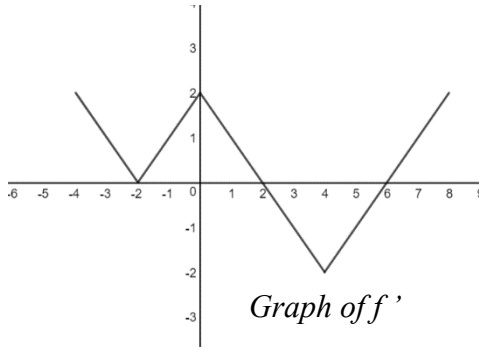


1.



a. Let  $g(x) = f(3x)$ . Calculate  $g'(-1)$ .

$$g'(x) = f'(3x) \cdot 3$$

$$g'(-1) = f'(-3) \cdot 3 = 1 \cdot 3 = 3$$

b. Let  $h(x) = \int_{-10}^{\frac{x}{2}} f'(x) dx$ . Calculate  $h'(8)$ .

$$h'(x) = f'\left(\frac{x}{2}\right) \cdot \frac{1}{2}$$

$$h'(8) = f'(4) \cdot \frac{1}{2} = -2 \cdot \frac{1}{2} = -1$$

c. Let  $m(x) = x^2 f'\left(\frac{x}{2}\right)$ . Calculate  $m'(2)$ .

$$m'(x) = x^2 \cdot f''\left(\frac{x}{2}\right) \cdot \frac{1}{2} + f'\left(\frac{x}{2}\right) \cdot 2x$$

$$m'(2) = 4 \cdot -1 \cdot \frac{1}{2} + 1 \cdot 2 \cdot 2 = 2$$

2. Given the graph of  $f'$ , write an expression for  $f(x)$  if  $f(3) = 11$ .

$$\int_3^x f'(t) dt = f(x) - f(3)$$

$$f(x) = 11 + \int_3^x f'(t) dt$$

3. Given the graph of  $f'$ , write an expression for  $f(x)$  if  $f(-2) = -5$ .

$$\int_{-2}^x f'(t) dt = f(x) - f(-2)$$

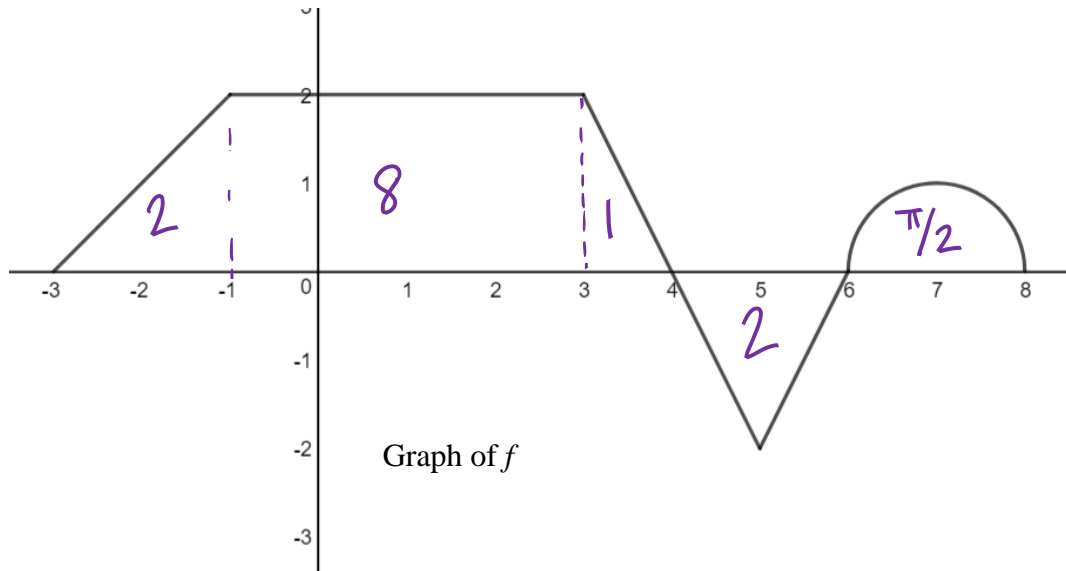
$$f(x) = -5 + \int_{-2}^x f'(t) dt$$

4. Let  $g(x)$  be the antiderivative of  $f(x)$ . If  $g(17) = 2.301$ , write an expression for  $g(x)$ .

$$\int_{17}^x f(t) dt = g(x) - g(17)$$

$$g(x) = 2.301 + \int_{17}^x f(t) dt$$

5. Let  $g$  be the antiderivative of  $f$  and let  $g(6) = 5$ .  $f$  is piecewise defined with line segments and a semicircle.



- a. For what interval(s) is  $g$  increasing? Justify your answer.

$$[-3, 4] \cup [6, 8] \text{ because } g' = f > 0$$

- b. For what interval(s) is  $g$  concave down? Justify your answer.

$$(3, 5) \cup (7, 8) \text{ because } g'' = f' < 0$$

- c. Write an expression for  $g(x)$ .

$$g(x) = 5 + \int_6^x f(t) dt$$

- d. Find  $g(-3)$ .

$$g(-3) = 5 + \int_6^{-3} f(t) dt = 5 - 9 = -4$$

- e. Find  $g(3)$ .

$$g(3) = 5 + \int_6^3 f(t) dt = 5 + 1 = 6$$

- f. Find  $g(8)$ .

$$g(8) = 5 + \int_6^8 f(t) dt = 5 + \frac{\pi}{2}$$

- g. Find the absolute maximum value of  $g$ . Justify your answer.

$x$	$g(x)$
-3	-4
4	7
8	$5 + \pi/2$

$$g(4) = 5 + \int_6^4 f(t) dt = 5 + 2 = 7$$

The max value is 7 at  $x=4$   
because  $g' = f$  changes from  $\oplus$  to  $\ominus$ .