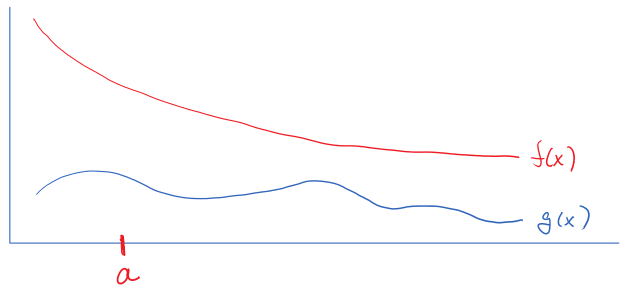
9.4 Day 2 Improper Integrals and the Comparison Test

Thursday, January 30, 2020 7:54 AM

Think About!



Let $f(x) \geq g(x) \geq 0$ for $a \leq x < \infty$

f & g are continuous

What can you say...

1. If $\int_a^\infty f(x)\,dx = 5$, then $\int_a^\infty g(x)\,dx$ ≤ 5 converge

2. If $\int_a^\infty g(x)\,dx$ diverges, then $\int_a^\infty f(x)\,dx$ diverges

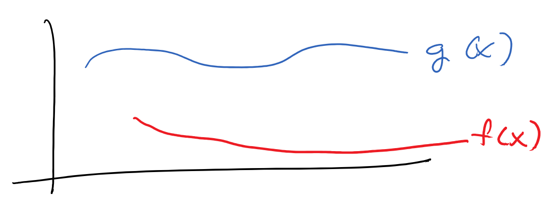
3. If $\int_a^\infty g(x)\,dx = 3$, then $\int_a^\infty f(x)\,dx$ ≥ 3 Inconclusive converge or diverge

4. If $\int_a^\infty f(x)\,dx$ diverges, then $\int_a^\infty g(x)\,dx$ Inconclusive converge or diverge.

Comparison Test

f & g are continuous on $[a, \infty)$ with

$0 \leq f(x) \leq g(x)$ for all $x \geq a$



Then...

1. $\int^\infty f(x)\,dx$ converges if $\int_a^\infty g(x)\,dx$ converges

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2. $\int_a^\infty g(x)\,dx$ diverges if $\int_a^\infty f(x)\,dx$ diverges

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Example:

Does $\int_1^\infty e^{-x^2}\,dx$ converge?

Compare to $\int_1^\infty e^{-x}\,dx$

$$\underline{\int_1^\infty e^{-x}\,dx} \geq \int_1^\infty e^{-x^2}\,dx$$

$$= \lim_{b\to\infty}\int_1^b e^{-x}\,dx = \lim_{b\to\infty}\left(-e^{-x}\right)\Big|_1^b = \lim_{b\to\infty}\left[\underline{-e^{-b}} + e^{-1}\right]$$

$$0 + \frac{1}{e}$$

$\therefore \int_1^\infty e^{-x^2}\,dx$ converges by the comparison test.

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you try... Does $\int_1^\infty \frac{1}{x^4+1}\,dx$ converge or diverge? Justify.

$$\int_1^\infty \frac{1}{\underline{x^4+1}}\,dx \leq \int_1^\infty \frac{1}{x^4}\,dx$$

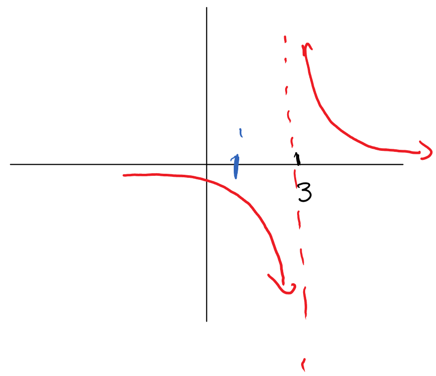
$$\lim_{b\to\infty}\int_1^b \frac{1}{x^4}\,dx = \lim_{b\to\infty}\frac{-1}{3x^3}\Big|_1^b = \lim_{b\to\infty}\left[\frac{-1}{3b^3} + \frac{1}{3}\right]$$

$$0 + \frac{1}{3}$$

$\int_1^\infty \frac{1}{x^4+1}\,dx$ converges by the comparison test

ex: more w/ Improper $\int$

Deal with V. A.



$$\int_0^4 \frac{1}{x-3}\,dx$$

$$= \lim_{b\to 3^-}\int_0^b \frac{1}{x-3}\,dx + \lim_{a\to 3^+}\int_a^4 \frac{1}{x-3}\,dx$$

$$= \lim_{b\to 3^-} \ln|x-3| \Big|_0^b + \lim_{a\to 3^+} \ln|x-3| \Big|_a^4$$

$$= \lim_{b\to 3^-}\left[\ln|b-3| - \ln|-3|\right] + \lim_{a\to 3^+}\left[\underset{\ln 1}{\ln|4-3|} - \ln|a-3|\right]$$

$$= \lim_{b\to 3^-}\left[\ln|b-3| - \ln 3\right] + \lim_{a\to 3^+}\left[0 - \ln|a-3|\right]$$

$$= \lim_{\substack{b\to 3^- \\ a\to 3^+}}\left[\ln\left|\frac{b-3}{a-3}\right| - \ln 3\right]$$

$$0 - \ln 3$$

side note

$$\ln|b-3| - \ln 3 - \ln|a-3|$$

$$\ln\left|\frac{b-3}{a-3}\right| - \ln 3$$

$$\boxed{\text{converges to } -\ln 3}$$

Try ish . . . . — — —

$$\int_1^{\infty} x \ln x \, dx$$

$$\boxed{uv - \int v\,du}$$

LIPET

$u = \ln x \qquad v = \frac{1}{2}x^2$

$du = \frac{1}{x}\,dx \qquad dv = x\,dx$

$$= \frac{1}{2}x^2 \ln x - \int_1^{\infty} \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{2}x^2 \ln x - \int_1^{\infty} \frac{1}{2}x\,dx$$

$$= \lim_{b \to \infty} \left[\frac{1}{2}x^2 \ln x - \left[\frac{1}{4}x^2\right]_1^b \right]$$

$$\lim_{b \to \infty} \left[\frac{1}{2}b^2 \ln b - \left[\frac{1}{4}b^2 - \frac{1}{4}\right] \right]$$

Diverges