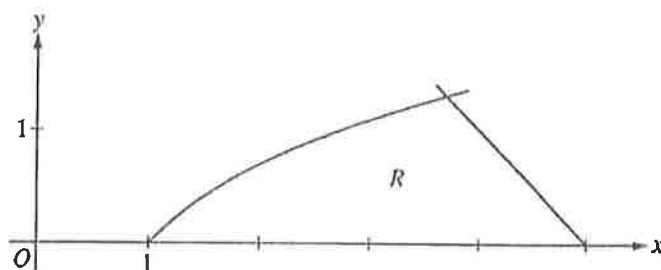


AP[®] CALCULUS AB
2012 SCORING GUIDELINES

Question 2

Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



- (a) Find the area of R .
- (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
- (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .

$$\ln x = 5 - x \Rightarrow x = 3.69344$$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

$$\begin{aligned} \text{(a) Area} &= \int_0^B (5 - y - e^y) dy \\ &= 2.986 \text{ (or } 2.985) \end{aligned}$$

OR

$$\begin{aligned} \text{Area} &= \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx \\ &= 2.986 \text{ (or } 2.985) \end{aligned}$$

$$\text{(b) Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$$

$$\text{(c) } \int_0^k (5 - y - e^y) dy = \frac{1}{2} \cdot 2.986 \text{ (or } \frac{1}{2} \cdot 2.985)$$

3 : $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{array} \right.$

3 : $\left\{ \begin{array}{l} 2 : \text{integrands} \\ 1 : \text{expression for total volume} \end{array} \right.$

3 : $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{equation} \end{array} \right.$