

AP[®] CALCULUS AB
2017 SCORING GUIDELINES

Question 5

(a) $x'_P(t) = \frac{2t - 2}{t^2 - 2t + 10} = \frac{2(t - 1)}{t^2 - 2t + 10}$

$t^2 - 2t + 10 > 0$ for all t .

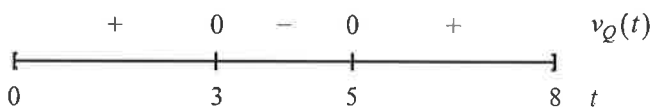
$x'_P(t) = 0 \Rightarrow t = 1$

$x'_P(t) < 0$ for $0 \leq t < 1$.

Therefore, the particle is moving to the left for $0 \leq t < 1$.

(b) $v_Q(t) = (t - 5)(t - 3)$

$v_Q(t) = 0 \Rightarrow t = 3, t = 5$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \leq 8$ since $v_P(t) = x'_P(t)$ and $v_Q(t)$ have the same sign on these intervals.

(c) $a_Q(t) = v'_Q(t) = 2t - 8$

$a_Q(2) = 2 \cdot 2 - 8 = -4$

$a_Q(2) < 0$ and $v_Q(2) = 3 > 0$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

(d) Particle Q first changes direction at time $t = 3$.

$$x_Q(3) = x_Q(0) + \int_0^3 v_Q(t) dt = 5 + \int_0^3 (t^2 - 8t + 15) dt$$

$$= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t \right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23$$

2 : $\begin{cases} 1 : x'_P(t) \\ 1 : \text{interval} \end{cases}$

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{analysis using } v_P(t) \text{ and } v_Q(t) \end{cases}$

Note: 1/2 if only one interval with analysis

Note: 0/2 if no analysis

2 : $\begin{cases} 1 : a_Q(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$