90 Minutes—Scientific Calculator

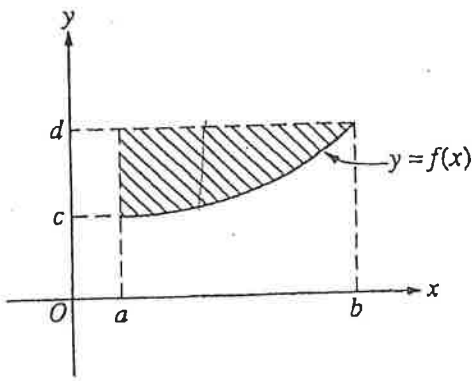
Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$f'(x) = \frac{3}{2}x^{1/2}$

1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) = \frac{3}{2}(4)^{1/2}$

(A) -6 (B) -3 (C) 3 (D) 6 (E) 8



$\int_a^b (d - f(x))\,dx$

2. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)dy$ (B) $\int_a^b (d - f(x))dx$ (C) $f'(b) - f'(a)$

(D) $(b-a)[f(b) - f(a)]$ (E) $(d-c)[f(b) - f(a)]$

3. $\lim_{n\to\infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is graphed | placed large x values in table

(A) -5 (B) -2 (C) 1 (D) 3 (E) nonexistent

4. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{x^2+y}{x+2y^2}$

(B) $-\frac{x^2+y}{x+y^2}$

(C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$

(E) $\frac{-x^2}{1+2y^2}$

$3x^2 + 3xy' + 3y + 6y^2y' = 0$

$3xy' + 6y^2y' = -3x^2 - 3y$

$y'(3x + 6y^2) = -3x^2 - 3y$

$y' = \frac{-3x^2 - 3y}{3x + 6y^2} = \frac{-3(x^2+y)}{3(x+2y^2)}$

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-4}{x+2}$ when $x \neq -2$, then $f(-2) =$

(A) -4 (B) -2 (C) -1 (D) 0 (E) 2

6. The area of the region enclosed by the curve $y = \frac{1}{(x-1)}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

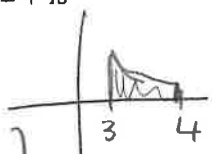
graph

use calculator to check answer

$\int_3^4 \frac{1}{x-1}\,dx$

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

$\ln(x-1)\Big|_3^4 = \ln(4-1) - \ln(3-1)$ $\ln 3 - \ln 2$ $\ln(3/2)$



7. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1,5)$ is

point

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

$y' = \frac{(3x-2)(2) - (2x+3)(3)}{(3x-2)^2}$

$y' = \frac{(3-2)(2) - (2+3)(3)}{(3-2)^2} = \frac{2-15}{1} = -13$

$y - 5 = -13(x-1)$

$y - 5 = -13x + 13$

$y - 5 = -13x + 13$

$13x + y = 18$

8. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$ $\sec^2 x - (-\csc^2 x)$

(A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

9. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$ $3|x|^2 - 1$

(A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$ $3x^2 - 1$

10. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$ $f'(x) = \sin x\,(2(x-1)\cdot 1) + (x-1)^2(\cos x)$

$\sin 0\,(2(0-1) + (0-1)^2 \cdot \cos 0 = 1$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

11. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

$v(3) = 25$ $s(1) = 10$ $v(t) = 3t^2 - 2t + 4$

(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

(C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

$v(t) = 3t^2 - 2t + C$

$25 = 3(3)^2 - 2(3) + C$

$25 = 27 - 6 + C$

$4 = C$

$s(t) = t^3 - t^2 + 4t + C$

$10 = 1 - 1 + 4 + C$

$C = 6$

$\boxed{s(t) = t^3 - t^2 + 4t + 6}$

12. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

I. $\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$

II. $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

III. $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

13. The fundamental period of $2\cos(3x)$ is $\frac{2\pi}{3}$

(A) $\frac{2\pi}{3}$ (B) 2π (C) 6π (D) 2 (E) 3

14. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1}+C$

(B) $\frac{3}{2}\sqrt{x^3+1}+C$

(C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$

(E) $\ln(x^3+1)+C$

$u = x^3+1$

$du = 3x^2\,dx$

$\int \frac{1}{u^{1/2}}\,du = \int u^{-1/2}\,du$

$2u^{1/2}+C$

$2(x^3+1)^{1/2}+C$

15. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

graph or calculus

$f'(x) = (x-3)^2 + (x-2)(2(x-3)(1))$

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

16. The slope of the line normal to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

perpendicular to tangent

$y' = \frac{2}{\cancel{\sec x}} \cdot \cancel{\sec x}\tan x$

$f'(x) = (x-3)^2 + 2(x-2)(x-3)$

$= (x-3)[(x-3)+2(x-2)]$ $x-3+2x-4$

$= (x-3)(3x-7)$

$x=3 \quad x = 7/3$

$+ \quad - \quad +$

$7/3 \quad 3$