Rectangular Approximation Method Practice Name(s)____

When we estimate distances from velocity data it is sometimes necessary to use times $t_0, t_1, t_2, t_3, \ldots$ that are not equally spaced. We can still estimate distances using the time periods $\Delta t_i = t_i - t_{i-1}$. For example, on May 7, 1992, the space shuttle *Endeavour* was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters.

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

Use these data to estimate the height above Earth's surface of the space shuttle *Endeavour*, $\boxed{\text{62 seconds}}$ after liftoff.

Using LRAM

$10 \cdot 0 + 5 \cdot 185 + 5 \cdot 319 + 12 \cdot 447 + 27 \cdot 742 + 3 \cdot 1325 = 31{,}893$ ft

Using RRAM

$10 \cdot 185 + 5 \cdot 319 + 5 \cdot 447 + 12 \cdot 742 + 27 \cdot 1325 + 3 \cdot 1445 = 54{,}694$ ft

The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

.5 .5 .5 .5 .5 .5

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

Upper estimate (RRAM)

$.5(6.2) + .5(10.8) + .5(14.9) + .5(18.1) + .5(19.4) + .5(20.2) = 44.8$ ft

Lower estimate (LRRAM)

$.5(0) + .5(6.2) + .5(10.8) + .5(14.9) + .5(18.1) + .5(19.4) = 34.7$ ft

From the given data, estimate the area between the curves for $0 \le x \le 2$.

x	0.0	0.2	0.4	0.6	0.8	1.0
$f(x)$	3.2	3.6	3.8	3.7	3.2	3.4
$g(x)$	1.2	1.5	1.6	2.2	2.0	2.4

x	1.2	1.4	1.6	1.8	2.0
$f(x)$	3.0	2.8	2.4	2.9	3.4
$g(x)$	2.2	2.1	2.3	2.8	2.4

$0.2\,(f(x) - g(x))$

LRAM

$.2(2) + .2(2.1) + .2(2.2) + \ldots + .2(0.1) = 2.34$

RRAM

$.2(2.1) + .2(2.2) + .2(1.5) + \ldots + .2(1) = 2.50$

The cross-sectional areas of an underwater object are given. Estimate the volume.

$V = x \cdot A(x)$

x	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2
$A(x)$	0.4	1.4	1.8	2.0	2.1	1.8	1.1	0.4	0

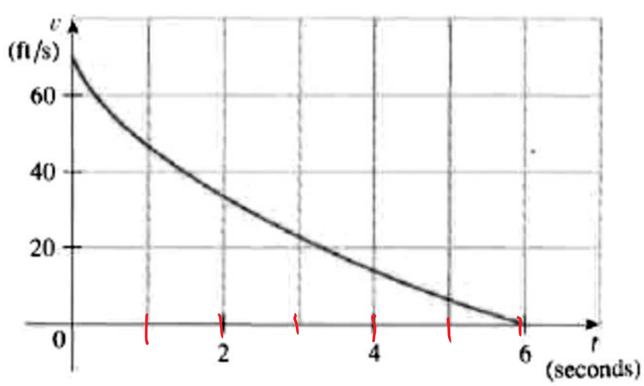
Using LRAM

$V \approx 0.4(.4) + 0.4(1.4) + 0.4(1.8) + \dots + 0.4(0.4) = 4.4$

Using RRAM

$V \approx 0.4(1.4) + 0.4(1.8) + 0.4(2.0) + \dots + 0.4(0) = 4.24$

The velocity graph of a braking car is shown. Use it to estimate the distance traveled by the car while the brakes are applied.



$MRAM_6$:

$1(55 + 40 + 30 + 18 + 10 + 5) = 158$ ft

$LRAM_6$:

$1(70 + 48 + 35 + 22 + 15 + 8) = 198$ ft

$RRAM_6$:

$1(48 + 35 + 22 + 15 + 8 + 0) = 128$ ft