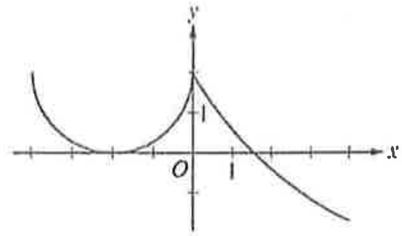
AP® CALCULUS AB
2009 SCORING GUIDELINES

Question 6

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}.$$

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.



Graph of f'

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

(a) f' changes from decreasing to increasing at $x = -2$ and from increasing to decreasing at $x = 0$. Therefore, the graph of f has points of inflection at $x = -2$ and $x = 0$.

$2: \begin{cases} 1: \text{identifies } x = -2 \text{ or } x = 0 \\ 1: \text{answer with justification} \end{cases}$

(b) $f(-4) = 5 + \int_0^{-4} g(x)\,dx$

$= 5 - (8 - 2\pi) = 2\pi - 3$

$f(4) = 5 + \int_0^4 \left(5e^{-x/3} - 3\right)dx$

$= 5 + \left(-15e^{-x/3} - 3x\right)\Big|_{x=0}^{x=4}$

$= 8 - 15e^{-4/3}$

$5: \begin{cases} 2: f(-4) \\ \quad 1: \text{integral} \\ \quad 1: \text{value} \\ 3: f(4) \\ \quad 1: \text{integral} \\ \quad 1: \text{antiderivative} \\ \quad 1: \text{value} \end{cases}$

(c) Since $f'(x) > 0$ on the intervals $-4 < x < -2$ and $-2 < x < 3\ln\left(\frac{5}{3}\right)$, f is increasing on the interval $-4 \le x \le 3\ln\left(\frac{5}{3}\right)$.

Since $f'(x) < 0$ on the interval $3\ln\left(\frac{5}{3}\right) < x < 4$, f is decreasing on the interval $3\ln\left(\frac{5}{3}\right) \le x \le 4$.

Therefore, f has an absolute maximum at $x = 3\ln\left(\frac{5}{3}\right)$.

$2: \begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$