AP® CALCULUS AB
2013 SCORING GUIDELINES

Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) Solve $|v(t)| = 2$ on $2 \le t \le 4$.
$t = 3.128$ (or 3.127) and $t = 3.473$

2 : { 1 : considers $|v(t)| = 2$; 1 : answer }

(b) $s(t) = 10 + \int_0^t v(x)\,dx$

$s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$

2 : { 1 : $s(t)$; 1 : $s(5)$ }

(c) $v(t) = 0$ when $t = 0.536033,\ 3.317756$
$v(t)$ changes sign from negative to positive at time $t = 0.536033$.
$v(t)$ changes sign from positive to negative at time $t = 3.317756$.

Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).

3 : { 1 : considers $v(t) = 0$; 2 : answers with justification }

(d) $v(4) = -11.475758 < 0,\ a(4) = v'(4) = -22.295714 < 0$

The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.

2 : conclusion with reason