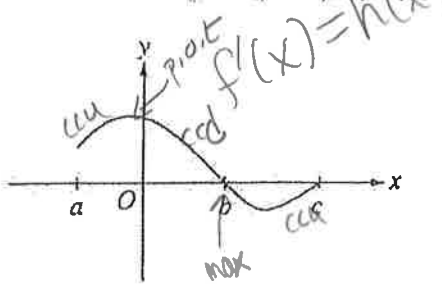
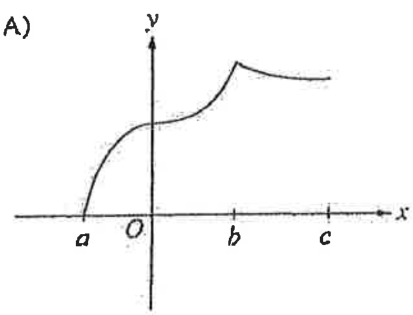
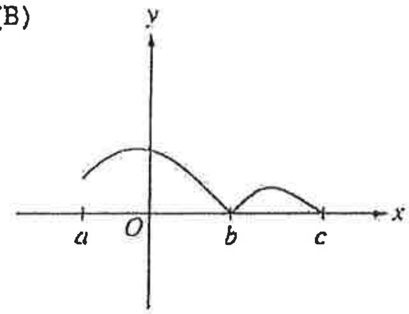


88. Let $f(x)=\int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

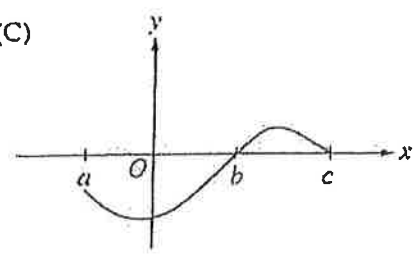
(A)



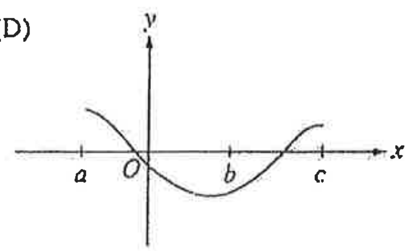
(B)



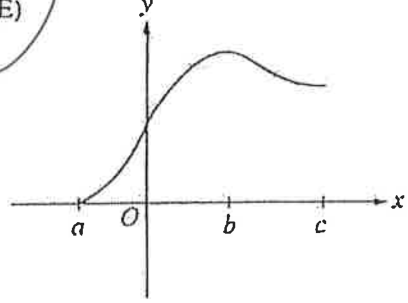
(C)



(D)



(E)



x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

$\frac{1}{2}\cdot\frac{1}{2}(3+6+10+16+13)$

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

---

90. Which of the following are antiderivatives of $f(x)=\sin x\cos x$?

I. $F(x)=\dfrac{\sin^2 x}{2}$

II. $F(x)=\dfrac{\cos^2 x}{2}$

III. $F(x)=\dfrac{-\cos(2x)}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

$u=\sin x \quad du=\cos x\,dx$

$\int \sin x\cos x\,dx$

$\int u\,du=\frac{u^2}{2}+C$

$=\frac{\sin^2 x}{2}+C$

$u=\cos x \quad du=-\sin x\,dx$

$-\int u\,du=-\frac{u^2}{2}+C$

$-\frac{\cos^2 x}{2}+C$

$2\sin x\cos x=\sin 2x$

$\int \sin x\cos x\,dx=\int \frac{1}{2}\cdot 2\sin x\cos x\,dx$

$u=2x \quad du=2dx$

$\frac{1}{2}du=dx$

$\frac{1}{2}\int \sin 2x\,dx$

$\frac{1}{2}\cdot\frac{1}{2}\int \sin u\,du=-\frac{1}{4}\cos 2x\,dx$

55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$y' = x^2 + 10x$

$y'' = 2x + 10$

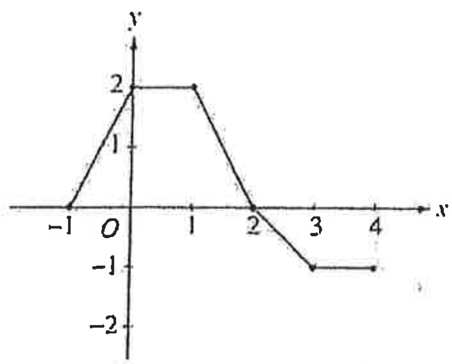
$x = -5$

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

y'' + −

−5



$\frac{1}{2}(2)(3+1) = 4$

$-\frac{1}{2}(1)(2+1)$

$= -\frac{3}{2}$

2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

---

3. $\int_{1}^{2} \frac{1}{x^2}\,dx =$ $\left. -\frac{1}{x} \right|_1^2 = -\frac{1}{2} + 1$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2 \ln 2$

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ for some c such that $a < c < b$. T

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

(E) $\int_a^b f(x)dx$ exists.

Though EVT applies the extrema may be @ the endpoints ∴ $f'(c) \neq 0$

---

5. $\int_0^x \sin t\, dt =$ $-\cos x - (-\cos 0)$ $\quad -\cos x + 1$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

---

6. If $x^2 + xy = 10$, then when $x = 2$, $\dfrac{dy}{dx} =$

$4 + 2y = 10 \quad 2y = 6 \quad y = 3$

$2x + xy' + y = 0 \quad xy' = -2x - y$

$y' = \dfrac{-2x-y}{x} = \dfrac{-2(2)-3}{2} = -7/2$

(A) $-\dfrac{7}{2}$ (B) -2 (C) $\dfrac{2}{7}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{7}{2}$

---

7. $\int_1^e \left(\dfrac{x^2-1}{x}\right)dx = \int_1^e x - \frac{1}{x}\,dx = \frac{x^2}{2} - \ln x \Big|_1^e = \left(\frac{e^2}{2} - \ln e\right) - \left(\frac{1}{2} - \ln 1\right)$

$\frac{e^2}{2} - 1 - \frac{1}{2}$

$\frac{e^2}{2} - 3/2$

(A) $e - \dfrac{1}{e}$ (B) $e^2 - e$ (C) $\dfrac{e^2}{2} - e + \dfrac{1}{2}$ (D) $e^2 - 2$ (E) $\dfrac{e^2}{2} - \dfrac{3}{2}$

---

8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x
(ii) $f(0) = 1$ $\quad f(x) \neq 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

$h'(x) = f(x)g'(x) + g(x) \cdot f'(x)$

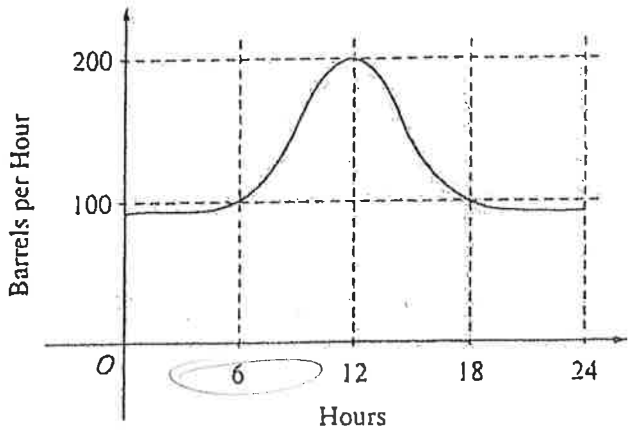
$f(x)g'(x) = f(x)g'(x) + g(x)f'(x)$

$0 = g(x)f'(x)$

$+$

must $f'(x) = 0$ happen

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

---

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

$f'(x) = \dfrac{(x-1)(2x) - (x^2-2)}{(x-1)^2}$

$\dfrac{4-2}{1} = 2$

(A) -2 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{2}$ (D) 2 (E) 6

---

11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx = f'(b) - f'(a)$

f' must be a constant

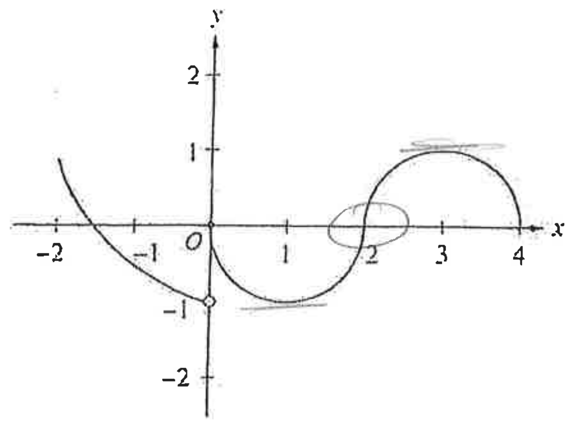
(A) 0 (B) 1 (C) $\dfrac{ab}{2}$ (D) $b - a$ (E) $\dfrac{b^2 - a^2}{2}$

---

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

$\ln 2$, $4\ln 2$ don't equal

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2<x<4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

---

14. A particle moves along the x-axis so that its position at time t is given by $x(t)=t^2-6t+5$. For what value of t is the velocity of the particle zero?

$v(t) = 2t - 6$ $2t-6=0$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

15. If $F(x)=\int_0^x \sqrt{t^3+1}\,dt$, then $F'(2)=$ $F'(x) = \sqrt{x^3+1}\,(1)$ $F'(2) = \sqrt{9} = 3$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

---

16. If $f(x)=\sin\left(e^{-x}\right)$, then $f'(x)=$ $\cos(e^{-x})\,(-e^{-x})$

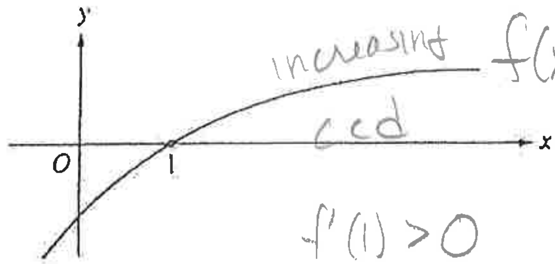
(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x})+e^{-x}$

(C) $\cos(e^{-x})-e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$



increasing $f(x)$

ccd

$f(1)=0$

$f'(1)>0$ $f''(1)<0$

17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1)<f'(1)<f''(1)$

(B) $f(1)<f''(1)<f'(1)$

(C) $f'(1)<f(1)<f''(1)$

(D) $f''(1)<f(1)<f'(1)$

(E) $f''(1)<f'(1)<f(1)$

$f''(1) < f(1) < f'(1)$

---

18. An equation of the line tangent to the graph of $y=x+\cos x$ at the point $(0,1)$ is

(A) $y=2x+1$ (B) $y=x+1$ (C) $y=x$ (D) $y=x-1$ (E) $y=0$

$y' = 1-\sin x$

$y' = 1 - \sin 0$

$y-1 = 1(x-0)$

$y = x+1$

---

19. If $f''(x)=x(x+1)(x-2)^2$, then the graph of f has inflection points when $x=$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

$+ \quad - \quad + \quad +$ (-1, 0, 2)

---

20. What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$?

(A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3

$\frac{x^3}{3}\Big|_{-3}^{k}$ $\quad \frac{k^3}{3}+9=0$ $\quad \frac{k^3}{3}=-9$

$k^3=-27$

---

21. If $\frac{dy}{dt}=ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt}+3$ (D) $kty+5$ (E) $\frac{1}{2}ky^2+\frac{1}{2}$

exponential functions

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

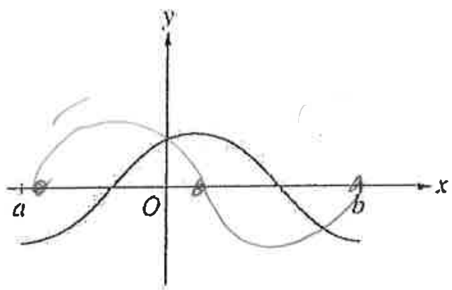
(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

$f'(x) = 4x^3 + 2x$

$0 = 4x^3 + 2x$
$= 2x(2x^2 + 1)$

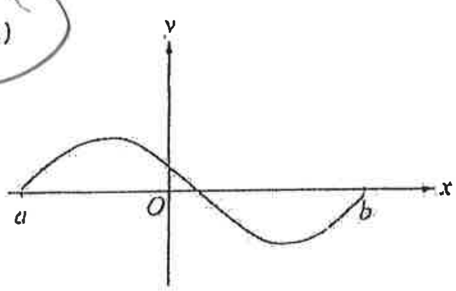
$x = 0$

$f'(x)$ — $-$ | $+$; 0

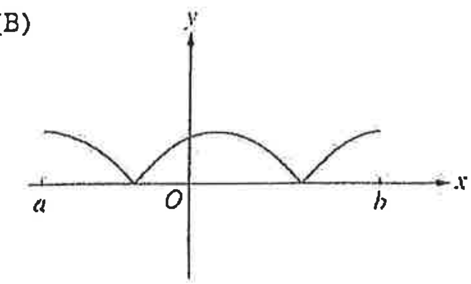


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

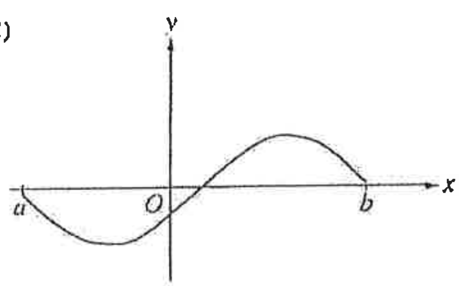
(A)



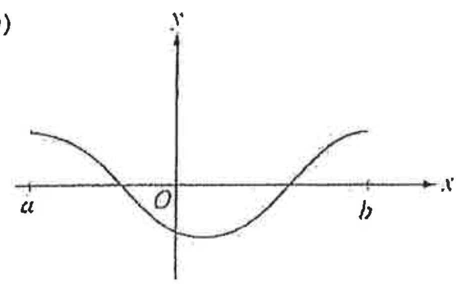
(B)



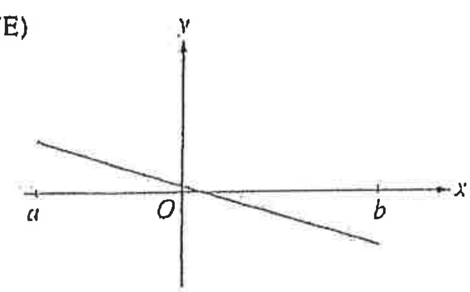
(C)



(D)



(E)



need to $a' = v''$

→ output y value

24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

$v(0) = 4$
$v(3) = 27 - 27$

$a(t) = 3t^2 - 6t + 12$ $\quad a'(t) = 6t - 6$

$a(0) = 12$ $\quad t = 1$

$a(3) = 27 - 18 + 12$

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

$\int_0^2 x^2 + x \, dx = \frac{x^3}{3} + \frac{x^2}{2} \Big|_0^2 \quad \frac{8}{3} + 2$

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

x	0	1	2
$f(x)$	1	k	2

(0,1) (2,2) $f(x) = \frac{1}{2}$

26. The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k =$

→ must be < 1/2

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

27. What is the average value of $y = x^2\sqrt{x^3 + 1}$ on the interval $[0,2]$?

$\frac{1}{2-0} \int_0^2 x^2 \sqrt{x^3+1} \, dx$

$u = x^3 + 1$

$du = 3x^2 \, dx$

$\frac{1}{3} du = x^2 \, dx$

$u(2) = 9$

$u(0) = 1$

$\frac{1}{2} \cdot \frac{1}{3} \int_1^9 u^{1/2} \, du$

$\frac{1}{6} \cdot \frac{2}{3} u^{3/2} \Big|_1^9$

$\frac{1}{9} 9^{3/2} - \frac{1}{9}$

$\frac{1}{9}(27) - \frac{1}{9}$

$\frac{26}{9}$

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

$f'(x) = 2\sec^2(2x)$

$= 2\left(\sec\left(\frac{\pi}{3}\right)\right)^2$

$= 8$