AP Calculus AB
Review Chapter 6

Name Key

Estimating with Finite sums (6.1 and 6.5)

- Left Reimann Sum LRAM
- Right Reimann Sum RRAM
- Midpoint Reimann Sum MRAM
- Trapezoidal Rule

1. Given $\int_0^8 f'(x)dx$ and $f'(x)$ is a continuous positive function that is strictly increasing and concave up, determine if the following would be an under or overestimate with the given conditions. Justify your answer.

a. Using LRAM to estimate $\int_0^8 f'(x)dx$

underestimate b/c $f'(x)$ is increasing

b. Using RRAM to estimate $\int_0^8 f'(x)dx$

overestimate b/c $f'(x)$ is increasing

c. Using the Trapezoidal Rule to estimate $\int_0^8 f'(x)dx$

overestimate b/c f' is CCU.

2. State in your own words what the integral represents.

a. $\int_0^8 f'(x)dx$

Value of the integral from 0 to 8

b. $\frac{1}{8}\int_0^8 f'(x)dx$

average value of $f'(x)$ from 0 to 8

c. $\int_0^8 |f'(x)|dx$

the total area between the curve and x-axis from 0 - 8

d. Now assume $f'(x)$ is the rate of snow removal inches per hour over the time interval in [0,8].
$\int_2^6 f'(x)dx$ The total amount of snow in inches from 2-6 hours

3. Write the following integrals using the limit definition.

a. $\int_1^5 sinx\, dx$

$b-a=5-1=4$

$\frac{b-a}{n}=\frac{4}{n}$

$a=1$

$c_k=1+\frac{4}{n}$

$$\boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(1+\frac{4k}{n}\right)\cdot\frac{4}{n}}$$

b. $\int_0^7 x^4\, dx$

$\frac{b-a}{n}=\frac{7}{n}$

$c_k=0+\frac{7}{n}$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{7k}{n}\right)^4\cdot\frac{7}{n}$$

4. Write the following limit as a definite integral

a. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{5}{n}\ln\left(2+\frac{5k}{n}\right)$

$\frac{b-a}{n}=\frac{5}{n}$

$b-a=5$

$a=2 \quad b=7$

$$\int_2^7 \ln x\, dx$$

b. $\lim_{n\to\infty}\frac{\pi}{3n}\sum_{k=1}^{n}\tan\left(\frac{k\pi}{3n}\right)$

$\frac{b-a}{n}=\frac{\pi/3}{n}$

$b-a=\frac{\pi}{3}$

$a=0$

$b=\pi/3$

$$\int_0^{\pi/3}\tan x\, dx$$

5. Select values of $p'(t)$ are given in the table below.

t minutes	10	20	30	40
$p'(t)$ feet per minute	4	-2	0	3

Approximate $\int_{10}^{40} p'(t)dt$ using a left Riemann sum with three subintervals.

$$\int_{10}^{40} p'(t)\, dt \approx 10[4+(-2)+0] = 20 \text{ feet}$$

6. Select values of $T(t)$ are given in the table below.

hours	0	2	3	7	10
$T(t)$ degrees per hour	3	5	2	0	-3

Approximate $\int_0^{10} T(t)dt$ using a right Riemann sum with four subintervals. Include units in your approximation.

$$\int_0^{10} T(t)dt \approx 5 \cdot 2 + 2 \cdot 1 + 0 \cdot 4 + (-3) \cdot 3$$

$$= 10 + 2 + 0 - 9 = 3 \text{ degrees}$$

7. Select values of $r'(t)$ are given in the table below.

t seconds	2	5	7	11
$r'(t)$ meters per second	10	6	2	8

Approximate $\int_2^{11} r'(t)dt$ using a trapezoidal sum with three subintervals. Include units in your approximation.

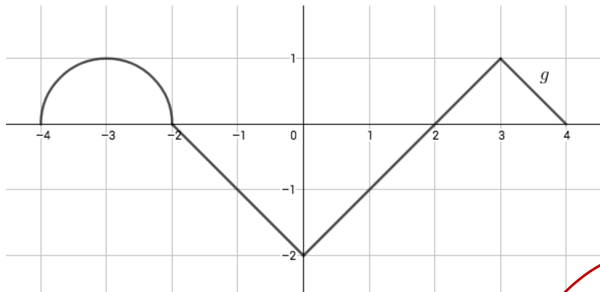
$$\int_2^{11} r'(t)dt \approx \frac{1}{2}[(10 + 6) \cdot 3 + (6 + 2) \cdot 2 + (2 + 8) \cdot 4]$$

$$= \frac{1}{2}[16 \cdot 3 + 8 \cdot 2 + 10 \cdot 4] = \frac{1}{2}[48 + 16 + 40]$$

$$= \frac{1}{2} \cdot 104 = 52 \text{ meters}$$

Using geometry to integrate and FTC (6.2 and 6.4):

8. The graph of g below is made up of line segments and a semi-circle.



+ − +

−4 −2 2 4

x	$f(x)$
−4	$-\pi/2$
−2	0
2	−4
4	−3

$f(x) = \int_{-2}^{x} g(t)dt$ over the interval $[-4,4]$

Find each of the following.

a.) $f(2) = \int_{-2}^{2} g(t)dt = -4$

b.) $f(-4) = \int_{-2}^{-4} g(t)dt - \int_{-4}^{-2} g(t)dt = -\frac{\pi}{2}$

c.) $f(4) = \int_{-2}^{4} g(t)dt = -4 + 1 = -3$

d. $f'(-1) =$ -1

e. Write the equation to the tangent line to $f(x)$ at x=0

$f(0) = \int_{-2}^{0} g(t)dt = -\frac{1}{2}(2)(2) = -2$ p.o.t $(0,-2)$ $f'(0) = g(0) = -2$

$\boxed{y+2 = -2(x-0)}$

f. Does $f(x)$ have a relative max, relative min, or neither at x= -2? Justify your answer.

f(x) has a relative max @ x= -2 b/c f'(x) = g(x) goes from + to − @ x = -2.

g. Find the absolute max on $f(x)$.

see work by graph f(x) has an abs max of 0 @ x=-2

h. Find all of the points of inflections on $f(x)$ and justify your answer.

f(x) has p.o.i.s @ x = -3, 0, 3 b/c f''(x) = g'(x) changes signs.

The following problems will cover

- FTC part 1 (the derivative of an integral)
- FTC part 2 (integrating using antiderivatives)
- Properties of integration
- Average value
- Mean value theorem for integrals
- Total area

9. Given that $\int_{-1}^{1} f(x)dx = 0$ and $\int_{0}^{1} f(x)dx = 5$, find

a) $\int_{-1}^{0} f(x)dx$

$\int_{-1}^{1} f(x)dx - \int_{1}^{0} f(x)\,dx$

$0 - 5 = \boxed{-5}$

b) $\int_{-1}^{1} (f(x)+2)dx$

$\int_{-1}^{1} f(x)\,dx + \int_{1}^{1} 2\,dx$

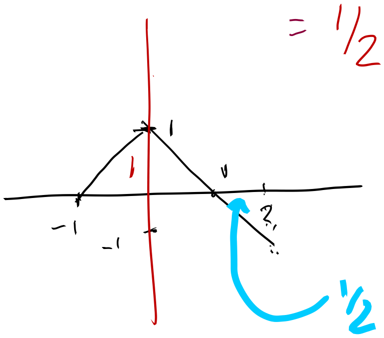
$0 + 2(1-(-1)) = \boxed{4}$

c) $\int_{1}^{0} 2f(x)dx$ $\quad -2\int_{0}^{1} f(x)dx = \boxed{-10}$

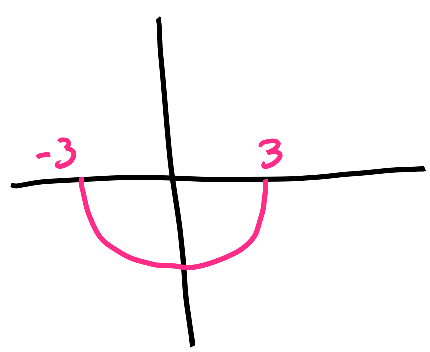
10. Use geometry to evaluate each integral.

a. $\int_{-1}^{2} (1-|x|)\,dx$ $\quad 1 - \frac{1}{2}$

$= \frac{1}{2}$



b. $\int_{-3}^{3} -\sqrt{9-x^2}\,dx$ $\quad = -\frac{1}{2}\pi(3)^2 = \boxed{-\frac{9}{2}\pi}$



11. Evaluate the given integrals.

a. $\int_{-4}^{2} (x^3 - 5)dx$

$\frac{x^4}{4} - 5x \Big|_{-4}^{2} = -90$

b. $\int_{1}^{3} \frac{3-x^2}{x} dx = \int_1^3 \frac{3}{x} - \frac{x^2}{x} dx$

$= \int_1^3 \left(\frac{3}{x} - x\right) dx = 3\ln x - \frac{x^2}{2}\Big|_1^3$

$\left(3\ln 3 - \frac{9}{2}\right) - \left(3\ln 1 - \frac{1}{2}\right)$

$\boxed{3\ln 3 - 4}$

c. $\int_{\frac{\pi}{4}}^{0} \sec^2 x\, dx = -\int_0^{\pi/4} \sec^2 x\, dx$

$-\tan x \Big|_0^{\pi/4} = -1-(0)$

$= \boxed{-1}$

d. $\int_{0}^{1} 2^x dx$

$= \frac{2^x}{\ln 2}\Big|_0^1 = \frac{2}{\ln 2} - \frac{2^0}{\ln 2}$

$= \frac{2}{\ln 2} - \frac{1}{\ln 2}$

$= \boxed{\frac{1}{\ln 2}}$

e. $\int_{0}^{1} \frac{1}{1+x^2} dx$

$= \tan^{-1} x \Big|_0^1 = \tan^{-1} 1 - \tan^{-1} 0$

$\pi/4 - 0 = \boxed{\pi/4}$

f. $\int_{2}^{-3} 5\, dx$

$= -\int_{-3}^{2} 5\, dx$

$= -5x \Big|_{-3}^{2} = -5(2) - (-5(-3))$

$= -10 - 15$

$= \boxed{-25}$

g. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc x \cot x dx$

$-\csc x \Big|_{\pi/4}^{\pi/2}$

$-\csc \pi/2 - (-\csc \pi/4) = -1 + \sqrt{2}$

h. $\int_{1}^{2} \frac{1}{2x} dx$

$\frac{1}{2}\int_1^2 \frac{1}{x} dx$

$\frac{1}{2}\ln x \Big|_1^2 = \frac{1}{2}\ln 2 - \frac{1}{2}\ln 1$

$\boxed{= \frac{1}{2}\ln 2}$

i. $\int_{1}^{2} \frac{3}{x^2} dx$

$\int_1^2 3x^{-2} dx = -3x^{-1}\Big|_1^2 = \frac{-3}{2} + 3$

$= \boxed{3/2}$

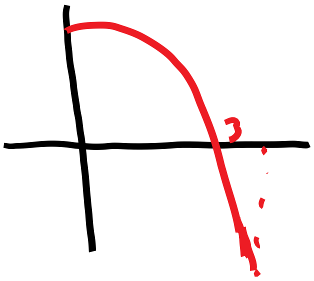
j. $\int_{1}^{32} x^{-\frac{6}{5}} dx$

$-5x^{-1/5}\Big|_1^{32}$

$-5(32)^{-1/5} - (-5)$

$-5(1/2) + 5 = -\frac{5}{2} + 5 = \boxed{5/2}$

12. Find the **total area** between the curve $y=9-x^2$ and the x-axis when $0 \le x \le 5$.



$$\int_0^3 (9-x^2)\,dx = 9x-\frac{x^3}{3}\Big|_0^3 = 18$$

$$\int_3^5 (9-x^2)\,dx = 9x-\frac{x^3}{3}\Big|_3^5 = |-14.667|$$

$$18+14.667 = \boxed{32.667}$$

13. Find dy/dx if $y=\int_{3x}^{x^2} \sqrt{1+t+t^2}\,dt$

$$\frac{dy}{dx} = \left(\sqrt{1+x^2+x^4}\right)(2x) - \left(\sqrt{1+3x+9x^2}\right)(3)$$

14. A particle is moving along the x-axis with velocity $v(t) = 2t - 5$. At $t = 0$ the particle is at $x = 3$.

(t, x) starting @ $(0,3)$

a. Find the position of the particle at $t = 7$.

$$\int_0^7 v(t)\,dt = x(7) - x(0) \qquad x(0) + \int_0^7 v(t)\,dt = x(7)$$

$$3 + \left[t^2 - 5t\right]_0^7 = 3 + 49 - 35 - 0 \qquad \boxed{x(7) = 17}$$

b. Find the particles average velocity from t = 0 to t = 7.

$$\frac{1}{7}\int_0^7 (2t-5)\,dt = 2$$

c. Find the time when the particle reaches the average velocity.

$$2t - 5 = 2 \qquad 2t = 7 \qquad \boxed{t = 7/2}$$