

**AP<sup>®</sup> CALCULUS AB**  
**2009 SCORING GUIDELINES**

**Question 3**

Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is  $x$  meters from the beginning of the cable is  $6\sqrt{x}$  dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

- (a) Find Mighty's profit on the sale of a 25-meter cable.
- (b) Using correct units, explain the meaning of  $\int_{25}^{30} 6\sqrt{x} \, dx$  in the context of this problem.
- (c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is  $k$  meters long.
- (d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

(a) Profit =  $120 \cdot 25 - \int_0^{25} 6\sqrt{x} \, dx = 2500$  dollars

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b)  $\int_{25}^{30} 6\sqrt{x} \, dx$  is the difference in cost, in dollars, of producing a cable of length 30 meters and a cable of length 25 meters.

1 : answer with units

(c) Profit =  $120k - \int_0^k 6\sqrt{x} \, dx$  dollars

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{expression} \end{cases}$

(d) Let  $P(k)$  be the profit for a cable of length  $k$ .

$$P'(k) = 120 - 6\sqrt{k} = 0 \text{ when } k = 400.$$

This is the only critical point for  $P$ , and  $P'$  changes from positive to negative at  $k = 400$ .

Therefore, the maximum profit is  $P(400) = 16,000$  dollars.

4 :  $\begin{cases} 1 : P'(k) = 0 \\ 1 : k = 400 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$