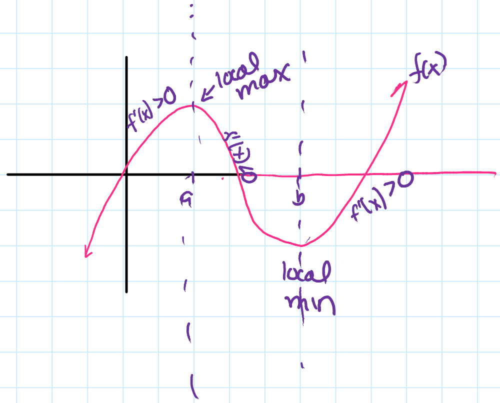
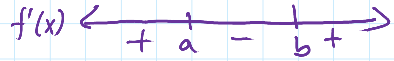
5.3 Day 1

Wednesday, November 2, 2016 12:35 PM

First Derivative Test

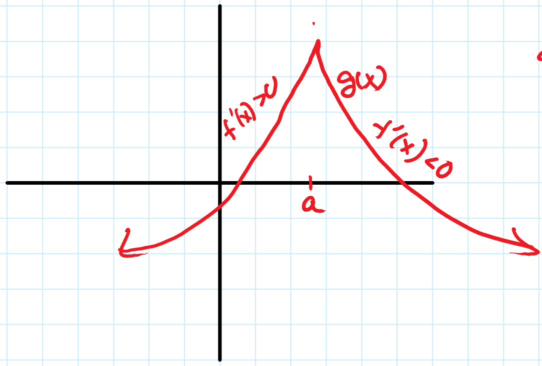


$f'(a)=0$ $f'(b)=0$

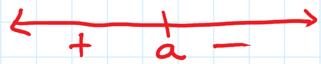


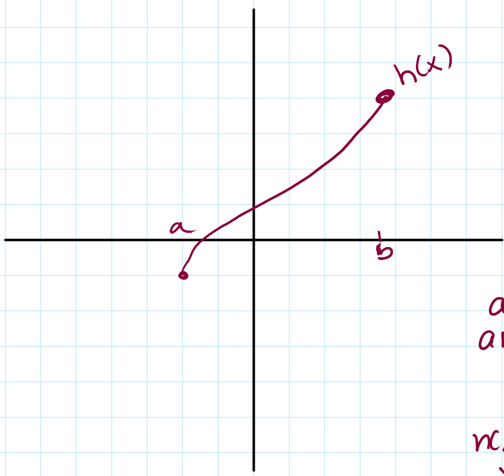
f(x) has a local min @ x = b, b/c f'(x) goes from − to + @ x = b.

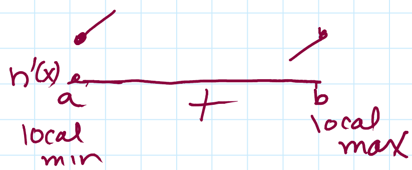
f(x) has a local max @ x = a b/c f'(x) goes from + to − @ x = a.



$g'(a)=$ dne





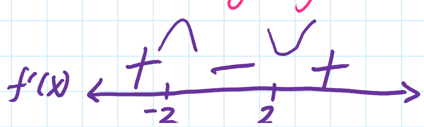


h(x) has a local min @ x=a, b/c x=a is an endpt. and h'(x) >0 to the right of x=a.

h(x) has a local max @ x=b, b/c x=b is an endpt and f'(x)>0 to the left of x=b.

example: Find all local extrema using FDT on $f(x)=x^3-12x-5$. Justify your answer.

$f'(x)=3x^2-12$



$3x^2-12=0$

$3x^2=12$

$x^2=4$

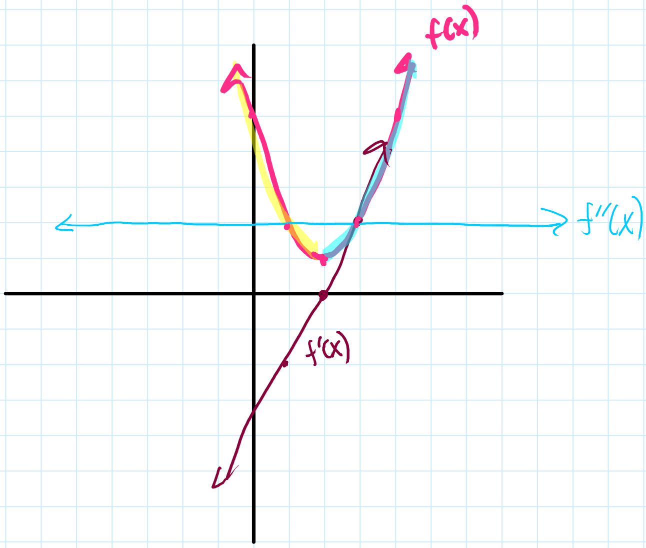
$x=\pm 2$

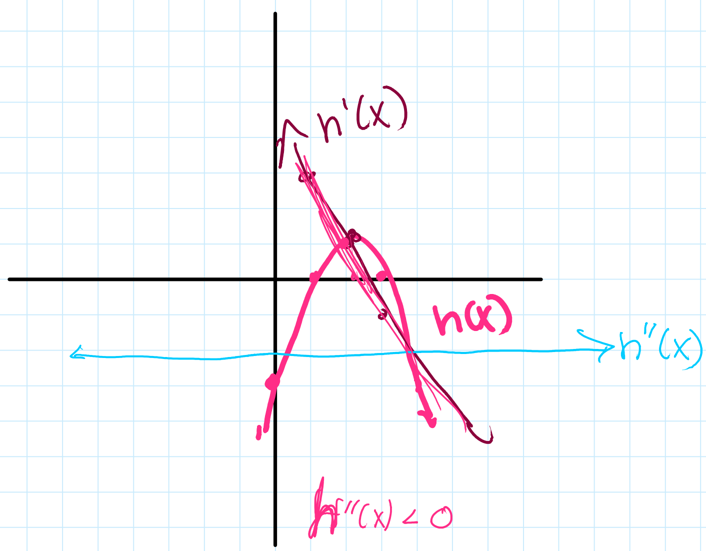
$f(-2) = -8+24-5$
$= 11$

$f(2) = 8-24-5$
$= -21$

$f(x)$ has a local max of 11 @ $x=-2$, b/c $f'(x)$ goes from + to − @ $x=-2$.

$f(x)$ has a local min of −21 @ $x=2$, b/c $f'(x)$ goes from − to + @ $x=2$.





$f''(x)>0$ $f(x)$ concave up
ccu

$h''(x)<0$
$h(x)$ concave down
ccd

2nd derivative

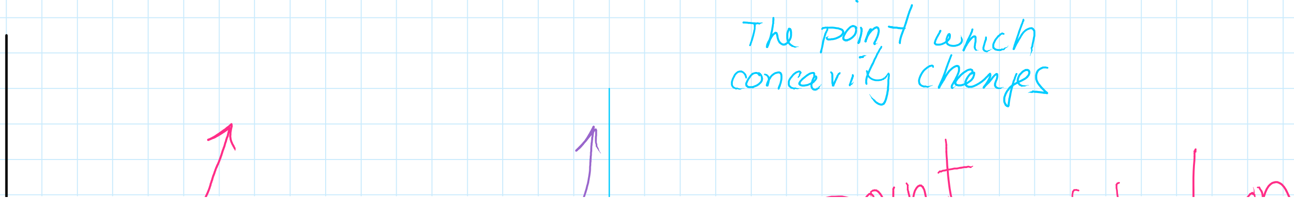
$f''(x)>0$ then the graph of $f(x)$ is concave up

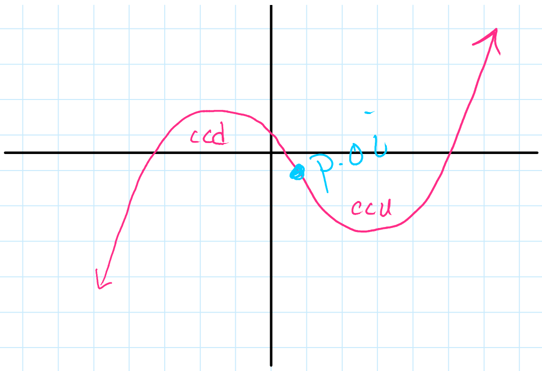
$f''(x)<0$ then the graph of $f(x)$ is concave down

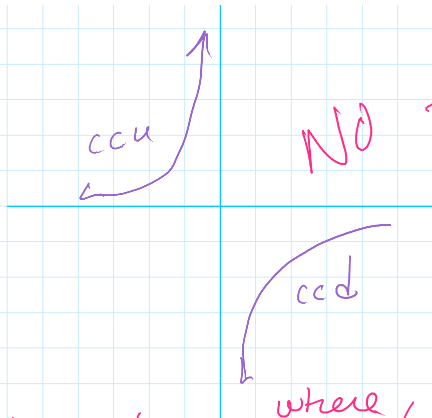
$f''(x)=0$ and $f''(x)$ changes signs

then there is a point of inflection
@ x. (p.o.i)

The point which concavity changes







No Point Of inflection

Determine ~~the~~ concavity of $f(x)$ and ~~any~~ where points of inflection.

$$f(x) = 4x^3 + 21x^2 + 36x - 20$$

$$f'(x) = 12x^2 + 42x + 36$$

$$f''(x) = 24x + 42$$

$$24x = -42$$

$$x = -42/24 = -7/4$$

$f''(x)$: N, ccd | $-7/4$ | P, ccu

ccd $(-\infty, -7/4)$

ccu $(-7/4, \infty)$

p.o.i occurs @ $x = -7/4$

ex find ~~the~~ all p.o.i:

$$g(x) = x^{1/3}(x - 4)$$

$$= x^{4/3} - 4x^{1/3}$$

$$g'(x) = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3}$$

$$g''(x) = \frac{4}{9}x^{-2/3} + \frac{8}{9}x^{-5/3}$$

$$\frac{4}{9}x^{-2/3} + \frac{8}{9}x^{-5/3} = 0$$

$$\frac{4}{9}x^{-5/3}(x + 2) = 0$$

$x = 0$ $\quad$ $x = -2$

-2

$$-\frac{5}{3} + \boxed{\frac{3}{3}} = -\frac{2}{3}$$