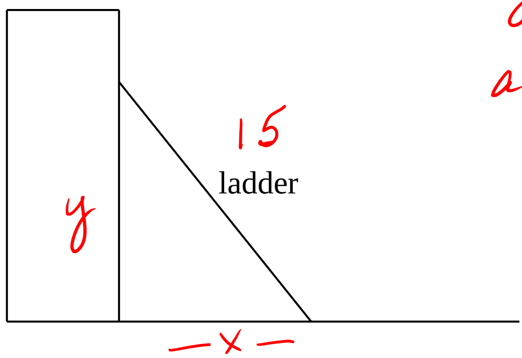
3) A ladder that is 15 feet long is leaning against a building. The bottom of the ladder is moving away from the wall at ½ foot per second.

a) Find the rate at which the top of the ladder is moving down the wall when the bottom is 9 feet from the wall.

b) Find the rate of change of the area of the triangle formed by the ladder and the wall when the bottom is 9 feet from the wall.

$$\frac{dx}{dt} = \frac{1}{2} \text{ ft/sec}$$

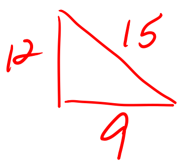
a) Find $\frac{dy}{dt}$ when $x = 9$ ft



$$x^2 + y^2 = 15^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$



$$9 \cdot \frac{1}{2} = -12\frac{dy}{dt}$$

$$-\frac{3}{4} \cdot \frac{1}{2} \text{ ft/sec} = \frac{dy}{dt}$$

$$-\frac{3}{8} \text{ ft/sec} = \frac{dy}{dt}$$

b) Find $\frac{dA}{dt}$ when $x = 9$ ft

$$A = \frac{1}{2} x \cdot y$$

$$\frac{dA}{dt} = \frac{1}{2} x \cdot \frac{dy}{dt} + \frac{1}{2} y \cdot \frac{dx}{dt}$$

$$= \frac{1}{2} \cdot 9\left(-\frac{3}{8}\right) + \frac{1}{2} \cdot 12 \cdot \frac{1}{2}$$

$$= -\frac{27}{16} + 3$$

$$= -\frac{27}{16} + \frac{48}{16}$$

$$= \frac{21}{16} \text{ ft}^2\text{/sec}$$