

AP[®] CALCULUS AB
2009 SCORING GUIDELINES

Question 5

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.

- (a) Estimate $f'(4)$. Show the work that leads to your answer.
- (b) Evaluate $\int_2^{13} (3 - 5f''(x)) dx$. Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.
- (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.

(a) $f'(4) \approx \frac{f(5) - f(3)}{5 - 3} = -3$

(b)
$$\int_2^{13} (3 - 5f''(x)) dx = \int_2^{13} 3 dx - 5 \int_2^{13} f''(x) dx$$

$$= 3(13 - 2) - 5(f(13) - f(2)) = 8$$

(c)
$$\int_2^{13} f(x) dx \approx f(2)(3 - 2) + f(3)(5 - 3)$$

$$+ f(5)(8 - 5) + f(8)(13 - 8) = 18$$

- (d) An equation for the tangent line is $y = -2 + 3(x - 5)$.
 Since $f''(x) < 0$ for all x in the interval $5 \leq x \leq 8$, the line tangent to the graph of $y = f(x)$ at $x = 5$ lies above the graph for all x in the interval $5 < x \leq 8$.
 Therefore, $f(7) \leq -2 + 3 \cdot 2 = 4$.

An equation for the secant line is $y = -2 + \frac{5}{3}(x - 5)$.

Since $f''(x) < 0$ for all x in the interval $5 \leq x \leq 8$, the secant line connecting $(5, f(5))$ and $(8, f(8))$ lies below the graph of $y = f(x)$ for all x in the interval $5 < x < 8$.

Therefore, $f(7) \geq -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$.

1 : answer

2 : $\left\{ \begin{array}{l} 1 : \text{uses Fundamental Theorem} \\ \quad \text{of Calculus} \\ 1 : \text{answer} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : \text{left Riemann sum} \\ 1 : \text{answer} \end{array} \right.$

4 : $\left\{ \begin{array}{l} 1 : \text{tangent line} \\ 1 : \text{shows } f(7) \leq 4 \\ 1 : \text{secant line} \\ 1 : \text{shows } f(7) \geq \frac{4}{3} \end{array} \right.$