AP Calculus AB
7.4 Day 2 HW

Homework

1. Write a differential equation for the statement.

a. The rate of change of y with respect to x is proportional to y^2.

$$\frac{dy}{dx} = ky^2$$

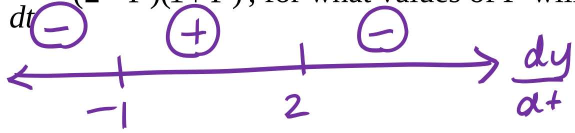
b. The rate of change of p with respect to r is proportional to the product of p and the difference between r and 4.

$$\frac{dp}{dr} = kp(r-4)$$

c. The rate of change of b with respect to t is inversely proportional to the square root of b.

$$\frac{db}{dt} = \frac{k}{\sqrt{b}}$$

2. If $\frac{dy}{dt} = (2-P)(1+P)$, for what values of P will $\frac{dy}{dt} < 0$?



$P < -1$ OR $P > 2$

3. Four months after it stops advertising, a manufacturing company notices that its sales have dropped from 100,000 units per month to 80,000 units per month. The sales follow the Law of Exponential Change. What will the sales be after another 2 months?

$80{,}000 = 100{,}000e^{k \cdot 4}$

$k = -0.0558$

$100{,}000e^{-0.0558 \cdot 6}$

$\approx 71{,}554$ units

The rate of decomposition of radioactive plutonium is proportional to the amount present at any time.

a. Write the differential equation for the statement.

$$\frac{dy}{dt} = ky$$

b. Using separation of variables, show that $y = y_0 e^{kt}$ is the solution to $\frac{dy}{dt} = ky$.

$$\int \frac{dy}{y} = \int k\, dt \qquad y = e^{kt+C} = e^{kt} \cdot e^{C} = y_0 e^{kt}$$

$$\ln|y| = kt + C$$

c. Ten grams of plutonium were released in a nuclear accident. If the half-life of plutonium is 24,100 years, how long will it take for the 10 grams to decay to 1 gram?

$$5 = 10e^{k \cdot 24100} \qquad 1 = 10e^{-.000029t}$$

$$k = -0.000029 \qquad t \approx 80{,}058 \text{ years}$$