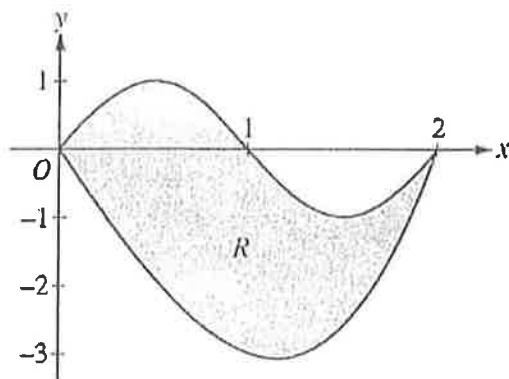


AP[®] CALCULUS AB
2008 SCORING GUIDELINES

Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

- Find the area of R .
- The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.
- The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.
- The region R models the surface of a small pond. At all points in R at a distance x from the y -axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$

$$\text{Area} = \int_0^2 (\sin(\pi x) - (x^3 - 4x)) dx = 4$$

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$

$$\text{The area of the stated region is } \int_r^s (-2 - (x^3 - 4x)) dx$$

$$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \end{cases}$$

(c) $\text{Volume} = \int_0^2 (\sin(\pi x) - (x^3 - 4x))^2 dx = 9.978$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(d) $\text{Volume} = \int_0^2 (3 - x)(\sin(\pi x) - (x^3 - 4x)) dx = 8.369$ or 8.370

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$