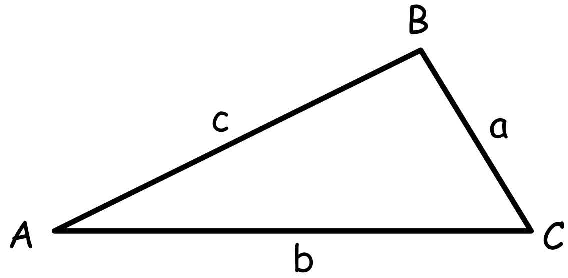
Law of Sines & Law of Cosines

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$



- To set up and solve a law of sines equation, you MUST know one set of "opposites" (angle and side across from that angle) plus one other measurement
- To use law of cosines, you MUST know at least 2 sides plus one other measurement (so either use 3 known sides to solve for an angle, or know 2 sides plus an angle and solve for the missing side)
- Law of Cosines is ALWAYS safe (barring user error)
- Law of Sines is always safe when used to solve for a side length. Always avoid using law of sines to solve for an angle that might be the largest angle in the triangle unless you determine the number of solutions in advance.

Examples. Solve for all missing values.

1. C=70°, a = 6, c = 10

$A \approx 34.32°$
$B = 75.68°$
$b \approx 10.31$

2. B = 61°, C = 38°, b = 9

$A \approx 81°$
$a \approx 10.16$
$c \approx 6.34$

3. A = 24°, a = 5, c = 12

Case #1
$B \approx 78.534°$
$C \approx 77.466°$
$b \approx 12.048$

Case #2
$B \approx 53.466°$
$C \approx 102.533°$
$b \approx 9.877$

4. B = 40°, a = 13, c = 8

$b \approx 8.583$
$A \approx 103.191°$
$C \approx 36.809°$

5. b=15, c=10, C=132°

NO SOLUTION

6. a = 16, b = 10, c = 8

$A \approx 125.10°$
$B \approx 30.753°$
$C \approx 24.147°$

7. A = 30°, a = 9, c = 18

$C = 90°$

$B = 60°$

$b = 9\sqrt{3} \approx 15.588$

8. B = 78°, b = 13, c = 14

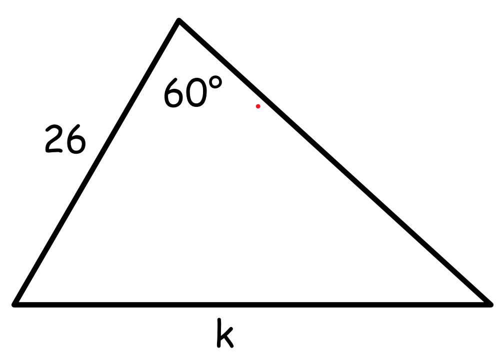
NO SOLUTION

9. Consider the diagram shown below. For what values of k would you have more than one possible triangle satisfying the information shown?

$\boxed{k \in (13\sqrt{3},\ 26)}$

or

$\boxed{13\sqrt{3} < k < 26}$



$(13\sqrt{3} \approx 22.517)$