17. $\int (x^2+1)^2\,dx =$

$\int (x^4+2x^2+1)\,dx$

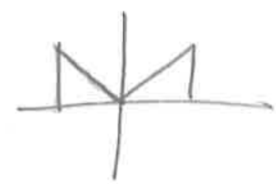
(A) $\dfrac{(x^2+1)^3}{3}+C$

(B) $\dfrac{(x^2+1)^3}{6x}+C$

(C) $\left(\dfrac{x^3}{3}+x\right)^2+C$

(D) $\dfrac{2x(x^2+1)^3}{3}+C$

(E) $\dfrac{x^5}{5}+\dfrac{2x^3}{3}+x+C$



$\dfrac{f(\frac{3\pi}{2})-f(\frac{\pi}{2})}{\frac{3\pi}{2}-\frac{\pi}{2}} = \dfrac{\sqrt{2}/2-\frac{\sqrt{2}}{2}}{\pi} = 0$

$f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right) = 0$

$\cos\left(\frac{x}{2}\right) = 0$

18. If $f(x)=\sin\left(\dfrac{x}{2}\right)$, then there exists a number c in the interval $\dfrac{\pi}{2}<x<\dfrac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\dfrac{2\pi}{3}$ (B) $\dfrac{3\pi}{4}$ (C) $\dfrac{5\pi}{6}$ (D) π (E) $\dfrac{3\pi}{2}$

19. Let f be the function defined by $f(x)=\begin{cases} x^3 & \text{for } x\le 0, \\ x & \text{for } x>0. \end{cases}$ Which of the following statements about f is true?

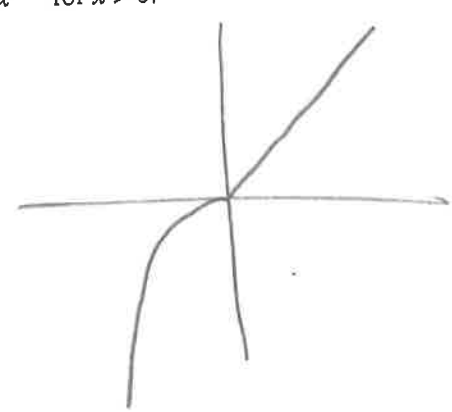
(A) f is an odd function.

(B) f is discontinuous at $x=0$.

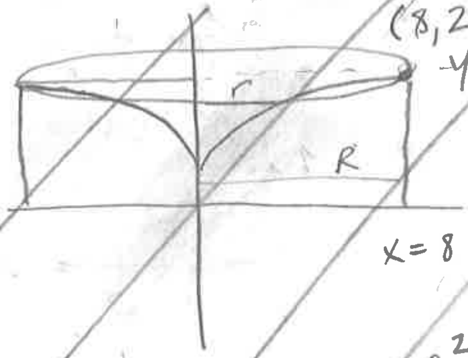
(C) f has a relative maximum.

(D) $f'(0)=0$

(E) $f'(x)>0$ for $x\ne 0$



20)



$y = (x+1)^{1/3} \Rightarrow y^3 - 1 = x$

$A(y) = \pi(R^2 - r^2) = \pi(4? -$

$V = \pi \int_0^2$

21) $y = x^{-2} - x^{-3}$ domain $(-\infty, 0) \cup (0, \infty)$

$y' = -2x^{-3} + 3x^{-4}$

$y'' = 6x^{-4} - 12x^{-5}$

$= 6x^{-5}(x - 2)$

$= \frac{6}{x^5}(x-2)$

y'': + | + | + at 0, 2

no tangent line @ $x = 0$

23) C.P. point in domain where $f' = 0$ or f' DNE

$f'(x) = 5(x+2)^4(x-3)^4 + 4(x+2)^5(x-3)^3$

$= (x+2)^4(x-3)^3(5(x-3) + 4(x+2))$

$= (x+2)^4(x-3)^3(9x-7)$

20. Let R be the region in the first quadrant enclosed by the graph of $y=(x+1)^{\frac{1}{3}}$, the line $x=7$, the x-axis, and the y-axis. The volume of the solid generated when R is revolved about the y-axis is given by

Cylindrical shells

(A) $\pi\int_0^7 (x+1)^{\frac{2}{3}}\,dx$ (B) $2\pi\int_0^7 x(x+1)^{\frac{1}{3}}\,dx$ (C) $\pi\int_0^2 (x+1)^{\frac{2}{3}}\,dx$

(D) $2\pi\int_0^2 x(x+1)^{\frac{1}{3}}\,dx$ (E) $\pi\int_0^7 (y^3-1)^2\,dy$

21. At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

22. An antiderivative for $\frac{1}{x^2-2x+2}$ is

$\frac{1}{x^2-2x+2} = \frac{1}{(x-1)^2+1}$

(A) $-(x^2-2x+2)^{-2}$

(B) $\ln(x^2-2x+2)$

$\int \frac{1}{(x-1)^2+1}\,dx = \tan^{-1}(x-1)+C$

(C) $\ln\left|\frac{x-2}{x+1}\right|$

(D) $\operatorname{arcsec}(x-1)$

(E) $\arctan(x-1)$

$\frac{d}{dx}\tan^{-1}(x-1)+C = \frac{1}{1+(x-1)^2} = \frac{1}{x^2-2x+2}$

23. How many critical points does the function $f(x)=(x+2)^5(x-3)^4$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

24. If $f(x)=(x^2-2x-1)^{\frac{2}{3}}$, then $f'(0)$ is

$f'(0) = \frac{2}{3}(x^2-2x-1)^{-1/3}(2x-2)\Big|_{x=0}$

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

$= \frac{2}{3}(-1)^{-1/3}(-2) = \frac{4}{3}$

26) $v(t) = s'(t) = 4\sin t - t$

$a(t) = v'(t) = 4\cos t - 1 = 0$

$\cos t = \frac{1}{4}$

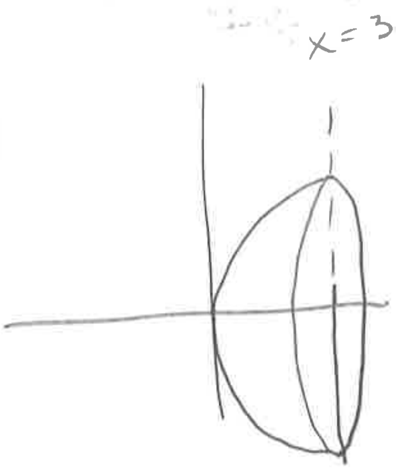
$t = 1.318$

$v(1.318) = 4\sin(1.318) - 1.318$

$= 2.554$

28) $\int_1^2 13^x\,dx - \int_1^2 11^x\,dx$

30)



$y = \sqrt{x}$

$V = \pi \int x^{\frac{1}{2}}\,dx$

$V =$

25. $\frac{d}{dx}\left(2^x\right) =$ $2^x / \ln 2$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

26. A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

27. The function f given by $f(x) = x^3 + 12x - 24$ is $f'(x) = 3x^2 + 12$ $3(x^2+4)$

(A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
(B) decreasing for $x < 0$, increasing for $x > 0$
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

28. $\int_1^{500}\left(13^x - 11^x\right)dx + \int_2^{500}\left(11^x - 13^x\right)dx =$ $\int_1^{500} 13^x dx - \int_2^{500} 13^x dx + \int_2^{500} 11^x dx - \int_1^{500} 11^x dx$

(A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364

29. $\lim_{\theta \to 0} \frac{1-\cos\theta}{2\sin^2\theta}$ is $\frac{1-\cos\theta}{2(1-\cos^2\theta)} = \frac{1-\cos\theta}{2(1+\cos\theta)(1-\cos\theta)} = \frac{1}{2(1+1)} = \frac{1}{4}$

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

30. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

31) $f(x) = e^{\ln x^6} = x^6$

$f'(x) = 6x^5$

33) $\frac{dy}{2y^2} = dx$

$\int \frac{1}{2} y^{-2} dy = \int dx$

$\frac{1}{2} \cdot -1 \cdot y^{-1} = x + C$

$-\frac{1}{2} y^{-1} = x + C$

$(1, -1)$

$-\frac{1}{2} \cdot (-1)^{-1} = 1 + C$

$C = -\frac{1}{2}$

$-\frac{1}{2} y^{-1} = 2 - \frac{1}{2}$

$y^{-1} = -2\left(\frac{3}{2}\right)$

$\frac{1}{y} = -3 \qquad y = -\frac{1}{3}$

31. If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

32. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$

33. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute

(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

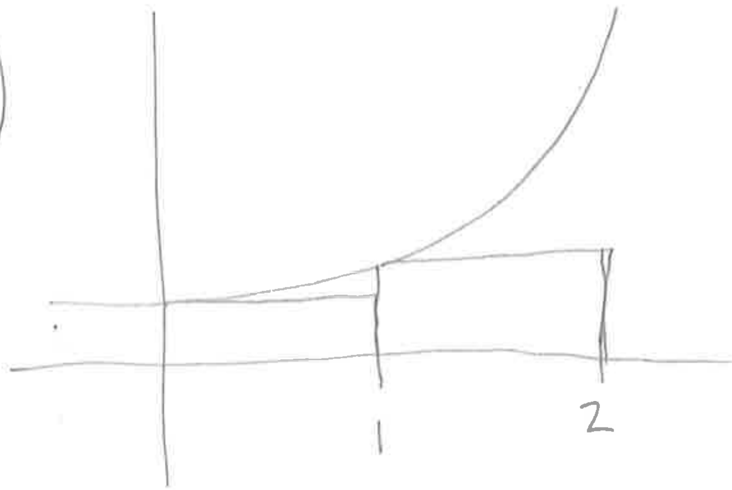
(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute

35. If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

(A) -5 (B) -1 (C) 0 (D) 1 (E) 5

36)



$LRAM = 1 \cdot e^0 + 1 \cdot e^1 = 1 + e \approx 3.718$

$TRAP = \frac{1}{2}(1+e) \cdot 1 + \frac{1}{2}(e + e^4) \cdot 1 \approx$

$TRAP - LRAM \approx 26.799$

39) $A = \pi r^2$ $\quad C = 2\pi r$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\quad \frac{dc}{dt} = 2\pi \frac{dr}{dt}$

$2\pi r \frac{dr}{dt} = 2\pi \frac{dr}{dt}$

$r = 1$

36. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

(A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

37. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

II. $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

III. $\lim_{x\to a} \frac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

38. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

$f'(x) = \int f''(x) + C = x^2 - \sin x + C_1$

$f(x) = \int f'(x) = \frac{1}{3}x^3 + \cos x + C_1 x + C_2$

(A) $\frac{x^3}{3} + \cos x - x + 1$

(B) $\frac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

(D) $x^2 - \sin x + 1$

(E) $x^2 + \sin x + 1$

39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

(A) $\frac{1}{\pi}$ (B) $\frac{1}{2}$ (C) $\frac{2}{\pi}$ (D) 1 (E) 2

42) $\frac{dy}{dt} = ky$

$\frac{1}{y} dy = k dt$

$\ln|y| + C = kt$

$y = e^{kt - C} = e^{kt} \cdot e^{-C}$

$y = y_0 e^{kt}$

$y_0 = 2$

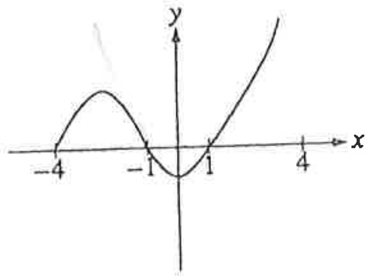
$3.5 = 2e^{2k}$

$k = \frac{\ln\left(\frac{3.5}{2}\right)}{2}$

$k \approx .2798$

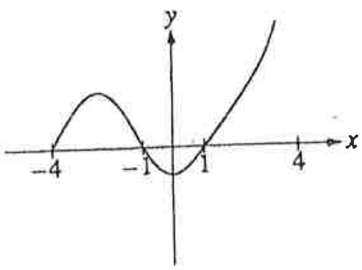
$y(3) = 2e^{3k}$

$y(3) \approx$

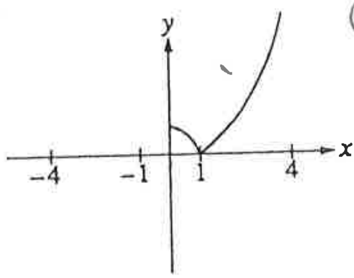


40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?

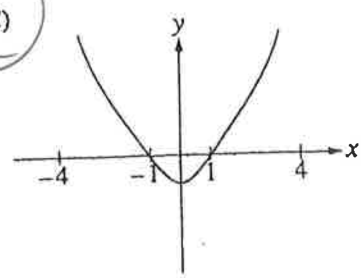
(A)



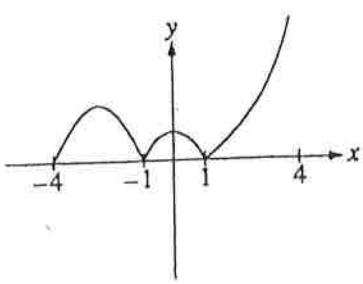
(B)



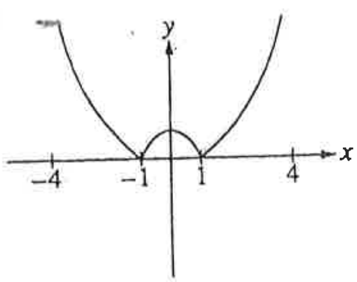
(C)



(D)



(E)



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41. $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

2π cos (2πx)

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

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42. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

43. $\int x f(x)\,dx =$

(A) $x f(x) - \int x f'(x)\,dx$

(B) $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x)dx$

(C) $x f(x) - \frac{x^2}{2} f(x) + C$

(D) $x f(x) - \int f'(x)\,dx$

(E) $\frac{x^2}{2} \int f(x)\,dx$

44. What is the minimum value of $f(x) = x \ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

45. If Newton's method is used to approximate the real root of $x^3 + x - 1 = 0$, then a first approximation $x_1 = 1$ would lead to a third approximation of $x_3 =$

(A) 0.682 (B) 0.686 (C) 0.694 (D) 0.750 (E) 1.637

44. DOMAIN $(0, \infty)$

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

let $f'(x) = 0 \Rightarrow \ln x = -1$

$x = e^{-1} = \frac{1}{e}$

TRY e^{-2} $f'(x) < 0$

try e^{0} $f'(x) > 0$

$f'(x)$ ——+—— e^{-1}

$\therefore f_{MIN}(e^{-1}) = \frac{1}{e} \cdot \ln e^{-1} = \frac{-1}{e}$