9.8 A Power Series

Thursday, February 27, 2020 8:08 AM

Exploration:

a. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}$ $= e^{-x}$

b. $\sum_{n=0}^{\infty} \frac{(-1) x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$ $\cos x - 1$

c. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ $\tan x$

These are examples of power series

Def: $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n + \ldots$

is a power series centered @ $x=0$

$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + \ldots + a_n (x-c)^n + \ldots$

power series centered @ $x = c$

* The function and the series will converge for certain values of x, called the interval of convergence.

* The radius of convergence (R) is the distance from the center to the endpts. of the interval

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Ex: Find the radius of convergence of

$$\sum_{n=0}^{\infty} 4(x-3)^n$$

center @ 3

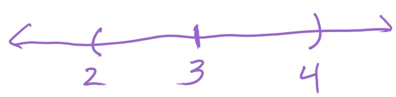
Use the ratio test

$$\lim_{n\to\infty}\left|\frac{\cancel{4}(x-3)^{n+1}}{\cancel{4}(x-3)^{n}}\right| = \lim_{n\to\infty}|x-3|$$

$|x-3| < 1$ ☆

$-1 < x-3 < 1$

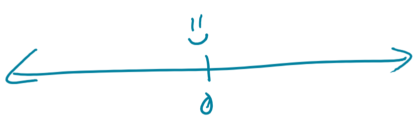
$2 < x < 4$



$$\boxed{R=1}$$

Ex: $\displaystyle\sum_{n=0}^{\infty} \frac{n!\,x^n}{2n}$ center $= 0 = x$

$$\lim_{n\to\infty}\left|\frac{(n+1)!\,x^{n+1}}{2n+2}\cdot\frac{2n}{n!\,x^n}\right| = \lim_{n\to\infty}\frac{(n+1)\,x\cdot 2n}{2n+2} \quad \infty > 1$$



$$\boxed{R=0}$$

you try . . . $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2n+2}}{(2n+2)!}\cdot\frac{(2n)!}{(-1)^n x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{-x^2}{(2n+2)(2n+1)}\right| = 0 < 1$$

hmmm......

$R = \infty$