

AP[®] CALCULUS AB
2011 SCORING GUIDELINES

Question 6

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

- (a) Show that f is continuous at $x = 0$.
 (b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.
 (c) Find the average value of f on the interval $[-1, 1]$.

(a) $\lim_{x \rightarrow 0^-} (1 - 2\sin x) = 1$

$$\lim_{x \rightarrow 0^+} e^{-4x} = 1$$

$$f(0) = 1$$

$$\text{So, } \lim_{x \rightarrow 0} f(x) = f(0).$$

Therefore f is continuous at $x = 0$.

(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$$-2\cos x \neq -3 \text{ for all values of } x < 0.$$

$$-4e^{-4x} = -3 \text{ when } x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0.$$

$$\text{Therefore } f'(x) = -3 \text{ for } x = -\frac{1}{4}\ln\left(\frac{3}{4}\right).$$

(c)
$$\begin{aligned} \int_{-1}^1 f(x) dx &= \int_{-1}^0 f(x) dx + \int_0^1 f(x) dx \\ &= \int_{-1}^0 (1 - 2\sin x) dx + \int_0^1 e^{-4x} dx \\ &= \left[x + 2\cos x \right]_{x=-1}^{x=0} + \left[-\frac{1}{4}e^{-4x} \right]_{x=0}^{x=1} \\ &= (3 - 2\cos(-1)) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4} \right) \end{aligned}$$

$$\begin{aligned} \text{Average value} &= \frac{1}{2} \int_{-1}^1 f(x) dx \\ &= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4} \end{aligned}$$

2 : analysis

3 : $\begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

4 : $\begin{cases} 1 : \int_{-1}^0 (1 - 2\sin x) dx \text{ and } \int_0^1 e^{-4x} dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$