9.2 Day 2

Tuesday, January 28, 2020 8:28 AM

L'Hôpital's Rule

$\frac{0}{0}, \frac{\infty}{\infty}$ — go ahead use L'Hop

$\infty \cdot 0,\ \infty - \infty,\ 1^{\infty},\ 0^0,\ \infty^0$ — Rearrange... to get $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Ex: $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) \to \underline{\infty \cdot 0}$

$$\lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} \quad \frac{\to 0}{\to 0}$$

$$\lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot\cancel{\left(\frac{-1}{x^2}\right)}}{\cancel{\frac{-1}{x^2}}} = 1$$

Ex: $\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right) \to \infty - \infty$

1. $$\lim_{x\to 1} \frac{x-1-\ln x}{(x-1)\ln x} \quad \frac{\to 0}{\to 0}$$

2. $$\lim_{x\to 1} \frac{1-\frac{1}{x}}{(x-1)\cdot\frac{1}{x} + \ln x\,(1)} \quad \frac{\to 0}{\to 0}$$

$(x-1)\frac{1}{x}$

$x\cdot\frac{1}{x} - 1\left(\frac{1}{x}\right)$

$1 - \frac{1}{x}$

3. $$\lim_{x\to 1} \frac{1-\frac{1}{x}}{1-\frac{1}{x}+\ln x} \quad \frac{\to 0}{\to 0}$$

4 $\lim_{x \to 1} \dfrac{\frac{1}{x^2}}{\frac{1}{x^2} + \frac{1}{x}} = \dfrac{\frac{1}{1}}{\frac{1}{1} + \frac{1}{1}} = \dfrac{1}{1+1} = \dfrac{1}{2}$

Think.... -

recall $b^{\log_b x} = x$

$e^{\ln x} = x$

$$e^{\ln f(x)} = f(x)$$

so $\lim_{x \to a} \ln f(x) = L \Rightarrow \lim_{x \to a} f(x) = \lim_{x \to a} e^{\ln f(x)} = e^{L}$

Ex: $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x \to 1^\infty$!!

$f(x) = \left(1 + \frac{1}{x}\right)^x$

$\ln f(x) = \ln \left(1 + \frac{1}{x}\right)^x$

$\ln f(x) = x \ln\left(1 + \frac{1}{x}\right)$

$\lim_{x \to \infty} \overset{\nearrow \infty}{x} \overset{\nearrow 0}{\ln\left(1 + \frac{1}{x}\right)} \to \infty \cdot 0$

$\lim_{x \to \infty} \dfrac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}}$ $\nearrow 0$ $\searrow 0$

$\dfrac{1}{\quad} \cdot -\cancel{1}$

$$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = 1$$

$$e^1$$