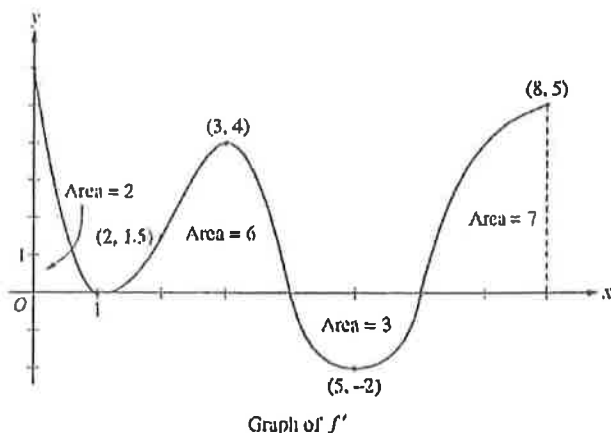


**AP<sup>®</sup> CALCULUS AB**  
**2013 SCORING GUIDELINES**

**Question 4**

The figure above shows the graph of  $f'$ , the derivative of a twice-differentiable function  $f$ , on the closed interval  $0 \leq x \leq 8$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$ ,  $x = 3$ , and  $x = 5$ . The areas of the regions between the graph of  $f'$  and the  $x$ -axis are labeled in the figure. The function  $f$  is defined for all real numbers and satisfies  $f(8) = 4$ .



- (a) Find all values of  $x$  on the open interval  $0 < x < 8$  for which the function  $f$  has a local minimum. Justify your answer.
- (b) Determine the absolute minimum value of  $f$  on the closed interval  $0 \leq x \leq 8$ . Justify your answer.
- (c) On what open intervals contained in  $0 < x < 8$  is the graph of  $f$  both concave down and increasing? Explain your reasoning.
- (d) The function  $g$  is defined by  $g(x) = (f(x))^3$ . If  $f(3) = -\frac{5}{2}$ , find the slope of the line tangent to the graph of  $g$  at  $x = 3$ .

(a)  $x = 6$  is the only critical point at which  $f'$  changes sign from negative to positive. Therefore,  $f$  has a local minimum at  $x = 6$ .

(b) From part (a), the absolute minimum occurs either at  $x = 6$  or at an endpoint.

$$\begin{aligned} f(0) &= f(8) + \int_8^0 f'(x) \, dx \\ &= f(8) - \int_0^8 f'(x) \, dx = 4 - 12 = -8 \end{aligned}$$

$$\begin{aligned} f(6) &= f(8) + \int_8^6 f'(x) \, dx \\ &= f(8) - \int_6^8 f'(x) \, dx = 4 - 7 = -3 \end{aligned}$$

$$f(8) = 4$$

The absolute minimum value of  $f$  on the closed interval  $[0, 8]$  is  $-8$ .

(c) The graph of  $f$  is concave down and increasing on  $0 < x < 1$  and  $3 < x < 4$ , because  $f'$  is decreasing and positive on these intervals.

(d)  $g'(x) = 3[f(x)]^2 \cdot f'(x)$

$$g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$$

1 : answer with justification

3 :  $\begin{cases} 1 : \text{considers } x = 0 \text{ and } x = 6 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

2 :  $\begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

3 :  $\begin{cases} 2 : g'(x) \\ 1 : \text{answer} \end{cases}$