

AP[®] CALCULUS AB
2007 SCORING GUIDELINES

Question 6

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

- (a) Find $f'(x)$ and $f''(x)$.
- (b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k , determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.
- (c) For a certain value of the constant k , the graph of f has a point of inflection on the x -axis. Find this value of k .

(a) $f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$

$$f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$$

$$2: \begin{cases} 1: f'(x) \\ 1: f''(x) \end{cases}$$

(b) $f'(1) = \frac{1}{2}k - 1 = 0 \Rightarrow k = 2$

When $k = 2$, $f'(1) = 0$ and $f''(1) = -\frac{1}{2} + 1 > 0$.

f has a relative minimum value at $x = 1$ by the Second Derivative Test.

$$4: \begin{cases} 1: \text{sets } f'(1) = 0 \text{ or } f'(x) = 0 \\ 1: \text{solves for } k \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$$

(c) At this inflection point, $f''(x) = 0$ and $f(x) = 0$.

$$f''(x) = 0 \Rightarrow \frac{-k}{4x^{3/2}} + \frac{1}{x^2} = 0 \Rightarrow k = \frac{4}{\sqrt{x}}$$

$$f(x) = 0 \Rightarrow k\sqrt{x} - \ln x = 0 \Rightarrow k = \frac{\ln x}{\sqrt{x}}$$

$$3: \begin{cases} 1: f''(x) = 0 \text{ or } f(x) = 0 \\ 1: \text{equation in one variable} \\ 1: \text{answer} \end{cases}$$

Therefore, $\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$
 $\Rightarrow 4 = \ln x$
 $\Rightarrow x = e^4$
 $\Rightarrow k = \frac{4}{e^2}$