

**AP[®] CALCULUS AB
2013 SCORING GUIDELINES**

Question 6

Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

- (a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.
- (b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) $\left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = e^0(3 \cdot 1^2 - 6 \cdot 1) = -3$

An equation for the tangent line is $y = -3(x - 1)$.

$$f(1.2) \approx -3(1.2 - 1) = -0.6$$

(b) $\frac{dy}{e^y} = (3x^2 - 6x) dx$

$$\int \frac{dy}{e^y} = \int (3x^2 - 6x) dx$$

$$-e^{-y} = x^3 - 3x^2 + C$$

$$-e^{-0} = 1^3 - 3 \cdot 1^2 + C \Rightarrow C = 1$$

$$-e^{-y} = x^3 - 3x^2 + 1$$

$$e^{-y} = -x^3 + 3x^2 - 1$$

$$-y = \ln(-x^3 + 3x^2 - 1)$$

$$y = -\ln(-x^3 + 3x^2 - 1)$$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$$3 : \begin{cases} 1 : \frac{dy}{dx} \text{ at the point } (x, y) = (1, 0) \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$$

$$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables