AP Calculus AB
AP Review #2

Name:

Find $\frac{dy}{dx}$.

1. $3xy + 4x^2 + 5y^2 = 17$

$3x \cdot \frac{dy}{dx} + y \cdot 3 + 8x + 10y\frac{dy}{dx} = 0$

$(3x + 10y)\frac{dy}{dx} = -3y - 8x$

$\frac{dy}{dx} = \frac{-3y - 8x}{3x + 10y}$

2. $(3x + y)(2 - x) = 5$

$(2-x)(3 + \frac{dy}{dx}) + (3x+y)(-1) = 0$

$3(2-x) + \frac{dy}{dx}(2-x) - 3x - y = 0$

$\frac{dy}{dx}(2-x) = 3x + y - 3(2-x)$

$\frac{dy}{dx}(2-x) = 3x + y - 6 + 3x$

$\boxed{\frac{dy}{dx} = \frac{6x + y - 6}{2 - x}}$

3. $\frac{1}{2}x - 3\sqrt{y} = 14y$

$\frac{1}{2} - 3 \cdot \frac{1}{2} y^{-1/2} \frac{dy}{dx} = 14\frac{dy}{dx}$

$1 - \frac{3}{\sqrt{y}}\frac{dy}{dx} = 28\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{28 + \frac{3}{\sqrt{y}}}$

4. $(2x - y)^2 + x^3 = 8x$

$2(2x-y)(2 - \frac{dy}{dx}) + 3x^2 = 8$

$2 - \frac{dy}{dx} = \frac{8 - 3x^2}{2(2x-y)}$

$\frac{dy}{dx} = 2 - \frac{8 - 3x^2}{2(2x-y)}$

5. Find $\frac{dy}{dx}$ at the point (-1, 4) for $y = (8 - 2x^2)(x + 3y)$.

$\frac{dy}{dx} = (8 - 2x^2)(1 + 3\frac{dy}{dx}) + (x + 3y)(-4x)$

$\frac{dy}{dx} = (8 - 2)(1 + 3\frac{dy}{dx}) + (-1 + 12) \cdot 4$

$\frac{dy}{dx} = 6 + 18\frac{dy}{dx} + 44$

$\frac{dy}{dx} = \frac{50}{-17}$

6. Find $\frac{d^2y}{dx^2}$ at (2, -3) if $y\frac{dy}{dx} = 3x + y$.

$\frac{dy}{dx} = \frac{3x + y}{y}$ @ $(2,-3)$ $\frac{dy}{dx} = -1$

$\frac{d^2y}{dx^2} = \frac{y(3 + \frac{dy}{dx}) - (3x+y)\frac{dy}{dx}}{y^2}$

$= \frac{-3(3 + -1) - (6-3) \cdot -1}{9}$

$= \frac{-6 + 3}{9} = \frac{-1}{3}$

7. Find an expression for f if $\dfrac{df}{dx} = \dfrac{x+5}{2f}$ and $f(3) = -8$.

$2f\,df = (x+5)\,dx$

$f^2 = \frac{x^2}{2} + 5x + C$

$64 = \frac{9}{2} + 15 + C$

$44.5 = C$

$f^2 = \frac{x^2}{2} + 5x + 44.5$

$f = \pm\sqrt{\frac{x^2}{2} + 5x + 44.5}$

choose $(-)$

$f = -\sqrt{\frac{x^2}{2} + 5x + 44.5}$

8. Find an expression for T if $\dfrac{dT}{dx} = e^T(\sec^2 x)$ and $T(0) = -\ln 3$.

$\int \frac{dT}{e^T} = \int \sec^2 x \, dx$

$-e^{-T} = \tan x + C$

$-e^{\ln 3} = 0 + C$

$-3 = C$

$-e^{-T} = \tan x - 3$

$e^{-T} = -\tan x + 3$

$-T = \ln|3 - \tan x|$

$T = -\ln|3 - \tan x|$

9. Find an expression for J if $\dfrac{dJ}{dx} = (4x^2+5)(10-J)$ and $J(0) = 11$.

$\int \frac{dJ}{10-J} = \int 4x^2 + 5$

$-\ln|10-J| = \frac{4x^3}{3} + 5x + C$

$-\ln|10 - -3| = 0 + C \qquad C = -\ln 13$

$\ln|10-J| = -\frac{4}{3}x^3 - 5x + \ln 13$

$10 - J = e^{-4/3x^3 - 5x + \ln 13}$

$J = 10 - 13e^{-4/3x^3 - 5x}$

10. Find an expression for H if $\dfrac{dH}{dx} = -3x(H-5)^{\frac{1}{5}}$ and $H(1) = 37$.

$\int \frac{dH}{(H-5)^{1/5}} = \int -3x\,dx$

$\frac{5}{4}(H-5)^{4/5} = -\frac{3x^2}{2} + C$

$\frac{5}{4}(37-5)^{4/5} = \frac{-3}{2} + C \qquad C = 21.5$

$5(H-5)^{4/5} = -6x^2 + 86$

$H = \left(\frac{-6x^2+86}{5}\right)^{5/4} + 5$