

**AP<sup>®</sup> CALCULUS AB**  
**2012 SCORING GUIDELINES**

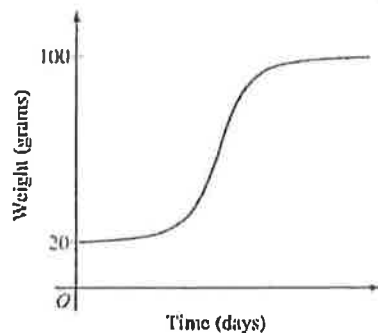
**Question 5**

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time  $t = 0$ , when the bird is first weighed, its weight is 20 grams. If  $B(t)$  is the weight of the bird, in grams, at time  $t$  days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let  $y = B(t)$  be the solution to the differential equation above with initial condition  $B(0) = 20$ .

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.
- (b) Find  $\frac{d^2B}{dt^2}$  in terms of  $B$ . Use  $\frac{d^2B}{dt^2}$  to explain why the graph of  $B$  cannot resemble the following graph.
- (c) Use separation of variables to find  $y = B(t)$ , the particular solution to the differential equation with initial condition  $B(0) = 20$ .



(a)  $\left. \frac{dB}{dt} \right|_{B=40} = \frac{1}{5}(60) = 12$

$$\left. \frac{dB}{dt} \right|_{B=70} = \frac{1}{5}(30) = 6$$

Because  $\left. \frac{dB}{dt} \right|_{B=40} > \left. \frac{dB}{dt} \right|_{B=70}$ , the bird is gaining weight faster when it weighs 40 grams.

(b)  $\frac{d^2B}{dt^2} = -\frac{1}{5} \frac{dB}{dt} = -\frac{1}{5} \cdot \frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of  $B$  is concave down for  $20 \leq B < 100$ . A portion of the given graph is concave up.

(c)  $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$$\int \frac{1}{100 - B} dB = \int \frac{1}{5} dt$$

$$-\ln|100 - B| = \frac{1}{5}t + C$$

Because  $20 \leq B < 100$ ,  $|100 - B| = 100 - B$ .

$$-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$$

$$100 - B = 80e^{-t/5}$$

$$B(t) = 100 - 80e^{-t/5}, \quad t \geq 0$$

2 :  $\left\{ \begin{array}{l} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{array} \right.$

5 :  $\left\{ \begin{array}{l} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{array} \right.$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables