

**AP<sup>®</sup> CALCULUS AB  
2008 SCORING GUIDELINES**

**Question 5**

Consider the differential equation  $\frac{dy}{dx} = \frac{y-1}{x^2}$ , where  $x \neq 0$ .

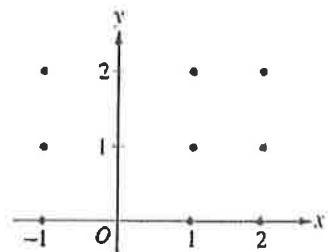
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

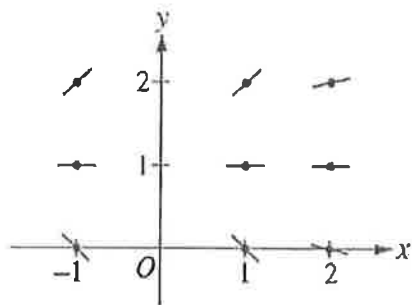
(b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(2) = 0$ .

(c) For the particular solution  $y = f(x)$  described in part (b), find

$$\lim_{x \rightarrow \infty} f(x).$$



(a)



2 :  $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b)  $\frac{1}{y-1} dy = \frac{1}{x^2} dx$

$$\ln|y-1| = -\frac{1}{x} + C$$

$$|y-1| = e^{-\frac{1}{x} + C}$$

$$|y-1| = e^C e^{-\frac{1}{x}}$$

$$y-1 = ke^{-\frac{1}{x}}, \text{ where } k = \pm e^C$$

$$-1 = ke^{-\frac{1}{2}}$$

$$k = -e^{\frac{1}{2}}$$

$$f(x) = 1 - e^{\left(\frac{1}{2} - \frac{1}{x}\right)}, x > 0$$

6 :  $\begin{cases} 1 : \text{separates variables} \\ 2 : \text{antidifferentiates} \\ 1 : \text{includes constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(c)  $\lim_{x \rightarrow \infty} 1 - e^{\left(\frac{1}{2} - \frac{1}{x}\right)} = 1 - \sqrt{e}$

1 : limit