

**AP[®] CALCULUS AB
2010 SCORING GUIDELINES**

Question 6

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

- (a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.
- (b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.
- (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f'(1) = \left. \frac{dy}{dx} \right|_{(1,2)} = 8$

An equation of the tangent line is $y = 2 + 8(x - 1)$.

(b) $f(1.1) \approx 2.8$

Since $y = f(x) > 0$ on the interval $1 \leq x < 1.1$,

$$\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2) > 0 \text{ on this interval.}$$

Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.

(c) $\frac{dy}{dx} = xy^3$

$$\int \frac{1}{y^3} dy = \int x dx$$

$$-\frac{1}{2y^2} = \frac{x^2}{2} + C$$

$$-\frac{1}{2 \cdot 2^2} = \frac{1^2}{2} + C \Rightarrow C = -\frac{5}{8}$$

$$y^2 = \frac{1}{\frac{5}{4} - x^2}$$

$$f(x) = \frac{2}{\sqrt{5 - 4x^2}}, \quad \frac{-\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$$

$$2 : \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{conclusion with explanation} \end{cases}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables