Quest 9.7-9.10 Review HW Solutions

Tuesday, March 19, 2019 9:56 AM

pg 677: 69, 72, 73, 75, 77, ~~78, 85, 88, 96, 100, 105~~; AP Review: 5, 8, 9

(69) $f(x) = e^{-2x}$, $n=3$ (use e^x)

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$$\boxed{e^{-2x} = 1 - 2x + \frac{4x^2}{2!} - \frac{8x^3}{3!}}$$

(72) $f(x) = \tan x \qquad c = -\frac{\pi}{4}$

$f(-\frac{\pi}{4}) = -1$

$f'(x) = \sec^2 x \qquad f'(-\frac{\pi}{4}) = (\sqrt{2})^2 = 2$

$f''(x) = 2\sec x \cdot \sec x \tan x \qquad f''(-\frac{\pi}{4}) = 2(\sqrt{2})^2 \cdot (-1) = -4$

$f'''(x) = 2\sec^2 x \cdot \sec^2 x + 2\tan x \cdot 2\sec^2 x \tan x$

$f'''(-\frac{\pi}{4}) = 2(\sqrt{2})^4 + 4(\sqrt{2})^2 \cdot (-1)^2$

$= 8 + 8 = 16$

$$\boxed{P_3(x) = -1 + 2(x+\tfrac{\pi}{4}) - \frac{4(x+\frac{\pi}{4})^2}{2!} + \frac{16(x+\frac{\pi}{4})^2}{3!}}$$

(73) $\cos(0.75)$

$[0, 0.75]$ max of 1

$|R_n(0.75)| \le \left| \frac{f^{(n+1)}(z) \cdot (0.75)^{n+1}}{(n+1)!} \right|$

$= \frac{(0.75)^{n+1}}{(n+1)!} < 0.001$ ← Graph to solve

$$\boxed{n = 5}$$

(75) $\sum_{n=0}^{\infty} \left(\frac{x}{10}\right)^n$ ← Geometric

$\left|\frac{x}{10}\right| < 1$

$-1 < \frac{x}{10} < 1$

$-10 < x < 10$

Conv. $(-10, 10)$
Conv. abs. $(-10, 10)$

(77) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{(n+1)^2}$ ← not geometric

$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} \cdot (x-2)^{n+1}}{(n+2)^2} \cdot \frac{(n+1)^2}{(-1)^n \cdot (x-2)^n} \right| = \lim_{n\to\infty} \left| \frac{-(x-2)(n+1)^2}{(n+2)^2} \right|$ (the $\frac{(n+1)^2}{(n+2)^2}$ → 1)

$|-(x-2)| < 1$

$-1 < x-2 < 1$

$1 < x < 3$ ← conv. abs

$x = 3$: $\sum \frac{(-1)^n \cdot 1^n}{(n+1)^2} = \sum \frac{(-1)^n}{(n+1)^2}$

$\sum \left| \frac{(-1)^n}{(n+1)^2} \right| = \sum \frac{1}{(n+1)^2} \le \sum \frac{1}{n^2}$ ← conv. by p-series test

$\sum \frac{1}{(n+1)^2}$ conv. by direct comp test

→ Conv. abs. by abs. conv. theorem.

$x = 1$: $\sum \frac{(-1)^n \cdot (-1)^n}{(n+1)^2} = \sum \frac{1}{(n+1)^2}$ (conv. by direct comp test)

Conv.: $[1, 3]$
Conv. abs.: $[1, 3]$
Conv. cond: NEVER

∞

Conv. cond: NEVER

(78) $\sum_{n=1}^{\infty} \frac{3^n (x-2)^n}{n}$

$\lim_{n\to\infty} \left| \frac{3^{n+1}(x-2)^{n+1}}{n+1} \cdot \frac{n}{3^n (x-2)^n} \right| = \lim_{n\to\infty} \left| \frac{3n(x-2)}{n+1} \right|$ → 3

$|3(x-2)| < 1$

$-1 < 3(x-2) < 1$

$-\frac{1}{3} < x-2 < \frac{1}{3}$

$\frac{5}{3} < x < \frac{7}{3}$ ← conv. abs.

$x = \frac{7}{3}$: $\sum \frac{3^n \cdot (\frac{1}{3})^n}{n} = \sum \frac{1}{n}$ ← Div. by p-series test.

$x = \frac{5}{3}$: $\sum \frac{3^n \cdot (-\frac{1}{3})^n}{n} = \sum \frac{(-1)^n}{n}$

↑ conv. cond

↑ conv. by alt. series test.

(1) $\frac{1}{n} \geq 0$ ✓
(2) $\frac{1}{n} \geq \frac{1}{n+1}$ ✓
(3) $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\sum \left| \frac{(-1)^n}{n} \right| = \sum \frac{1}{n}$

Converges: $[5/3, 7/3)$
Conv. abs: $(5/3, 7/3)$
Conv. cond: $x = 5/3$

(86) $g(x) = \frac{2}{3-x}$, $c = 0$

$\frac{2 \cdot \frac{1}{3}}{(3-x)\frac{1}{3}} = \frac{\frac{2}{3}}{1 - \frac{1}{3}x}$ $a = \frac{2}{3}$, $r = \frac{1}{3}x$

$\frac{2}{3} + \frac{2}{9}x + \frac{2}{27}x^2 + \ldots = \sum_{n=0}^{\infty} \frac{2}{3}\left(\frac{1}{3}x\right)^n$

$\left| \frac{x}{3} \right| < 1$

$-1 < \frac{x}{3} < 1$ $-3 < x < 3$

(88) $f(x) = \frac{1}{3-2x}$, $c = 0$

$\frac{1 \cdot \frac{1}{3}}{(3-2x)\frac{1}{3}} = \frac{\frac{1}{3}}{1 - \frac{2x}{3}}$ $a = \frac{1}{3}$, $r = \frac{2x}{3}$

$\frac{1}{3} + \frac{2x}{9} + \frac{4x^2}{27} + \ldots = \sum_{n=0}^{\infty} \frac{1}{3}\left(\frac{2x}{3}\right)^n$

$\left| \frac{2x}{3} \right| < 1$

$-3 < 2x < 3$

$-\frac{3}{2} < x < \frac{3}{2}$

make sure you can also do these if $c \neq 0$

(100) $f(x) = \sqrt{x}$, $c = 4$

$f(4) = 2$

$f'(x) = \frac{1}{2}x^{-1/2}$ $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$f''(x) = -\frac{1}{4}x^{-3/2}$ $f''(4) = \frac{-1}{4(2^2)^{3/2}} = \frac{-1}{4 \cdot 8} = \frac{-1}{32}$

$f'''(x) = \frac{3}{8}x^{-5/2}$ $f'''(4) = \frac{3}{8 \cdot 4^{5/2}} = \frac{3}{8 \cdot 32}$

$f^{(4)}(x) = \frac{-15}{16}x^{-7/2}$ $f^{(4)}(4) = \frac{-15}{16(4)^{7/2}} = \frac{-15}{16 \cdot 128}$ [illegible]

∞ $(-1)^{n+1}$ [illegible] $(2n-1)(x-4)^n$

$f'''(x) = \frac{3}{8}x^{-5/2}$ $\quad f'''(4) = \frac{3}{8\cdot 4^{5/2}} = \frac{3}{8\cdot 32}$

$f^{(4)}(x) = \frac{-15}{16}x^{-7/2}$ $\quad f^{(4)}(4) = \frac{-15}{16(4)^{7/2}} = \frac{-15}{16\cdot 128}$ ($2^4\cdot 2^7 = 2^{11}$)

$$\sqrt{x} = 2 + \frac{(x-4)}{4\cdot 1!} - \frac{1\cdot 1(x-4)^2}{32\cdot 2!} + \frac{3(x-4)^3}{8\cdot 32\cdot 3!} - \frac{15(x-4)^4}{16\cdot 128\cdot 4!} + \dots = 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+1)(2n-1)(x-4)^n}{2^{3n-1}\cdot n!}$$

(2^2, 2^5, 2^8, 2^{11}; $1\cdot 3$, $3\cdot 5$)

(105) $f(x) = e^{6x}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{6x} = 1 + 6x + \frac{36x^2}{2!} + \dots = \sum_{n=0}^{\infty} \frac{(6x)^n}{n!}$$

(5) $f(x) = xe^{-x}$

a) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$

$xe^{-x} = x - x^2 + \frac{x^3}{2!} - \frac{x^4}{3!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{n+1}}{n!}$

b) $\lim_{x\to 0}\left(\frac{f(x) - x + x^2}{x^3}\right)$ $\to \frac{0}{0}$

$\lim_{x\to 0} \frac{f'(x) - 1 + 2x}{3x^2}$ $\to \frac{0}{0}$ $\quad f'(x) = -xe^{-x} + e^{-x}$, $f'(0) = 1$

Seriously...

$\lim_{x\to 0} \frac{f''(x) + 2}{6x}$ $\to \frac{0}{0}$ $\quad f''(x) = xe^{-x} - e^{-x} - e^{-x}$, $f''(0) = -2$

$\lim_{x\to 0} \frac{f'''(x)}{6}$ $\quad f'''(x) = -xe^{-x} + e^{-x} + 2e^{-x}$, $f'''(0) = 3$

$= \frac{3}{6} = \boxed{\frac{1}{2}}$ Yay!

$f(0) = 0$ so $C = 0$

c) $\int xe^{-x}dx = \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4\cdot 2!} - \frac{x^5}{5\cdot 3!} + \dots \quad \sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n\cdot(n-2)!}$

$g(\frac{1}{3}) \approx \frac{(\frac{1}{3})^2}{2} - \frac{(\frac{1}{3})^3}{3} + \frac{(\frac{1}{3})^4}{8}$ $\quad$ 1st neglected term

d) $\left|R_4(\frac{1}{5})\right| \le \left|\frac{-(\frac{1}{5})^5}{5\cdot 3!}\right| = \frac{1}{5^5\cdot 5\cdot 3!}$

$= \frac{1}{25\cdot 25\cdot 25\cdot 6}$

$= \frac{1}{625\cdot 150}$

$= \frac{1}{93750} < \frac{1}{90000}$ ✓

625×15: $3125 + 6250 = 9375$

(8) $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

$\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!}$

$x^2\cos(x^2) = x^2 - \frac{x^6}{2!} + \frac{x^{10}}{4!} - \frac{x^{14}}{6!} + \dots$ $\quad$ (C)

(9) $e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots$

$x = \ln$

$e^{\ln 3} = 1 + \ln 3 + \frac{(\ln 3)^2}{2!} + \dots + \frac{(\ln 3)^n}{n!} + \dots$

3 $\quad$ (C)