AP Calculus AB
First Semester Review #1

Name________________________________

YOU MUST SHOW ALL WORK TO RECEIVE CREDIT!!

1. The graph of $y = 3x^2 - x^3$ has a relative maximum at

(A) (0, 0) only

(B) (1, 2) only

(C) (2, 4) only

(D) (4, -16) only

(E) (0, 0) and (2, 4)

$y' = 6x - 3x^2$
$= 3x(2-x)$

$y' = 0$ when $x = 0, 2$

y': $-$ | 0 | $+$ | 2 | $-$

$y(2) = 3\cdot 2^2 - 2^3 = 4$

y has a relative max of 4 when $x = 2$ because y' changes sign from pos to neg

$\boxed{(2,4)}$

2. If $f(x) = \dfrac{x^2 - 9}{x+3}$ is continuous at x = -3, then $f(-3) =$

$x \neq -3$

(A) 3

(B) -3

(C) 0

(D) 6

(E) -6

$f(x) = \dfrac{\cancel{(x+3)}(x-3)}{\cancel{x+3}}$

$f(x) = x - 3$

$f(-3) = -3 - 3$
$= -6$

3. $\lim\limits_{x \to \infty} \dfrac{10^8 x^5 + 10^6 x^4 + 10^4 x^2}{10^9 x^6 + 10^7 x^5 + 10^5 x^3} =$

(A) 0

(B) 1

(C) -1

(D) $\dfrac{1}{10}$

(E) $-\dfrac{1}{10}$

THE MODEL FOR FUNCTION

$y = \dfrac{10^8 x^5}{10^9 x^6}$

$y = \dfrac{1}{10x}$

MODELS THE END BEHAVIOR OF OUR FUNCTION

$\frac{1}{10} \lim\limits_{x \to \infty} \frac{1}{x} = 0$

4. The equation of the tangent line to the curve $x^2 + y^2 = 169$ at the point (5, -12) is

(A) $5y - 12x = -120$

(B) $5x - 12y = 119$

(C) $5x - 12y = 169$

(D) $12x + 5y = 0$

(E) $12x + 5y = 169$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}\Bigg|_{(5,-12)} = \frac{5}{12}$$

slope

$$y + 12 = \frac{5}{12}(x-5)$$

$$12y + 144 = 5(x-5)$$

$$12y + 144 = 5x - 25$$

$$169 = 5x - 12y$$

5. If $e^y = x$, then $\frac{dy}{dx} =$

(A) 1

(B) $\frac{1}{x}$

(C) $\frac{1}{y}$

(D) $\ln x$

(E) $\ln y$

$$\ln x = y$$

$$\frac{1}{x} = \frac{dy}{dx}$$

OR

$$e^y = x$$

$$e^y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$\frac{dy}{dx} = \frac{1}{x}$$

6. If $f(x) = \frac{(\ln x)^2}{2}$, then $f'(e) =$

(A) e^2

(B) $\frac{1}{e}$

(C) $\frac{1}{e^2}$

(D) e

(F) 0

$$f'(x) = \frac{2\ln x}{2} \cdot \frac{1}{x}$$

$$f'(x) = \frac{\ln x}{x}$$

$$f'(e) = \frac{\ln e}{e}$$

$$f'(e) = \frac{1}{e}$$

$\ln e = 1$

why?

7. If the graph of $f(x) = 2x^2 + \frac{k}{x}$ has a point of inflection $\boxed{\text{at } x = -1,}$ then the value of k is

(A) 1

(B) -1

(C) 2

(D) -2

(E) 0

$f'(x) = 4x - k\frac{1}{x^2}$

$f''(x) = 4 + 2k \cdot \frac{1}{x^3}$

$f''(-1) = 4 - 2k = 0$

$-2k = -4$

$k = 2$

8. If $f(x) = 3x^2 - 8x^{-2}$, then $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} =$ $f'(2)$

(A) 10

(B) 14

(C) 20

(D) -14

(E) -20

$f'(x) = 6x + 16x^{-3}$

$f'(2) = 12 + 16\left(\frac{1}{8}\right)$

$= 14$

9. For what values of x is the graph of $y = \frac{2}{4-x}$ concave downward?

(A) No values of x

(B) $x < 4$

(C) $x > -4$

(D) $x < -4$

(E) $x > 4$

$y' = \frac{2}{(4-x)^2} \Longrightarrow \frac{d}{dx}\, 2(4-x)^{-2}$

$-4(4-x)^{-3} \cdot -1$

chain rule

$y'' = \frac{4}{(4-x)^3}$

y'': $+$ | 4 | $-$

concave up 4 concave down

10. $\frac{d}{dx}\left(e^{3\ln x}\right) =$

(A) $e^{3\ln x}$

(B) $\frac{e^{3\ln x}}{x}$

(C) x^3

(D) $3x^2$

(F) 3

THIS WAY

$= e^{3\ln x} \cdot \frac{3}{x}$ CHAIN RULE

$= e^{\ln x^3} \cdot \frac{3}{x}$

$= x^3 \cdot \frac{3}{x}$

$= 3x^2$

OR

THIS WAY

$= \frac{d}{dx} e^{3\ln x}$

$= \frac{d}{dx} e^{\ln x^3}$

$= \frac{d}{dx} x^3$

$= 3x^2$

11. A particle moves along the x-axis in such a way that its position at time t is given by $x(t) = \frac{1-t}{1+t}$.

What is the acceleration of the particle at time $t = 0$?

(A) $-\frac{3}{5}$

(B) -4

(C) 4

(D) 2

(E) -2

$\frac{dx}{dt} = \frac{(1+t)(-1) - (1-t)(1)}{(1+t)^2}$

$\frac{dx}{dt} = \frac{-2}{(1+t)^2} = -2(1+t)^{-2} \cdot 1$

$\frac{d^2x}{dt^2} = \frac{4}{(1+t)^3}\Big|_0 = \frac{4}{1} = 4$

12. Let f and g be differentiable functions such that

$$f(1) = 4,\ g(1) = 3,\ f'(3) = -5,\ f'(1) = -4,\ g'(1) = -3,\ g'(3) = 2$$

If $h(x) = f(g(x))$, then $h'(1) =$

(A) -9

(B) 15

(C) 0

(D) -5

(E) -12

$\boxed{h'(x) = f'(g(x)) \cdot g'(x)}$

$h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(3) \cdot -3$

$= -5 \cdot -3$

$= 15$

13. A point moves along the curve $y = x^2 + 1$ in such a way that when $x = 4$, the x-coordinate is increasing at the rate of 5 ft/sec. At what rate is the y-coordinate changing at that time?

(A) 80 ft/sec

(B) 45 ft/sec

(C) 32 ft/sec

(D) 85 ft/sec

(E) 40 ft/sec

$\frac{dx}{dt} = 5$ ft/sec $\quad \frac{dy}{dt} = ?$ when $x = 4$

$y = x^2 + 1$

$\frac{dy}{dt} = 2x \frac{dx}{dt} + 0$

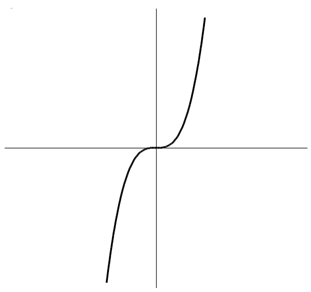
$\frac{dy}{dt} = 2(4)(5) = 40$

14. If, for all values of x, $f'(x) < 0$ and $f''(x) > 0$, which of the following curves could be part of the graph of f?

DEC

concave up

(A)



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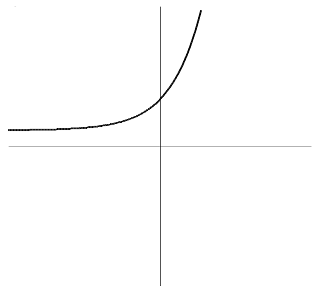

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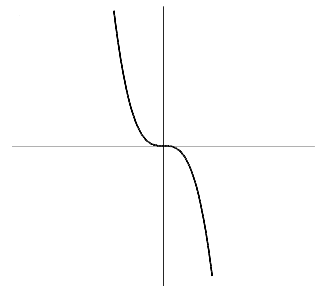
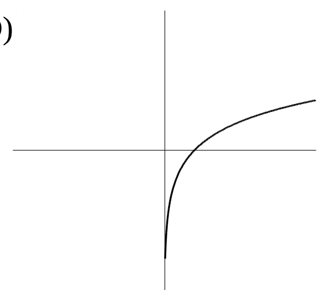
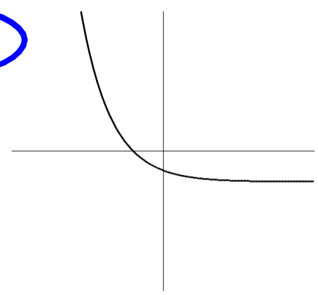


(D)



(E)



15. If the graph of $f(x) = x^3 + ax^2 + bx - 8$ has an inflection point at (2, 0), what is the value of b?

(A) 0

(B) 4

(C) 8

(D) 12

(E) The value of b cannot be determined from the given information

$f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$

$f''(x) = 0$ when $x = 2$

$0 = 6(2) + 2a$

$-12 = 2a$

$-6 = a$

$f(x) = x^3 - 6x^2 + bx - 8$

(2, 0) is on $f(x)$

$0 = 2^3 - 6(2)^2 + b(2) - 8$

$0 = 8 - 24 + 2b - 8$

$24 = 2b$

$12 = b$

16. If $y = x^{(x^3)}$ for $x > 0$, then $\frac{dy}{dx} =$

(A) $x^3 \cdot x^{(x^3-1)}$

(B) $4x^3$

(C) $x^2 + 3x^2 \ln x$

(D) $x^{(x^3+2)}(1+3\ln x)$

(E) $3x^{(x^3+2)} \ln x$

$\ln y = \ln x^{x^3}$

$\ln y = x^3 \cdot \ln x$

IMPLICIT DIFFERENTIATION
Take der. both sides w.r.t. to x remembering y is a function of x (Chain Rule!)

$\frac{1}{y} \cdot \frac{dy}{dx} = x^3 \cdot \frac{1}{x} + (\ln x)(3x^2)$

$\frac{dy}{dx} = y\left(x^2 + 3x^2 \ln x\right)$

$\frac{dy}{dx} = x^{x^3}\left(x^2 + 3x^2 \ln x\right)$

$\frac{dy}{dx} = x^{x^3} \cdot x^2 \left(1 + 3\ln x\right)$

17. The maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 1$ on [-1, 2] is

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$f'(x) = 6x^2 - 18x + 12$

$= 6(x^2 - 3x + 2)$

$= 6(x-2)(x-1)$

f': -1 min, $+$, 1 max, $-$, 2 min

Abs. max at $x = 1$ because f' changes sign from pos. to neg.

(Endpts are where min values occur)

$f(1) = 4$

OPTIMIZATION

18. The shortest distance from the curve $xy = 4$ to the origin is

(A) 2

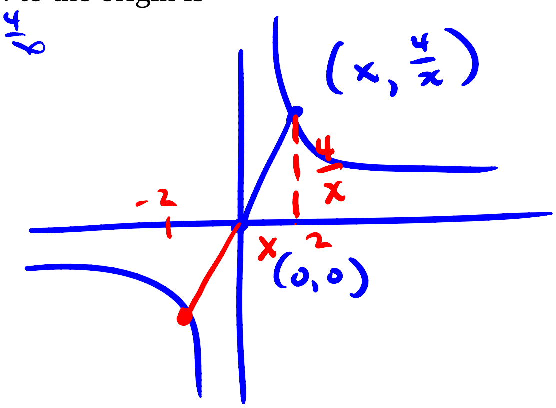
(B) 4

(C) $\sqrt{2}$

(D) $2\sqrt{2}$

(E) $\frac{1}{2}\sqrt{2}$

$y = \frac{4}{x}$



$16x^{-2}$

$d = \sqrt{\left(\frac{4}{x}\right)^2 + x^2}$

$d^2 = \frac{16}{x^2} + x^2$

$(d^2)' = -\frac{32}{x^3} + 2x$

$0 = -\frac{32}{x^3} + 2x$

$\frac{32}{x^3} = 2x$

$32 = 2x^4$

$16 = x^4$

$\pm 2 = x$

$(d^2)'$: $-$, -2 min, $+$, 0 max, $-$, 2 min, $+$

Rel min is at $x = 2$ because $(d^2)'$ changes sign from neg to pos.

$d(2) = \sqrt{8} = 2\sqrt{2}$