AP Calculus AB Name Key

7.2 Practice: Integrals and Anti-Derivatives

Watch out!! U-substitution will work on some of these. Others just need to be simplified/rearranged with basic algebra, and then they will have an easy anti-derivative.

1. $\int \frac{1}{x^6}\,dx = \int x^{-6}\,dx$

$\boxed{= \frac{-x^{-5}}{5} + C}$

2. $\int x^{\frac{4}{7}}\,dx$

$= \boxed{\frac{7}{11} x^{11/7} + C}$

3. $\int r(r+3)^2\,dr$

$\int r(r^2+6r+9)\,dr$

$\int (r^3+6r^2+9r)\,dr$

$\boxed{= \frac{r^4}{4} + 2r^3 + \frac{9}{2}r^2 + C}$

4. $\int (1-t)^2\,dt$

$u = 1-t \quad du = -1dt$

$-du = dt$

$-\int u^2\,dt$

$-\frac{u^3}{3} + C = \boxed{-\frac{(1-t)^3}{3} + C}$

5. $\int \frac{x^5+3x+4}{x}\,dx$

$\int \left(x^4 + 3 + \frac{4}{x}\right)dx$

$\boxed{= \frac{x^5}{5} + 3x + 4\ln|x| + C}$

6. $\int \frac{5}{\sqrt[3]{x}}\,dx = \int 5x^{-1/3}$

$\boxed{= \frac{15}{2} x^{2/3} + C}$

7. $\int \tan t \sec^2 t\,dt$

$u = \tan t$

$du = \sec^2 t\,dt$

$\int u\,du$

$= \frac{u^2}{2} + C = \frac{\tan^2 t}{2} + C$

8. $\int 5x^4\left(2x^5+70\right)^{10}\,dx$

$u = 2x^5 + 70$

$du = 10x^4\,dx$

$\frac{1}{2}du = 5x^4\,dx$

$\frac{1}{2}\int u^{10}\,du$

$\frac{1}{2} \cdot \frac{u^{11}}{11} + C$

$\boxed{= \frac{(2x^5+70)^{11}}{22} + C}$

9. $\int \frac{\sqrt{\ln x}}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u^{1/2}\,du$

$\frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{3}\ln^{3/2}x + C}$

10. $\int \sin x \cdot e^{\cos x}\,dx$

$u = \cos x \qquad du = -\sin x\,dx$

$-\int e^{u}\,du = \boxed{-e^{\cos x} + C}$

11. $\int \frac{5}{\sqrt{5t}}\,dt$

$u = 5t \qquad du = 5\,dt$

$\int 5(5t)^{-1/2}\,dt$

$\int u^{-1/2}\,du = 2u^{1/2} + C$

$\boxed{2\sqrt{5t} + C}$

12. $\int \frac{x^3+5}{x^2}\,dx$

$\int \left(x + 5x^{-2}\right)dx$

$= \frac{x^2}{2} + -5x^{-1} + C$

$\boxed{= \frac{x^2}{2} - \frac{5}{x} + C}$

Evaluate each integral.

13. $\int_0^{\pi} (1+\cos 5t)^2 \sin 5t\,dt$

$u = 1 + \cos 5t$

$du = -\sin 5t \cdot 5\,dt$

$du = -5\sin 5t\,dt$

$-\frac{1}{5}\,du = \sin 5t\,dt$

$u = 1 + \cos 5\pi$

$u = 0$

$u(0) = 1 + \cos 0$

$= 2$

$-\frac{1}{5}\int_2^0 u^2\,du$

$-\frac{1}{5} \cdot \frac{u^3}{3}\Big|_2^0$

$-\frac{u^3}{15}\Big|_2^0$

$0 + \frac{8}{15} = 8/15$

14. $\int_0^1 \left(8x^2 - x + 1\right)^{-1/3}(48x-3)\,dx$

$u = 8x^2 - x + 1$

$du = (16x - 1)\,dx$

$3\,du = 48x - 3\,dx$

$u(1) = 8$

$u(0) = 1$

$3\int_1^8 u^{-1/3}\,du$

$3 \cdot \frac{3}{2}u^{2/3}\Big|_1^8$

$= \frac{9}{2}(8)^{2/3} - \frac{9}{2}(1)^{2/3}$

$= \frac{9}{2}\left(2^3\right)^{2/3} - \frac{9}{2} = \frac{36}{2} - \frac{9}{2}$

$= \frac{27}{2}$