

10.4 (Tuesday 1/28)

Monday, January 27, 2020 8:44 AM

Precalculus Honors
10.4 Binomial Distribution

Name _____

DEFINITION Binomial Probability Distribution

Consider a simple event with these properties:

- Each trial has two possible outcomes, called *success* and *failure*.
- The probability of success on each trial is the same. (We denote the probability of success as p and the probability of failure as q . Note that $q = 1 - p$.)
- The trials are independent.

Let random variable $X =$ the number of successes in n trials. Then the probability model for X is called the **binomial distribution**, and the probability of getting k successes in the n trials is $P(X = k) = \binom{n}{k} p^k q^{n-k}$.

Examples:

1) If a six sided die is rolled 4 times, find the following probabilities:

a) P(exactly one 3)

$$4C_1 \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^3 \approx 0.376$$

$\binom{4}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3$
 ↑ success ↑ failure

b) P(exactly three 3s)

$$4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right) \approx 0.015$$

c) P(at most two 3s)

$$2(3s) \text{ or } 1(3s) \text{ or } 0(3s)$$

$$4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 + 4C_1 \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^3 + 4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4$$

d) P(at least three 3s)

$$4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right) + 4C_4 \left(\frac{1}{6}\right)^4$$

2) Suppose that when Jeff plays hockey his probability of scoring a goal when he takes a shot is $1/7$. In his most recent game, Jeff took 6 shots.

a) What is the probability that Jeff will score exactly 2 goals?

$$6C_2 \left(\frac{1}{7}\right)^2 \left(\frac{6}{7}\right)^4 = 0.165$$



b) What is the probability that he will score at least 2 goals?

$$1 - (1 \text{ goal or } 0 \text{ goals})$$

$$= 0.206$$

$$1 - (6C_1 \left(\frac{1}{7}\right) \left(\frac{6}{7}\right)^5 + 6C_0 \left(\frac{6}{7}\right)^6)$$

3) Fannie May makes boxes of assorted chocolates, 40% of which are dark chocolate on average. The production line mixes the chocolates randomly and packages 10 chocolates per box.

a) What is the probability that at least 3 chocolates in a given box are dark chocolates?



b) If 20% of the chocolates are white chocolates, what is the probability that at least one chocolate in a given box is a white chocolate?

$$1 - \text{none} \\ 1 - {}_{10}C_0 (.2)^0 (.8)^{10} \star$$

4) There are twenty questions on a test. We know that the probability you will get any individual question correct is 0.75 and the probability you will get any individual question incorrect is 0.25.

Find:

a) Probability all 20 correct.

b) Probability all 20 wrong.

c) Probability exactly 18 correct

d) Probability at least 18 correct.