

AP[®] CALCULUS AB
2008 SCORING GUIDELINES

Question 6

Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by

$$f'(x) = \frac{1 - \ln x}{x^2}.$$

- (a) Write an equation for the line tangent to the graph of f at $x = e^2$.
 (b) Find the x -coordinate of the critical point of f . Determine whether this point is a relative minimum, a relative maximum, or neither for the function f . Justify your answer.
 (c) The graph of the function f has exactly one point of inflection. Find the x -coordinate of this point.
 (d) Find $\lim_{x \rightarrow 0^+} f(x)$.

(a) $f(e^2) = \frac{\ln e^2}{e^2} = \frac{2}{e^2}$, $f'(e^2) = \frac{1 - \ln e^2}{(e^2)^2} = -\frac{1}{e^4}$

An equation for the tangent line is $y = \frac{2}{e^2} - \frac{1}{e^4}(x - e^2)$.

2 : $\begin{cases} 1 : f(e^2) \text{ and } f'(e^2) \\ 1 : \text{answer} \end{cases}$

- (b) $f'(x) = 0$ when $x = e$. The function f has a relative maximum at $x = e$ because $f'(x)$ changes from positive to negative at $x = e$.

3 : $\begin{cases} 1 : x = e \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$

(c) $f''(x) = \frac{-\frac{1}{x}x^2 - (1 - \ln x)2x}{x^4} = \frac{-3 + 2 \ln x}{x^3}$ for all $x > 0$

$f''(x) = 0$ when $-3 + 2 \ln x = 0$

$x = e^{3/2}$

The graph of f has a point of inflection at $x = e^{3/2}$ because $f''(x)$ changes sign at $x = e^{3/2}$.

3 : $\begin{cases} 2 : f''(x) \\ 1 : \text{answer} \end{cases}$

(d) $\lim_{x \rightarrow 0^+} \frac{\ln x}{x} = -\infty$ or Does Not Exist

1 : answer