pg 673: 2, 4, 7, 28, 36

$\frac{f^n(x)\, x^n}{n!}$

2. $f(x) = e^{-4x}$

☆ Recall $f(x) = e^x$

$\approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$f(0) = 1$
$f'(0) = 1$
$f''(0) = 1$

$f' = -4e^{-4x}$
$f'' = -4 \cdot -4 e^{-4x}$
$f''' = -4 \cdot -4 \cdot -4 e^{-4x}$
$f^{(n)} = (-4)^n e^{-4x}$

however we can think of substituten

$f(u) = e^u = \sum_{n=0}^{\infty} \frac{u^n}{n!}$ $\therefore$ $\sum_{n=0}^{\infty} \frac{(-4x)^n}{n!}$

4. $f(x) = \sin x$ $c = \pi/4$

Think about
$f(x) = \sin x$ centered $x = 0$

$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$
$f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$
$f''(\pi/4) = -\frac{\sqrt{2}}{2}$
$f'''(\pi/4) - \sqrt{2}/2$

$$\frac{\sqrt{2}}{2} + \frac{\frac{\sqrt{2}}{2}(x - \pi/4)}{1!} - \frac{\frac{\sqrt{2}}{2}(x - \pi/4)^2}{2!} - \frac{\frac{\sqrt{2}}{2}(x - \pi/4)^3}{3!} + \frac{\frac{\sqrt{2}}{2}(x - \pi/4)^4}{4!} + \frac{\frac{\sqrt{2}}{2}(x - \pi/4)}{5!} + \ldots$$

$$\underbrace{\frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} \frac{(-1)^n (x - \pi/4)^{2n}}{(2n)!}}_{\text{even degree \& degree of zero}} + \underbrace{\frac{\sqrt{2}}{2} \sum_{n=1} \frac{(-1)^{n-1} (x - \pi/4)^{2n-1}}{(2n-1)!}}_{\text{odd degree}}$$

7. $f(x) = \ln x$ $c = 1$ $f(1) = \ln 1 = 0$

$f' = \frac{1}{x}$ $f'' = -\frac{1}{x^2}$ $f''' = \frac{2}{x^3}$ $f^{(4)} = \frac{-6}{x^4}$ $f^{(5)} = \frac{24}{x^5}$

$$P_5 = 0 + 1(x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!} - \frac{6(x-1)^4}{4!} + \frac{24(x-1)^5}{5!}$$

$$= (x+1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5} + \ldots$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$$

28. $g(x) = e^{-3x}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots + \frac{x^n}{n!} + \ldots$

$$\sum_{n=0}^{\infty} \frac{(-3x)^n}{n!}$$

$$\sum_{n=0} \frac{(-2x)}{n!}$$

36. $g(x) = 2\sin x^3$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$2\sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n+1}}{(2n+1)!} = 2\sum_{n=0}^{\infty} \frac{(-1) x^{6n+3}}{(2n+1)!}$$