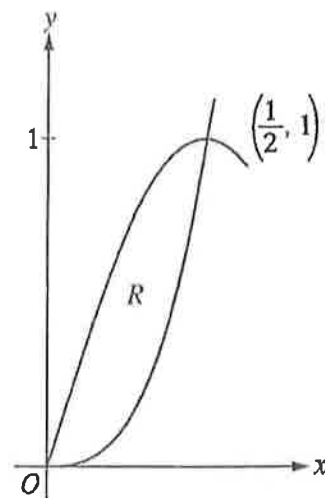


AP[®] CALCULUS AB
2011 SCORING GUIDELINES

Question 3

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.



- (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.
- (b) Find the area of R .
- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

(a) $f\left(\frac{1}{2}\right) = 1$

$$f'(x) = 24x^2, \text{ so } f'\left(\frac{1}{2}\right) = 6$$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

(b) Area = $\int_0^{1/2} (g(x) - f(x)) dx$
 $= \int_0^{1/2} (\sin(\pi x) - 8x^3) dx$
 $= \left[-\frac{1}{\pi} \cos(\pi x) - 2x^4 \right]_{x=0}^{x=1/2}$
 $= -\frac{1}{8} + \frac{1}{\pi}$

(c) $\pi \int_0^{1/2} ((1 - f(x))^2 - (1 - g(x))^2) dx$
 $= \pi \int_0^{1/2} ((1 - 8x^3)^2 - (1 - \sin(\pi x))^2) dx$

2: $\begin{cases} 1: f'\left(\frac{1}{2}\right) \\ 1: \text{answer} \end{cases}$

4: $\begin{cases} 1: \text{integrand} \\ 2: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

3: $\begin{cases} 1: \text{limits and constant} \\ 2: \text{integrand} \end{cases}$