

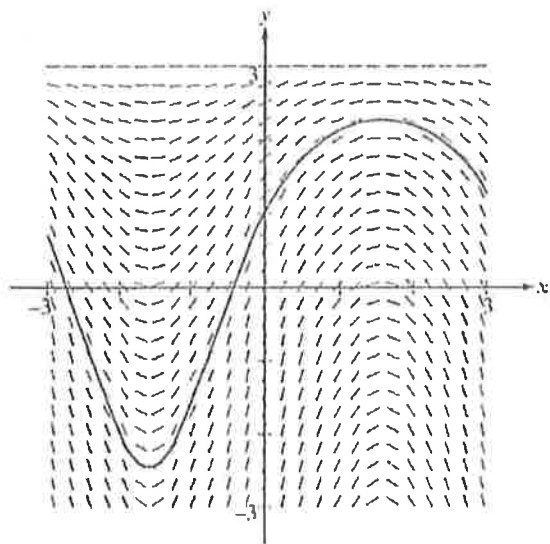
**AP<sup>®</sup> CALCULUS AB**  
**2014 SCORING GUIDELINES**

**Question 6**

Consider the differential equation  $\frac{dy}{dx} = (3 - y)\cos x$ . Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(0) = 1$ . The function  $f$  is defined for all real numbers.

- (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point  $(0, 1)$ .
- (b) Write an equation for the line tangent to the solution curve in part (a) at the point  $(0, 1)$ . Use the equation to approximate  $f(0.2)$ .
- (c) Find  $y = f(x)$ , the particular solution to the differential equation with the initial condition  $f(0) = 1$ .

(a)



- (b)  $\left. \frac{dy}{dx} \right|_{(x,y)=(0,1)} = 2 \cos 0 = 2$   
An equation for the tangent line is  $y = 2x + 1$ .  
 $f(0.2) \approx 2(0.2) + 1 = 1.4$

- (c)  $\frac{dy}{dx} = (3 - y)\cos x$   
 $\int \frac{dy}{3 - y} = \int \cos x \, dx$   
 $-\ln|3 - y| = \sin x + C$   
 $-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$   
 $-\ln|3 - y| = \sin x - \ln 2$   
Because  $y(0) = 1$ ,  $y < 3$ , so  $|3 - y| = 3 - y$   
 $3 - y = 2e^{-\sin x}$   
 $y = 3 - 2e^{-\sin x}$   
Note: this solution is valid for all real numbers.

1 : solution curve

2 :  $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

6 :  $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables