

AP[®] CALCULUS AB
2012 SCORING GUIDELINES

Question 6

For $0 \leq t \leq 12$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$$v(t) = \cos\left(\frac{\pi}{6}t\right). \text{ The particle is at position } x = -2 \text{ at time } t = 0.$$

- (a) For $0 \leq t \leq 12$, when is the particle moving to the left?
- (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.
- (c) Find the acceleration of the particle at time t . Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.
- (d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3, 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

2 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

(b) $\int_0^6 |v(t)| dt$

1 : answer

(c) $a(t) = -\frac{\pi}{6} \sin\left(\frac{\pi}{6}t\right)$

$$a(4) = -\frac{\pi}{6} \sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$$

$$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

3 : $\begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

(d) $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt$

$$= -2 + \left[\frac{6}{\pi} \sin\left(\frac{\pi}{6}t\right)\right]_0^4$$

$$= -2 + \frac{6}{\pi} \left[\sin\left(\frac{2\pi}{3}\right) - 0\right]$$

$$= -2 + \frac{6}{\pi} \cdot \frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi}$$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$