

AP[®] CALCULUS AB
2007 SCORING GUIDELINES

Question 3

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 1 | 6 | 4 | 2 | 5 |
| 2 | 9 | 2 | 3 | 1 |
| 3 | 10 | -4 | 4 | 2 |
| 4 | -1 | 3 | 6 | 7 |

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.

- (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
- (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.
- (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.
- (d) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$
 $h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$
 Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r , $1 < r < 3$, such that $h(r) = -5$.

(b) $\frac{h(3) - h(1)}{3 - 1} = \frac{-7 - 3}{3 - 1} = -5$
 Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c , $1 < c < 3$, such that $h'(c) = -5$.

(c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

(d) $g(1) = 2$, so $g^{-1}(2) = 1$.
 $(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5}$
 An equation of the tangent line is $y - 1 = \frac{1}{5}(x - 2)$.

2: $\begin{cases} 1 : h(1) \text{ and } h(3) \\ 1 : \text{conclusion, using IVT} \end{cases}$

2: $\begin{cases} 1 : \frac{h(3) - h(1)}{3 - 1} \\ 1 : \text{conclusion, using MVT} \end{cases}$

2: $\begin{cases} 1 : \text{apply chain rule} \\ 1 : \text{answer} \end{cases}$

3: $\begin{cases} 1 : g^{-1}(2) \\ 1 : (g^{-1})'(2) \\ 1 : \text{tangent line equation} \end{cases}$